Petro: a multi-agent model of historical warfare

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***Abstract*—Numerous mathematical and computer models were used to simulate a situation on a battlefield. We introduce *Petro*, a new agent-based model of historical warfare, designed to maintain a structure of command between a regiment and its soldiers. It can model various tactics of different kinds of military forces and is simple enough to build massive simulations.**

We use the model to simulate the warfare of 17th century Europe. Paper presents modeling of various types of military forces – Tatar riders, line infantry, hussars and reiters. We show how general patterns of combat are the result of basic rules directing the behavior of soldier and regiment agents.

Finally, we use modeled troops to simulate the Battle of Kokenhausen (1601), waged between armies of the Polish-Lithuanian Commonwealth and the Kingdom of Sweden. After producing results very close to historical data, alternative scenario of the battle is considered.

***Index Terms*—multiagent systems; warfare simulation; experimentation**

I. Introduction

For thousands of years war was one of the most important phenomena affecting human culture. Numerous mathematical and computer models were created in the hope of finding a way to predict the result of a struggle on a battlefield.

In 1916, during World War I, Frederick Lanchester presented a way of simulating armed combat through a system of differential equations [1], [2]. Lanchester Equations worked well for static trench warfare of World War I. Later, they were also adapted for maneuver warfare of World War II, including the Ardennes campaign [3] and the Battle of Kursk [4].

Washburn and Kress presented a number of stochastic models, concerning many different spheres of military combat, including minefield clearance and routing of Unmanned Aerial Vehicles [5]. Finnish system Sandis is another example of usage of mathematical model for simulating warfare [6]. It is based on the probability calculus, fault logic analysis and Markov chains. Number of soldiers of every regiment is represented by a probability distribution.

Multi-agent systems were also used as a warfare simulation tool. Ilachinski described benefits of simulating warfare with agent-based models, demonstrating them with multi-agent systems ISAAC [7] and EINSTein [8]. Cioppa et al. used agent-based simulations to study impact of degraded communications in the U.S. Army's Future Force, coordinate actions of Unmanned Surface Vehicles and choose the standard Army squad size [9]. Multi-agent systems can also be used to efficiently control Unmanned Aerial Vehicles [10], [11].

Multi-agent systems proved to be a useful tool for analyzing historical battles and conflicts. Trautteur and Virgilio [12] managed to create a model of the Battle of Trafalgar, fought by the British Royal Navy against the combined fleets of the French Navy and Spanish Navy during the Napoleonic Wars. Scogings and Hawick [13] presented a model of the Battle of Isandlwana, making use of over 20,000 agents and analyzing alternative scenario of the engagement.

Unfortunately, in most of the previous agent-based models of armed combat, agents act only on the basis of an inner set of rules [7], [12], [13]. In the real world all armies rely on the chain of command. Ilachinski proposed a multi-level agent hierarchy for EINSTein [8]. However, it would be hard to use for large-scale simulations, due to its complexity. In this article, we present a new way of modeling situation on a battlefield with the use of *Petro*, our multi-agent system. The goal of our work was to create model simple enough to allow simulations of battles with thousands of soldiers. At the same time, it had to be powerful enough to recreate tactics of a number of types of military forces. We introduce a two-layered agent hierarchy, that allows an agent to fight as a part of a regiment, cooperating with his brothers-in-arms to achieve better performance.

Capabilities of the model are presented on the example of European warfare of 17th century, historical period yet to be analyzed by computer scientists. We managed to recreate, with the use of simple rules, several distinctive styles of combat of different types of military forces. These tactics are used to simulate a major encounter from 17th century, the Battle of Kokenhausen (1601), with over 8,000 agents representing soldiers. Results achieved during simulations are very close to historical data in respect of the course of the battle, as well as losses of both sides. Analysis of an alternative scenario of the battle shows the cause of the Swedish defeat.

The general construction of the model and the flow of information between agents is presented in Section II. Section III describes of European warfare of 17th century and a few examples of specific types of modeled military forces. Section IV presents the simulation of the Battle of Kokenhausen. An alternative scenario of the battle is considered in Section V. A summary and possible ideas for future work are discussed in Section VI.

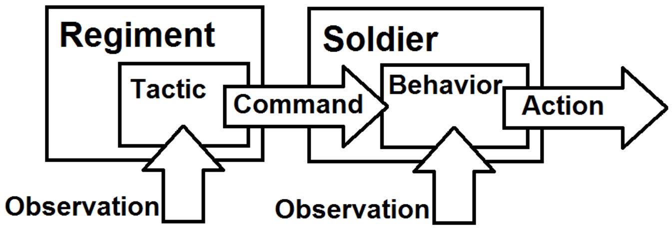

Fig. 1. Scheme of the flow of information in the model.

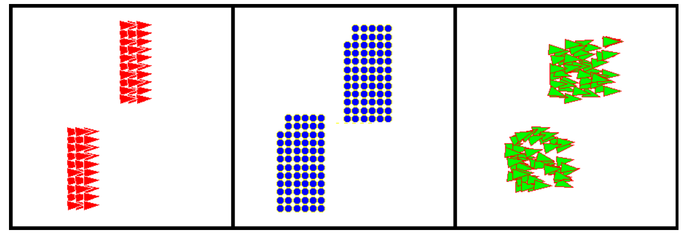

Fig. 2. Examples of formations.

II. The model

This section provides information about general construction of the model and a way of conducting combat. It describes agents representing a soldier and a regiment, as well as the flow of information between them.

A. *Model of conducting combat*

Two armies of agents take part in the combat. There are two types of agents: those representing soldiers of different types of military forces and those representing regiments. Every soldier is a member of exactly one regiment.

Previous historical agent-based simulation did not contain agent hierarchy [13], [12], thus making it difficult to model more advanced tactics. On the other hand, EINSTein included multi-level, highly complicated structure of connections between agents [8]. While appropriate for modern warfare, it does not fit well with the reality of historical battlefields, where communication between a general and his soldiers was difficult, if not impossible. What is more, maintaining such a vast structure makes preparing simulations with thousands of agent a hard task. We propose a system, that allows to conveniently model various tactics of different kinds of military forces and, at the same time, is simple enough to build massive simulations.

Combat takes place on a rectangular field of preset size. An agent that leaves the field of battle is considered routed and is removed from combat. Battle is conducted in consecutive rounds. Figure 1 presents the general flow of the information during single round.

At the beginning of each round, every regiment chooses a command for its soldiers. The regiment does not have knowledge about commands chosen by other regiments, neither hostile nor allied. Examples of commands are *attack certain enemy regiment*, *move to certain position across the battlefield* or *regroup to formation*. Section II-B describes the process of selecting a command by a regiment agent.

Subsequently, every soldier chooses his action. The soldier does not have knowledge about actions chosen by other soldiers, neither hostile nor allied. He knows the command chosen by his regiment, but not any other commands. Examples of actions are *attack certain enemy soldier*, *move across the battlefield* or *reload the weapon*. Section II-C describes the process of selecting an action by a soldier agent.

The purpose of this model is to be suitable for many different historical periods. To ensure that, we do not put any restrictions on commands and actions that can be used. Those used in modeling of 17th century warfare are described in Section III-B. Other historical periods would demand different types of commands and actions.

Finally, at the end of each round, the system gathers all actions of soldiers and executes them in random order. Battle is over when there is only one army with alive soldiers able to fight.

B. *Regiment agent*

Every regiment is a part of exactly one army, it also has assigned *tactic* and *formation*. While it can be convenient to think about regiment agent as a commander of a tactical unit, it is not represented by a single soldier taking part in the battle, unlike in Ilachinsky's EINSTein [8]. It simplifies the model and allows to do not consider effects of the commander's death.

Tactic represents regiment's behavior on the battlefield. It is an algorithm that selects a command for regiment's soldiers. The decision is based on observation of the environment and knowledge about the state of the regiment (number and location of soldiers etc.). The command is one of the main factors during the choice of an action by soldier agents. Tactic answers for a coordination of soldiers of a single regiment.

Introduction of a tactic allows to model cooperation more conveniently, while most systems force it to be done on a level of a single soldier [7], [13]. Unlike in EINSTein [8], regiments of a single army do not communicate with one supreme commander or with each other, according to historical realities of previous centuries.

Formation is a representation of arrangement of soldiers on the battlefield. One of possible commands to choose by a tactic is *regroup to formation*, that means for soldiers to take positions designated by the formation. Figure 2 presents examples of different formations.

In the simulation of the Battle of Isandlwana formation of regiments was determined either by initial positions on the battlefield for stationary British troops or by rules of behavior of a single soldier for Zulu warriors [13]. We introduce an explicit representation of the formation for purpose of modeling more advanced maneuvers, such as the countermarch or the caracole of reiters.

Agent hierarchy does not contain a single commander of an entire army, because in most historical periods communication during battle was very limited. However, modeling of more complicated encounters may require representation of strategy prepared by the commander. The most appropriate way to do it would be to prepare a simple script - representation of guidelines given by an army's commander to a regiment's commanding officer. Using that script as a part of a regiment's tactic would allow soldiers to act according to a general plan of battle.

C. Soldier agent

Every soldier agent is a part of exactly one regiment. A soldier is characterized by endurance and speed, as well as range, power and accuracy of the attack. When soldier agent chooses an attack action, a number is generated using uniform distribution on the interval [0,1]. If the number is lower than soldier's accuracy, the attack is successful. A successful attack is reducing endurance of the target by the value of attacker's power. When endurance of a soldier agent drops below zero, he dies.

Lanchester used differential equations for modeling the warfare of World War I [1], [2], where single battle could be fought by hundreds of thousands of soldiers. However, for most historical periods, actions of every single soldier were important, hence modeling all combatants as separate agents.

In most modern simulations direct hit results in death [7]. While it is true for new, deadly weapons, it does not have to be in historical scenarios. While single sword swing could easily kill simple infantryman, it would not do much against heavily armored knight. Thus introducing the concept of endurance points to model soldiers with different kinds of armor.

Every soldier has an assigned *behavior*. *Behavior* represents a way the agent acts during battle. Behavior selects action to be performed by the agent during current round, basing on observation of the environment and the command chosen by the agent's regiment. The type of behavior is usually determined by the type of armed forces the agent belongs to.

We do not put any explicit limitations on the class of an algorithm used as behavior of an agent. Behaviors described in Section III-C are series of simple rules, similarly to the work of Scogings and Hawick [13]. This way is efficient in terms of computation cost and helps building simulations with many agents.

III. Modeled troops

This section briefly describes the European warfare of 17th century and presents a few types of military forces modeled using scheme described in Section II.

A. The warfare of 17th century

Technological and tactical solutions from battlefields of 17th century Europe were a mix of those from much earlier and much later historical periods.

Territories of Eastern Europe were not very urbanized and consisted mainly of vast areas of steppe. As a result of the necessity of conducting swift military operations over huge distances, cavalry was still the main force on eastern battlefields. Armies of such countries as the Polish-Lithuanian Commonwealth, Tsardom of Russia and Crimean Khanate consisted mainly of horsemen, often fighting with melee weapons or bows. Although technologies changed, main concepts of the eastern art of war still had their roots in early Renaissance or even the Middle Ages.

At the same time, Western Europe was largely urbanized and had dense network of roads. Western warfare evolved in the direction of using large groups of infantry and common usage of artillery. Armies of the Holy Roman Empire, Kingdom of Sweden and Kingdom of France consisted mainly of infantry armed with firearms. Cavalry acted as scouts and support. These trends finally led to the warfare of the period of the Napoleonic Wars [14].

This mix of different approaches to warfare makes a 17th century European battlefield an interesting subject of modeling. Furthermore, many battles from this historic period are poorly documented and a well-calibrated model could fill gaps in historical knowledge.

B. Commands and actions

This section enlists types of regiments' commands and soldiers' actions used for the simulation the warfare of 17th century.

1) Commands: Regiments' commands used during simulation:

- *Attack certain enemy regiment*
- *Move to a certain position across battlefield*
- *Regroup to a certain formation*
- *Do nothing*

How different commands are executed depends on the type of military forces the regiment models.

2) Actions: Soldiers' actions used during simulation:

- *Attack certain enemy soldier*
- *Move to a certain position across battlefield*
- *Attack certain enemy soldier and move*
- *Attack certain coordinates*
- *Reload weapon*
- *Do nothing*

Some actions are limited a certain types of military forces. Only cavalry units can use *Attack certain enemy soldier and move* action. Only artillery units can use *Attack certain coordinates* action.

C. Types of military forces

This section contains information about modeling of various types of military forces from 17th century, according to schema from Section II.

1) Tatar riders: Warriors of Crimean Khanate, descendants of the Mongolian Golden Horde, usually fought on horseback. They primarily used short reflex bows and sabres. If possible, they avoided melee fight, willingly using hit-and-run tactics. During simulated escape in a crescent-shaped formation, they continued shooting at pursuers.

The tactic of a regiment of Tatar riders is simple, it gives a command to attack the nearest enemy regiment. The most important decisions are made at the level of a soldier.

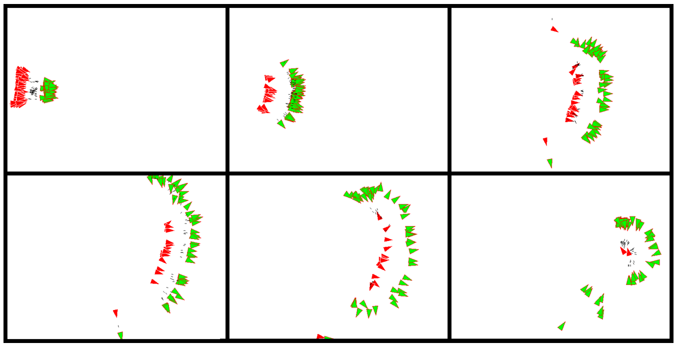

Fig. 3. Example of encounter between Tatar riders (green forces) and melee cavalry (red forces).

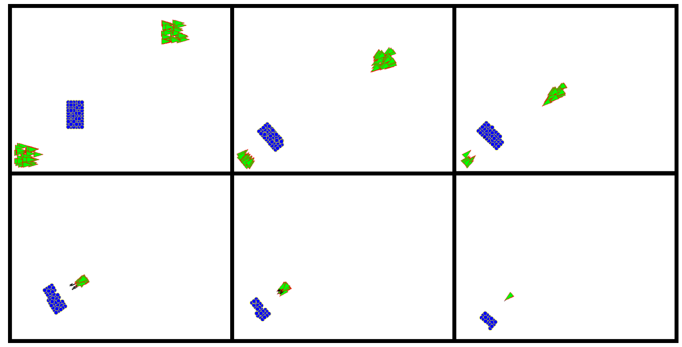

Fig. 4. Example of encounter between line infantry (blue forces) and Tatar riders (green forces).

Every Tatar rider has a distance, that he considers safe. If the closest enemy is closer than the safe distance, rider flees from the enemy with maximal speed. Otherwise, Tatar rider seeks for the closest enemy belonging to a regiment pointed in the command. Then he moves toward closest enemy until he will be in range. Whenever possible, Tatar rider shoots toward an enemy. As a cavalry unit, riders can both move and attack as a part of a single action.

Figure 3 shows how rules of behavior of a single agent affect self-organization of the regiment. Tatar riders encounter generic melee cavalry, that simply tries to attack the nearest enemy. Riders scatter under pressure of the enemy, assume crescent-shaped formation and continue firing. This very organized behavior is achieved without any direct coordination between members of a regiment. At the end of the battle, they turn to attack, finishing off weakened foes.

2) Line infantry: Line infantry was the most common type of armed force in western Europe. Regiments of line infantry consisted of soldiers armed with firearms – arquebuses and muskets. These types of weapons were characterized by long reload time. To maximize the effect of shooting, whole regiment fired volleys – shot at the same time.

Unlike Tatar riders, the most important decisions for line infantry are made at the level of regiment's tactic. If at least 90 percent of soldiers are ready to fire (i.e. with reloaded weapon and an enemy in range), regiment gives command to attack the target. Otherwise, regiment gives a command to regroup to a formation, facing the nearest enemy regiment. When regiment does not attack for too many turns, tactic gives command to move toward the nearest enemy regiment.

Behavior of soldier agent is not complicated, it simply follows commands, attacking and moving only when commanded. Whenever possible, line infantry behavior chooses to reload weapon.

Figure 4 shows an example of encounter between a regiment of line infantry and two regiments of Tatar riders. Line infantry fights in close formation, trying to face the nearest enemy and eliminate foes with volley fire. When first group of Tatar riders is defeated, infantry regiment turns formation to face remaining enemies. Placing decision-making power at the level of regiment's tactic results in a more orderly manner of unit self-organization.

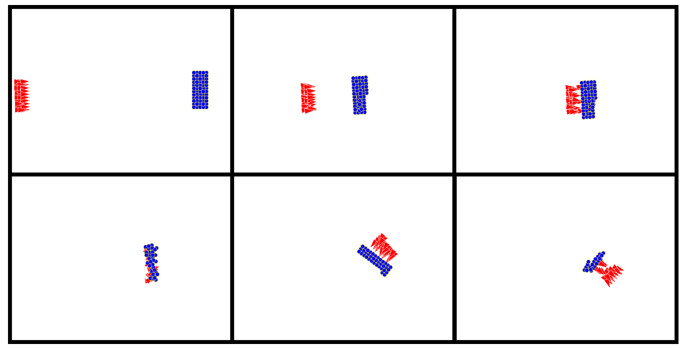

Fig. 5. Example of encounter between hussars (red forces) and line infantry (blue forces).

3) Hussars: Hussars were elite heavy cavalry in armies of the Polish-Lithuanian Commonwealth. Hussar rider started a battle wielding a lance. When it broke after a successful hit, hussar grabbed an estoc or a sabre. Their primary battle maneuver was impetuous charge. If the enemy wasn't destroyed after first charge, hussars regrouped (usually behind enemy lines) and attacked again [15].

The tactic of a hussar regiment chooses nearest enemy regiment and orders a charge. During the clash, tactic keeps count of number of rounds. When melee combat lasts longer than a certain number of rounds, tactic gives order to regroup behind enemy regiment. After regroup maneuver is completed, tactic orders another attack.

To model the ability of impetuous charge, hussar soldier can move with doubled speed and attack with greater damage every couple of rounds. Hussar behavior tries to make use of this ability whenever possible and act according to regiment's commands at the same time. To model the use of Hungarian lances, first successful strike of a hussar rider during the battle has doubled damage.

Figure 5 shows an example of encounter between a group of hussar riders and a regiment of line infantry. Hussars attack the enemy, making use of the ability of impetuous charge. When enemy survives the first attack, hussars regroup behind enemy lines and charge again. Hussars are an example of military forces, where both regiment's tactic and soldier's behavior play

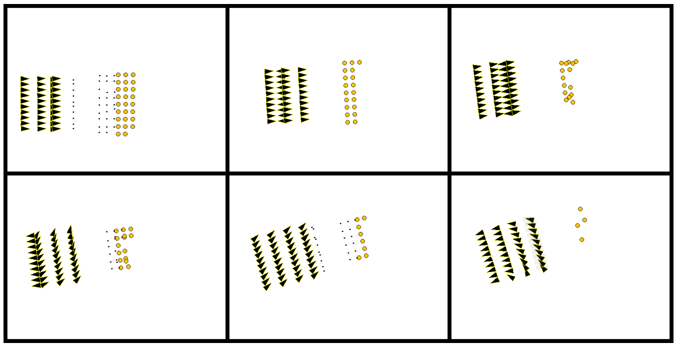

Fig. 6. Example of encounter between reiters (black forces) and line infantry (yellow forces).

an important role in modeling a style of fighting.

4) Reiters: Reiters were a type of cavalry armed with firearms. They were using a tactic called caracole, designed to face line infantry. Riders were fighting in a rectangular formation. Soldiers from the first row fired their weapons, then moved to the back of the formation to reload, with second row taking their place. Then second row fired, moved to the back, with third row taking their place etc. This allowed to maintain a steady rate of fire.

Reiters tactic makes intense use of formation. When close to the enemy, tactic gives command to attack. In the next round tactic modifies the formation, so that first row would move to the back and gives an order to regroup. After regrouping, another attack command is given. Tactic is trying to maintain constant distance to the enemy. As a result, regiment pursues fleeing foes and draws back under pressure.

Reiter behavior is simply following orders received from the tactic. Soldier attacks the enemy only when he is in the first row of the formation and reloads weapon whenever possible.

Figure 6 shows an example of encounter between a group of reiters and a regiments of line infantry. As a result of caracole tactic, damage is spread more evenly among members of the regiment, allowing reiters to maintain high numbers and finally defeat the enemy.

IV. The battle of Kokenhausen

This section contains information about using model described in Section II to simulate the Battle of Kokenhausen (1601). It describes the historical battle, building the model and achieved results.

A. The battle

At the beginning of 17th century the Polish-Lithuanian Commonwealth and the Kingdom of Sweden waged war over the territory of Livonia. In May of 1601 Polish forces under command of hetman Krzysztof “the Thunderbolt” Radziwiłł laid siege to the city of Kokenhausen controlled by the Swedes. Swedish army, under the command of Carl Gyllenhielm, arrived in Livonia and decided to break the Polish siege on the morning of 23 June. Polish commander decided to face them in the field.

TABLE I
Calibrated parameters of the simulation of the Battle of Kokenhausen.

	Hussar	Reiter	Infantry	Gunner	Cannon
Endurance	200	90	40	40	-
Speed (dist/round)	8	6	4	1	4
Accuracy	0.9	0.65	0.6	0.5	1
Reload (rounds)	0	10	10	0	400
Range (dist)	2	80	90	1	100
Damage	15	10	12	1	150

Swedish forces consisted of about 4000 cavalrymen (mainly reiters) evenly divided among two wings, 900 infantrymen and 17 cannons positioned in the middle. Hetman Radziwiłł had at his disposal about 2900 riders (mainly hussars) divided into three groups, 300 infantrymen and 9 cannons.

The battle was mainly a confrontation of riders. Polish riders first broke the left wing of Swedish cavalry. The Swedes counterattacked on the right flank and were initially successful, but finally group of Polish hussars under hetman Radziwiłł defeated them. Swedish infantry, left alone after the escape of reiters, was destroyed almost to the last man.

The Polish forces lost about 100 – 200 men, while the Swedes lost about 2000, including all the infantry. Swedish garrison of Kokenhausen surrendered after the battle and the city fell into Polish hands [16].

We chose the Battle of Kokenhausen as the subject of modeling because it is well documented, unlike many other encounters from 17th century. Thanks to documents prepared by hetman Radziwiłł before battle, we know exact size and setup of Polish forces, including commanders of all subgroups [16].

B. Building and calibrating the model

We built the simulation of the Battle of Kokenhausen with the use of the model described in Section II. Reiters, hussars and infantry acted according to tactics and behaviors described in Section III-C.

Artillery regiments consisted of cannons and gunners. Reload speed of the cannon was proportional to the number of alive gunners. To simulate the lack of accuracy of 17th century artillery, cannonball hit the ground in random place around intended target, dealing damage in small radius.

We introduced the simple mechanism of panic, because during massive land confrontation soldiers often run away from the battle when overwhelmed by the enemy. Regiment entered panic mode when it lost more than half of its soldiers and was in direct vicinity of the enemy. Panicked soldiers tried to reach borders of the battlefield and were removed from battle if successful.

Initial setup and number of soldiers of all regiments were selected according to historical knowledge [16].

Some characteristics of the model, such as numerical parameters, required calibration. We built the simulation using bottom-up design. We calibrated the model on the smaller, less complicated confrontations, such as fight of two regiments of cavalry on one of the flanks. Then we used so determined

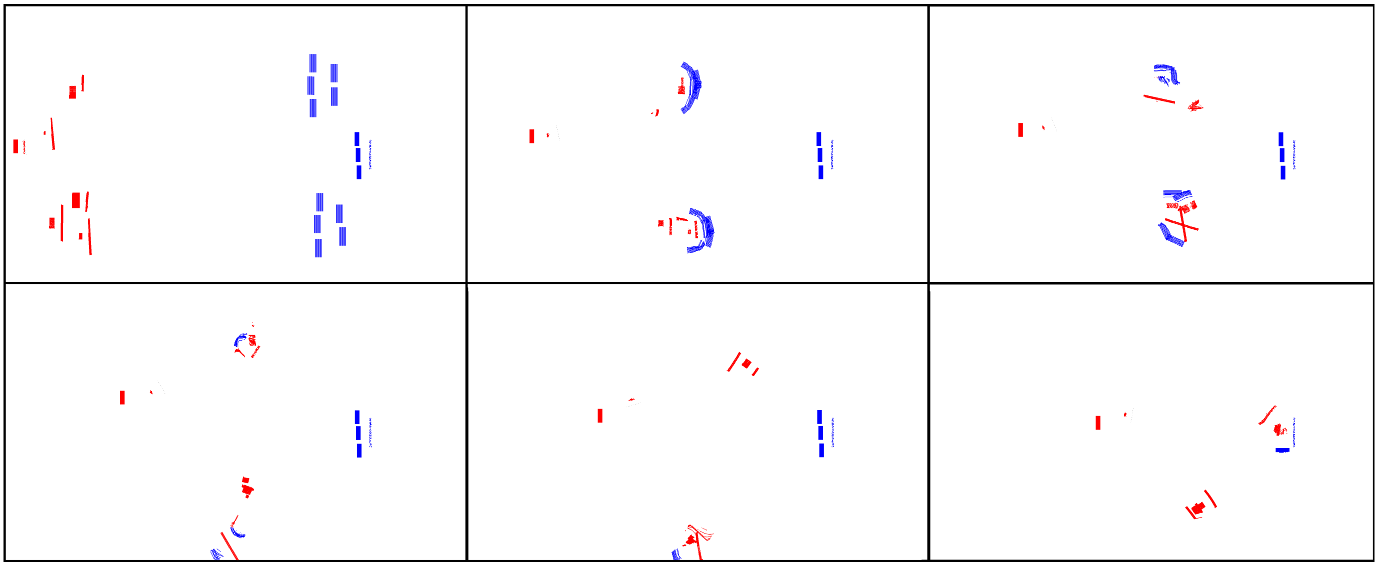

Fig. 7. Course of the simulation of the historical scenario of the Battle of Kokenhausen.

TABLE II
RESULTS OF THE SIMULATION OF THE BATTLE OF KOKENHAUSEN.

	Poland-Lithuania	Sweden
Before battle	3324	5070
Alive after battle	3100	0
Run away	130	2518
Died	94	2552
Died (historically) [16]	ca. 100-200	ca. 2000

initial parameters for more complicated simulations – for example confrontation of whole wings of armies – and finally for the whole battle. Table I contains values of calibrated parameters.

C. Results of the simulation

Figure 7 presents course of the simulation of the historical scenario of the Battle of Kokenhausen. Red army represents Polish-Lithuanian forces, blue army represents Swedes.

Course of simulation reenacts the actual battle, as described in Section IV-A. Cavalry of both armies clashes at the flanks. Left wing of Polish forces dominates the enemy after getting help from the main group. On the right wing Polish cavalry also manages to defeat the enemy. Battle ends with the destruction of abandoned Swedish infantry.

Table II contains numerical results of the simulation. Historical data allowed us to precisely set number of soldiers in each regiment of Polish army. We approximated the number of Swedish soldiers according to available information. Results are an average over ten runs of simulation.

Results of the simulation, in terms of number of fallen and routed soldiers, are very close to the historical data. This shows that described model can be a valuable tool in terms of reproducing results of historical battles. Running the simulation on a home-class PC with dual-core 2.93 GHz processor and 4 GB of RAM took less than 3 minutes.

V. AN ALTERNATIVE SCENARIO OF THE BATTLE

This section contains information about simulation of an alternative scenario of the Battle of Kokenhausen. It is designed to test historians hypothesis about the cause of the Swedish defeat.

A. Changing the model

Historians believe that one of the most important reasons of such severe Swedish defeat was tactic used by reiters [15]. Caracole was developed as a way of fighting enemy infantry. Complicated dance of caracole deprived reiters of their mobility, crucial when fighting against other horsemen. Using it against melee cavalry repeatedly proved to be ineffective (Kokenhausen (1601), Weissenstein (1604), Kircholm (1605)).

Alternative scenario of the Battle of Kokenhausen considers what would have happen, if Swedish reiters had used tactic designed to face melee cavalry. To check that, we replaced original tactic and behavior of reiters with tactic and behavior of Tatar riders, described in Section III-C1. All other parameters of the simulation remained the same, including numerical statistics of reiter soldiers and initial positions of both armies.

B. Results of the simulation

Figure 8 presents course of the simulation of an alternative scenario of the Battle of Kokenhausen. Red army represents Polish-Lithuanian forces, blue army represents Swedes.

Once again, the cavalry of both armies clashes on the flanks. This time however, reiters with modified behavior are able to much more effectively avoid close contact with Polish riders. Despite being slower than Polish riders, Swedish cavalrymen manage to keep distance from the enemy long enough to deal serious damage with their firearms. Finally, Polish-Lithuanian army is forced to retreat.

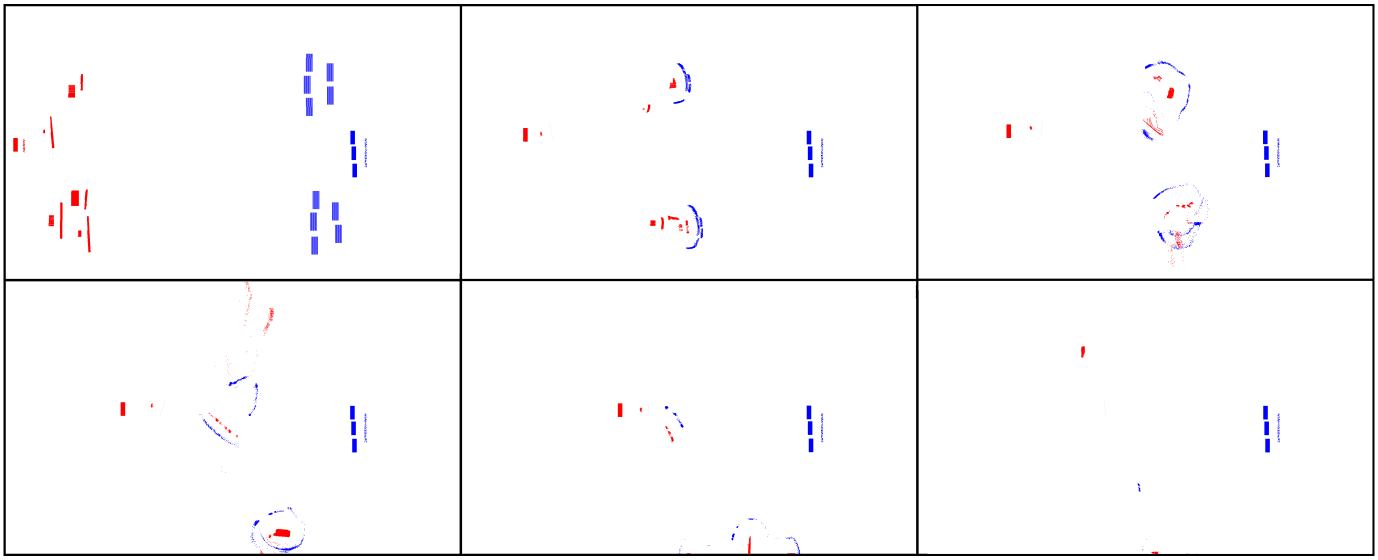

Fig. 8. Course of the simulation of the alternative scenario of the Battle of Kokenhausen.

TABLE III
RESULTS OF THE ALTERNATIVE SCENARIO OF THE BATTLE OF KOKENHAUSEN.

	Poland-Lithuania	Sweden
Before battle	3324	5070
Alive after battle	0	2250
Run away	2007	1647
Died	1317	1173

Table III contains numerical results of the simulation. Results are an average over ten runs of simulation. Despite sustaining severe losses, Swedish army forced adversaries to retreat. Results confirm change of reiters tactic to be crucial in turning the tide of battle, thus confirming assumptions of historians.

VI. Discussion and Future Work

The paper described an experiment in using agent-based model *Petro* to simulate historical warfare. Simple rules of combat made it possible to construct simulations with thousands of independent agents. Adding a second layer of agent hierarchy – the regiment agent – allowed us to model a tactic of various types of military forces from 17th century. We combined modeled troops together in a single simulation and recreated the course of the Battle of Kokenhausen (1601). Finally, simulating an alternative scenario of the battle confirmed thesis of historians, that using caracole tactic was one of the main reasons of Swedish defeat.

Results achieved during simulations show that our model can be useful in reconstructing course of historical battles. At the same time, a simulation with thousands of agents could easily be run on a home-class PC. It shows that there is a middle ground between simple systems designed only for a single battle and complicated, multi-level agent hierarchies.

Many battles from 17th century and even earlier times are poorly documented. A well-calibrated model, constructed for different encounters from similar time, could be used to test various theories about the real course of battle. Therefore agent-based simulations could become a valuable instrument for historical studies.

Future work could investigate how the model would react to adding a third level of agent hierarchy, a single commander of an entire army. Historically, a commander in 17th century and earlier ages had very limited influence on actions of his soldiers once the engagement started. Nevertheless, reconstruction of more complicated battles could demand representation of strategy created by army's commander.

Another possible direction of future work is checking, how restricting the class of algorithms used as tactics and behaviors affects effectiveness of troops.

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