

Blocks & declarations

For now: let's look at declarations of variables:

TINY⁺

$$S \in \mathbf{Stmt} ::= \dots \mid \mathbf{begin} D_V S \mathbf{end}$$
$$D_V \in \mathbf{VDecl} ::= \mathbf{var} x; D_V \mid \varepsilon$$

Locations

We have identified two roles of variables:

- identifiers, as used in programs
- names for memory cells, where values are stored

To separate them, the structure of the semantic domains has to be changed.

Splitting states into
environments and stores

Int = $\{0, 1, -1, 2, -2, \dots\}$

Bool = $\{\mathbf{tt}, \mathbf{ff}\}$

VEnv = **Var** \rightarrow (**Loc** + $\{??\}$)

Store = **Loc** \rightarrow (**Int** + $\{??\}$)

combine : **VEnv** \times **Store** \rightarrow **State**

combine(ρ_V, s) = $\lambda x:\mathbf{Var}.s(\rho_V(x))$

Inaccurate, but right...

Semantic functions

$$\mathcal{N}: \mathbf{Num} \rightarrow \mathbf{Int}$$

$$\mathcal{E}: \mathbf{Exp} \rightarrow \underbrace{\mathbf{VEnv} \rightarrow \mathbf{Store}}_{\mathbf{EXP}} \rightarrow (\mathbf{Int} + \{?\})$$

$$\mathcal{B}: \mathbf{BExp} \rightarrow \underbrace{\mathbf{VEnv} \rightarrow \mathbf{Store}}_{\mathbf{BEXP}} \rightarrow (\mathbf{Bool} + \{?\})$$

and then for instance

$$\mathcal{E}[[x]] = \lambda\rho_V:\mathbf{VEnv}.\lambda s:\mathbf{Store}.\text{ifte}(\rho_V x = ??, ??, \text{ifte}(s(\rho_V x) = ??, ??, s(\rho_V x)))$$

$$\mathcal{E}[[e_1 + e_2]] = \lambda\rho_V:\mathbf{VEnv}.\lambda s:\mathbf{Store}.\text{ifte}(\mathcal{E}[[e_1]] \rho_V s = ??, ??, \\ \text{ifte}(\mathcal{E}[[e_2]] \rho_V s = ??, ??, \\ \mathcal{E}[[e_1]] \rho_V s + \mathcal{E}[[e_2]] \rho_V s))$$

Looks horrible!
Reads even worse!

Bits of notation

- (re)move lambda-abstraction
- use **where**-notation, **let**-notation, explicit **if-then**-notation, etc
- assume that errors ?? propagate

Then:

$$\mathcal{E}[[x]] \rho_V s = s l \text{ where } l = \rho_V x$$

$$\mathcal{E}[[e_1 + e_2]] \rho_V s = n_1 + n_2 \text{ where } n_1 = \mathcal{E}[[e_1]] \rho_V s, n_2 = \mathcal{E}[[e_2]] \rho_V s$$

Relate this to the previous semantics using combine

Write down all the other rules for \mathcal{E} and \mathcal{B} .
Spell out their exact meaning
expanding all the notations in use.

Statements

One could work with “big states”

$$\mathcal{S}: \text{Stmt} \rightarrow \underbrace{\text{VEnv} \times \text{Store}}_{\text{STMT}} \rightarrow (\text{VEnv} \times \text{Store} + \{\text{??}\})$$

BUT:

Statements do not modify the environment!

Hence:

$$\mathcal{S}: \text{Stmt} \rightarrow \underbrace{\text{VEnv} \rightarrow \text{Store}}_{\text{STMT}} \rightarrow (\text{Store} + \{\text{??}\})$$

$$\mathcal{S}[\mathcal{S}] \rho_v: \text{Store} \rightarrow (\text{Store} + \{\text{??}\})$$

- environments flow “statically”, following program block structure
- stores change “dynamically”, following program execution

Semantic clauses

$\mathcal{S}[[x := e]] \rho_V s = s[l \mapsto n]$ where $l = \rho_V x, n = \mathcal{E}[[e]] \rho_V s$

$\mathcal{S}[[\text{skip}]] \rho_V s = s$

$\mathcal{S}[[S_1; S_2]] \rho_V s = \mathcal{S}[[S_2]] \rho_V s_1$ where $s_1 = \mathcal{S}[[S_1]] \rho_V s$

$\mathcal{S}[[\text{if } b \text{ then } S_1 \text{ else } S_2]] \rho_V s = \text{let } v = \mathcal{B}[[b]] \rho_V s \text{ in}$
if $v = \text{tt}$ then $\mathcal{S}[[S_1]] \rho_V s$
if $v = \text{ff}$ then $\mathcal{S}[[S_2]] \rho_V s$

$\mathcal{S}[[\text{while } b \text{ do } S]] \rho_V s = \text{let } v = \mathcal{B}[[b]] \rho_V s \text{ in}$
if $v = \text{ff}$ then s
if $v = \text{tt}$ then $\mathcal{S}[[\text{while } b \text{ do } S]] \rho_V s'$
where $s' = \mathcal{S}[[S]] \rho_V s$

fixed-point equation for
 $\mathcal{S}[[\text{while } b \text{ do } S]] \rho_V$
in $\text{Store} \rightarrow (\text{Store} + \{\?\})$

Relate this to the previous semantics using *combine*

More compact version

Relying on propagation of errors $??$ to be also built into composition of (partial) functions from **Store** to **Store** + $\{??\}$:

$$\mathcal{S}[[x := e]] \rho_V s = s[l \mapsto n] \text{ where } l = \rho_V x, n = \mathcal{E}[[e]] \rho_V s$$

$$\mathcal{S}[[\text{skip}]] \rho_V = id_{\text{Store}}$$

$$\mathcal{S}[[S_1; S_2]] \rho_V = \mathcal{S}[[S_1]] \rho_V; \mathcal{S}[[S_2]] \rho_V$$

$$\mathcal{S}[[\text{if } b \text{ then } S_1 \text{ else } S_2]] \rho_V = \text{cond}(\mathcal{B}[[b]] \rho_V, \mathcal{S}[[S_1]] \rho_V, \mathcal{S}[[S_2]] \rho_V)$$

$$\mathcal{S}[[\text{while } b \text{ do } S]] \rho_V = \text{cond}(\mathcal{B}[[b]] \rho_V, \mathcal{S}[[S]] \rho_V; \mathcal{S}[[\text{while } b \text{ do } S]] \rho_V, id_{\text{Store}})$$

The missing clause for blocks in a moment

Declarations modify environments

$$\mathcal{D}_V : \mathbf{VDecl} \rightarrow \underbrace{\mathbf{VEnv} \rightarrow \mathbf{Store} \rightarrow (\mathbf{VEnv} \times \mathbf{Store} + \{??\})}_{\mathbf{VDECL}}$$

$$\mathcal{D}_V[\varepsilon] \rho_V s = \langle \rho_V, s \rangle$$

$$\mathcal{D}_V[\mathbf{var} \ x; D_V] \rho_V s = \mathcal{D}_V[D_V] \rho'_V s'$$

$$\text{where } l = \text{newloc}(s), \rho'_V = \rho_V[x \mapsto l], s' = s[l \mapsto ??]$$

Trouble: We want $\text{newloc} : \mathbf{Store} \rightarrow \mathbf{Loc}$ to yield a new, unused location. This cannot be defined under the definitions given so far. Solution: more information in stores is needed to determine used and unused locations.

Simple solution

Take:

$$\mathbf{Loc} = \{0, 1, 2, \dots\}$$

Add to each store a pointer to the next unused location:

$$\mathbf{Store} = (\mathbf{Loc} + \{next\}) \rightarrow (\mathbf{Int} + \{??\})$$

Semantic clauses then:

$$\mathcal{D}_V[\varepsilon] \rho_V s = \langle \rho_V, s \rangle$$

$$\mathcal{D}_V[\mathbf{var} \ x; D_V] \rho_V s =$$

$$\mathcal{D}_V[D_V] \rho'_V s' \text{ where } l = s \ next, \rho'_V = \rho_V[x \mapsto l], s' = s[l \mapsto ??, next \mapsto l + 1]$$

Semantics of blocks

$$\mathcal{S}[\mathbf{begin} D_V S \mathbf{end}] \rho_V s = \mathcal{S}[S] \rho'_V s' \text{ where } \langle \rho'_V, s' \rangle = \mathcal{D}_V[D_V] \rho_V s$$

*The scope of a declaration is the block it occurs in
with holes resulting from redeclarations of the same variable within it*

For instance

begin var y ; var x $x := 1$; begin var x $y := 2$; $x := 5$ end; $y := x$ end

may be marked as follows to indicate the relevant declarations:

begin var y ; var x x := 1; begin var x y := 2; x := 5 end; y := x end

Procedures

TINY++

naming statements/blocks
for multiple use

$$S \in \mathbf{Stmt} ::= \dots \mid \mathbf{begin} D_V D_P S \mathbf{end} \mid \mathbf{call} p$$
$$D_V \in \mathbf{VDecl} ::= \mathbf{var} x; D_V \mid \varepsilon$$
$$D_P \in \mathbf{PDecl} ::= \mathbf{proc} p \mathbf{is} (S); D_P \mid \varepsilon$$

- binding of global variables
- recursion

Binding of global variables

Static binding

```
begin var y;  
  var x ;  
  proc p is (x := 1);  
  begin var x ;  
    x := 3;  
    call p;  
    y := x  
  end  
end
```

Dynamic binding

```
begin var y;  
  var x ;  
  proc p is (x := 1);  
  begin var x ;  
    x := 3;  
    call p; %%% with x  
    y := x  
  end  
end
```

Semantic domains and functions

Dynamic binding

$$\mathbf{PEnv} = \mathbf{IDE} \rightarrow (\mathbf{PROC}_0 + \{?\})$$

$$\mathbf{PROC}_0 = \mathbf{VEnv} \rightarrow \mathbf{PEnv} \rightarrow \mathbf{Store} \rightarrow (\mathbf{Store} + \{?\})$$

$$\mathcal{S}: \mathbf{Stmt} \rightarrow \underbrace{\mathbf{VEnv} \rightarrow \mathbf{PEnv} \rightarrow \mathbf{Store} \rightarrow (\mathbf{Store} + \{?\})}_{\mathbf{STMT}}$$

$$\mathcal{D}_P: \mathbf{PDecl} \rightarrow \underbrace{\mathbf{PEnv} \rightarrow (\mathbf{PEnv} + \{?\})}_{\mathbf{PDECL}}$$

Semantic clauses

$\mathcal{S}: \text{Stmt} \rightarrow \text{STMT}$

$\mathcal{S}[x := e] \rho_V \rho_P s = s[l \mapsto n]$ where $l = \rho_V x, n = \mathcal{E}[e] \rho_V s$

$\mathcal{S}[\text{skip}] \rho_V \rho_P = id_{\text{Store}}$

$\mathcal{S}[S_1; S_2] \rho_V \rho_P = \mathcal{S}[S_1] \rho_V \rho_P; \mathcal{S}[S_2] \rho_V \rho_P$

$\mathcal{S}[\text{if } b \text{ then } S_1 \text{ else } S_2] \rho_V \rho_P = \text{cond}(\mathcal{B}[b] \rho_V, \mathcal{S}[S_1] \rho_V \rho_P, \mathcal{S}[S_2] \rho_V \rho_P)$

$\mathcal{S}[\text{while } b \text{ do } S] \rho_V \rho_P =$
 $\text{cond}(\mathcal{B}[b] \rho_V, \mathcal{S}[S] \rho_V \rho_P; \mathcal{S}[\text{while } b \text{ do } S] \rho_V \rho_P, id_{\text{Store}})$

$\mathcal{S}[\text{call } p] \rho_V \rho_P = P \rho_V \rho_P$ where $P = \rho_P p$

$\mathcal{S}[\text{begin } D_V \ D_P \ S \ \text{end}] \rho_V \rho_P s =$
 $\mathcal{S}[S] \rho'_V \rho'_P s'$ where $\langle \rho'_V, s' \rangle = \mathcal{D}_V[D_V] \rho_V s, \rho'_P = \mathcal{D}_P[D_P] \rho_P$

$\mathcal{D}_P: \text{PDecl} \rightarrow \text{PDECL}$

$\mathcal{D}_P[\varepsilon] = id_{\text{PEnv}}$

$\mathcal{D}_P[\text{proc } p \text{ is } (S); D_P] \rho_P = \mathcal{D}_P[D_P] \rho_P[p \mapsto \mathcal{S}[S]]$

Recursion

```
begin var  $x$ ;  
  proc  $NO$  is (if  $101 \leq x$  then  $x := x - 10$   
    else ( $x := x + 11$ ; call  $NO$ ; call  $NO$ ) );  
   $x := 54$ ;  
  call  $NO$   
end
```

Semantic domains and functions

Static binding

$$\mathbf{PEnv} = \mathbf{IDE} \rightarrow (\mathbf{PROC}_0 + \{?\})$$

$$\mathbf{PROC}_0 = \mathbf{Store} \rightarrow (\mathbf{Store} + \{?\})$$

$$\mathcal{S}: \mathbf{Stmt} \rightarrow \underbrace{\mathbf{VEnv} \rightarrow \mathbf{PEnv} \rightarrow \mathbf{Store} \rightarrow (\mathbf{Store} + \{?\})}_{\mathbf{STMT}}$$

$$\mathcal{D}_P: \mathbf{PDecl} \rightarrow \underbrace{\mathbf{VEnv} \rightarrow \mathbf{PEnv} \rightarrow (\mathbf{PEnv} + \{?\})}_{\mathbf{PDECL}}$$

Semantic clauses

$\mathcal{S}: \text{Stmt} \rightarrow \text{STMT}$

$\mathcal{S}[x := e] \rho_V \rho_P s = s[l \mapsto n]$ where $l = \rho_V x, n = \mathcal{E}[e] \rho_V s$

$\mathcal{S}[\text{skip}] \rho_V \rho_P = id_{\text{Store}}$

$\mathcal{S}[S_1; S_2] \rho_V \rho_P = \mathcal{S}[S_1] \rho_V \rho_P; \mathcal{S}[S_2] \rho_V \rho_P$

$\mathcal{S}[\text{if } b \text{ then } S_1 \text{ else } S_2] \rho_V \rho_P = \text{cond}(\mathcal{B}[b] \rho_V, \mathcal{S}[S_1] \rho_V \rho_P, \mathcal{S}[S_2] \rho_V \rho_P)$

$\mathcal{S}[\text{while } b \text{ do } S] \rho_V \rho_P =$
 $\text{cond}(\mathcal{B}[b] \rho_V, \mathcal{S}[S] \rho_V \rho_P; \mathcal{S}[\text{while } b \text{ do } S] \rho_V \rho_P, id_{\text{Store}})$

$\mathcal{S}[\text{call } p] \rho_V \rho_P = P$ where $P = \rho_P p$

$\mathcal{S}[\text{begin } D_V \ D_P \ S \ \text{end}] \rho_V \rho_P s =$
 $\mathcal{S}[S] \rho'_V \rho'_P s'$ where $\langle \rho'_V, s' \rangle = \mathcal{D}_V[D_V] \rho_V s, \rho'_P = \mathcal{D}_P[D_P] \rho_V \rho_P$

$\mathcal{D}_P[\varepsilon] \rho_V = id_{\text{PEnv}}$

$\mathcal{D}_P[\text{proc } p \text{ is } (S); D_P] \rho_V \rho_P =$
 $\mathcal{D}_P[D_P] \rho_V \rho_P [p \mapsto P]$ where $P = \mathcal{S}[S] \rho_V \rho_P [p \mapsto P]$

$\mathcal{D}_P: \text{PDecl} \rightarrow \text{PDECL}$

Recursion

- Static binding, no recursive calls:

$$\mathcal{D}_P[\mathbf{proc} \ p \ \mathbf{is} \ (S); D_P] \rho_V \rho_P = \mathcal{D}_P[D_P] \rho_V \rho_P [p \mapsto P] \text{ where } P = \mathcal{S}[S] \rho_V \rho_P$$

- Static binding, recursive calls permitted:

$$\mathcal{D}_P[\mathbf{proc} \ p \ \mathbf{is} \ (S); D_P] \rho_V \rho_P = \mathcal{D}_P[D_P] \rho_V \rho_P [p \mapsto P] \text{ where } P = \mathcal{S}[S] \rho_V \rho_P [p \mapsto P]$$

or perhaps using fixed-point operator explicitly:

$$\mathcal{D}_P[\mathbf{proc} \ p \ \mathbf{is} \ (S); D_P] \rho_V \rho_P = \mathcal{D}_P[D_P] \rho_V \rho_P [p \mapsto \mathit{fix}(\Phi)] \text{ where } \Phi(P) = \mathcal{S}[S] \rho_V \rho_P [p \mapsto P]$$

- Dynamic binding, recursion “for free” (though only seemingly so):

$$\begin{aligned} \mathcal{S}[\mathbf{call} \ p] \rho_V \rho_P &= P \rho_V \rho_P \text{ where } P = \rho_P p \\ \mathcal{D}_P[\mathbf{proc} \ p \ \mathbf{is} \ (S); D_P] \rho_P &= \mathcal{D}_P[D_P] \rho_P [p \mapsto \mathcal{S}[S]] \end{aligned}$$

Static binding, procedure calls:
 $\mathcal{S}[\mathbf{call} \ p] \rho_V \rho_P = P \text{ where } P = \rho_P p$

Parameters

Parameter passing:

- call by value
- call by variable
- call by name

We will do **static binding only**

$S \in \mathbf{Stmt} ::= \dots \mid \mathbf{begin} D_V D_P S \mathbf{end}$

$\mid \mathbf{call} p \mid \mathbf{call} p(\mathbf{vl} e) \mid \mathbf{call} p(\mathbf{vr} x) \mid \mathbf{call} p(\mathbf{nm} e)$

$D_V \in \mathbf{VDecl} ::= \mathbf{var} x; D_V \mid \varepsilon$

$D_P \in \mathbf{PDecl} ::= \mathbf{proc} p \mathbf{is} (S); D_P \mid \mathbf{proc} p(\mathbf{vl} x) \mathbf{is} (S); D_P$

$\mid \mathbf{proc} p(\mathbf{vr} x) \mathbf{is} (S); D_P \mid \mathbf{proc} p(\mathbf{nm} x) \mathbf{is} (S); D_P \mid \varepsilon$

Call by name

```
begin var sum; var i
  proc ADD(vl a, b, vr x, nm step, f)
    is (sum := 0; x := a;
      while x ≤ b do (sum := sum + f; x := step) )
  call ADD(vl 1, 10, vr i, nm i + 1, i);      {sum = 55}
  call ADD(vl 1, 10, vr i, nm 2 * i, i);      {sum = 15}
  call ADD(vl 1, 10, vr i, nm i + 2, i * i) {sum = 165}
end
```

Semantic domains

$$\mathbf{PEnv} = \mathbf{IDE} \rightarrow (\mathbf{PROC}_0 + \mathbf{PROC}_1^{\text{vl}} + \mathbf{PROC}_1^{\text{vr}} + \mathbf{PROC}_1^{\text{nm}} + \{\text{??}\})$$

$$\mathbf{PROC}_0 = \mathbf{Store} \rightarrow (\mathbf{Store} + \{\text{??}\})$$

$$\mathbf{PROC}_1^{\text{vl}} = \mathbf{Int} \rightarrow \mathbf{PROC}_0$$

$$\mathbf{PROC}_1^{\text{vr}} = \mathbf{Loc} \rightarrow \mathbf{PROC}_0$$

$$\mathbf{PROC}_1^{\text{nm}} = (\mathbf{Store} \rightarrow (\mathbf{Int} + \{\text{??}\})) \rightarrow \mathbf{PROC}_0$$

Semantic functions

As before:

$$\begin{array}{l} \mathcal{S}: \text{Stmt} \rightarrow \text{VEnv} \rightarrow \text{PEnv} \rightarrow \text{Store} \rightarrow (\text{Store} + \{??\}) \\ \quad \underbrace{\hspace{15em}}_{\text{STMT}} \\ \mathcal{D}_P: \text{PDecl} \rightarrow \text{VEnv} \rightarrow \text{PEnv} \rightarrow (\text{PEnv} + \{??\}) \\ \quad \underbrace{\hspace{15em}}_{\text{PDECL}} \end{array}$$

Semantic clauses

No parameters

$\mathcal{S}[\mathbf{call} \ p] \ \rho_V \ \rho_P = P$ where $P = \rho_P \ p \in \mathbf{PROC}_0$

$\mathcal{D}_P[\mathbf{proc} \ p \ \mathbf{is} \ (S); \ D_P] \ \rho_V \ \rho_P =$

$\mathcal{D}_P[\ D_P] \ \rho_V \ \rho_P [p \mapsto P]$ where $P = \mathcal{S}[S] \ \rho_V \ \rho_P [p \mapsto P]$

Parameter called by value

$\mathcal{S}[\mathbf{call} \ p(\mathbf{vl} \ e)] \rho_V \rho_P s = P \ n \ s$ where $P = \rho_P p \in \mathbf{PROC}_1^{\mathbf{vl}}$, $n = \mathcal{E}[e] \rho_V s$

$\mathcal{D}_P[\mathbf{proc} \ p(\mathbf{vl} \ x) \ \mathbf{is} \ (S); D_P] \rho_V \rho_P =$

$\mathcal{D}_P[D_P] \rho_V \rho_P [p \mapsto P]$ where

$P \ n \ s = \mathcal{S}[S] \rho'_V \rho_P [p \mapsto P] s'$ where

$l = s \ next, \ \rho'_V = \rho_V [x \mapsto l], \ s' = s [l \mapsto n, \ next \mapsto l + 1]$

Parameter called by variable

$\mathcal{S}[\mathbf{call} \ p(\mathbf{vr} \ y)] \rho_V \rho_P = P l$ where $P = \rho_P p \in \mathbf{PROC}_1^{\mathbf{vr}}$, $l = \rho_V y$

$\mathcal{D}_P[\mathbf{proc} \ p(\mathbf{vr} \ x) \ \mathbf{is} \ (S); D_P] \rho_V \rho_P =$

$\mathcal{D}_P[D_P] \rho_V \rho_P [p \mapsto P]$ where $P l = \mathcal{S}[S] \rho_V [x \mapsto l] \rho_P [p \mapsto P]$

Parameter called by name

$\mathcal{S}[\text{call } p(\text{nm } e)] \rho_V \rho_P = P (\mathcal{E}[e] \rho_V)$ where $P = \rho_P p \in \mathbf{PROC}_1^{\text{nm}}$

$\mathcal{D}_P[\text{proc } p(\text{nm } x) \text{ is } (S); D_P] \rho_V \rho_P =$

$\mathcal{D}_P[D_P] \rho_V \rho_P [p \mapsto P]$ where $P E = \mathcal{S}[S] \rho_V [x \mapsto E] \rho_P [p \mapsto P]$

OOOPS!

$\rho_V [x \mapsto E] \notin \mathbf{VEnv}$

Corrections necessary!

$\mathbf{VEnv} = \mathbf{Var} \rightarrow (\mathbf{Loc} + (\mathbf{Store} \rightarrow (\mathbf{Int} + \{??\}))) + \{??\}$

$\mathcal{E}[x] \rho_V s = \text{let } v = \rho_V x \text{ in if } v \in \mathbf{Loc} \text{ then } s v$

$\text{if } v \in (\mathbf{Store} \rightarrow (\mathbf{Int} + \{??\})) \text{ then } v s$

This allows for evaluation of called-by-name parameters,
but not for assignments to variables passed in such a way