Automaty nieskończone

2024/25

Wykład 6

Petri nets reachability problem – – sketch of decidability proof

Input: A VASS and two configurations c, c'

Question: $\underset{(q,v)}{c} \longrightarrow^* \underset{(q',v')}{c'}$?

VASS:

$$\circ \xrightarrow{v \in \mathbb{Z}^d} \circ$$

Brief history: . . . . .

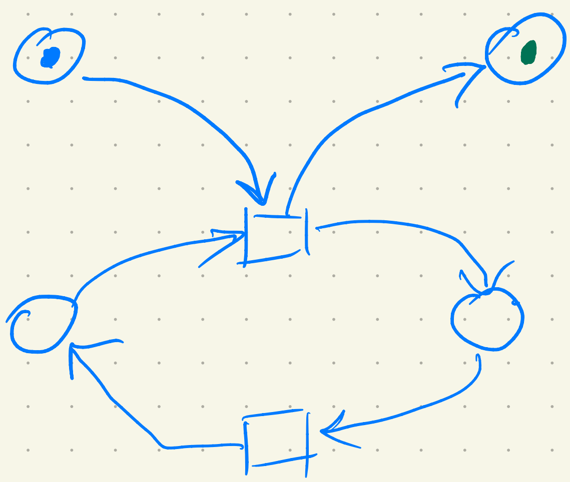
Pseudo runs: $c \dashrightarrow^* c'$

intermediate vectors may drop below 0 on some coordinates

Question: $c \dashrightarrow^* c' \implies c \longrightarrow^* c'$?

No!

- initial
- final



Sufficient condition for $c \longrightarrow^* c'$:

Θ_1 : For every $n \geq 1$, $(q, v) \dashrightarrow^* (q', v')$ using every transition $\geq n$ times.

Implies strong connectedness

Θ_2 : For some $\Delta, \Delta' \geq \vec{1}$,

a) $(q, v) \xrightarrow{\pi}{}^* (q, v + \Delta)$

b) $(q', v') \;{}^*\!\xleftarrow{\pi'} (q', v' + \Delta')$

Claim : $(q', \Delta) \dashrightarrow^* (q', \Delta')$

$$
\begin{tikzcd}
 & q', v' + n\Delta \arrow[dr, bend left, "\text{Claim} \leftarrow \Theta_1"] & \\
q, v + n\Delta \arrow[ur, bend left, "\Theta_1"] & & q', v' + n\Delta' \arrow[d, "\Theta_2", "*"'] \\
q, v \arrow[u, "*", "\Theta_2"'] & & q', v'
\end{tikzcd}
$$

<u>effect</u> of a pseudorun $e(\alpha)$: final vector − initial vector $\in \mathbb{Z}^d$

determines

<u>folding</u> of a pseudorun $F(\alpha)$: $\in \mathbb{N}^T$

<u>Observation:</u> Two pseudoruns

$$\begin{tikzcd} q \arrow[r, bend left, squiggly, "\alpha"] \arrow[r, bend right, squiggly, "\beta"'] & q' \end{tikzcd}$$

$$F(\alpha) - F(\beta) \geq \vec{1}.$$

For every non-isolated control state p there is a pseudorun $\begin{tikzcd} p \arrow[loop right, squiggly, "\gamma"] \end{tikzcd}$ s.t.

$$F(\gamma) = F(\alpha) - F(\beta).$$

<u>Proof</u>:

Proof of the claim: Pseudoruns (using Θ_1)

$$\begin{tikzcd} (q, v) \arrow[r, bend left, squiggly, "\alpha"] \arrow[r, bend right, squiggly, "\beta"'] & (q', v') \end{tikzcd}$$

such that

$$F(\alpha) - F(\beta) - F(\pi) - F(\pi') \geq \vec{1}$$

By Observation, there is a pseudorun $q' \overset{\gamma}{\circlearrowleft}$

s.t.

$$F(\gamma) = F(\alpha) - F(\beta) - F(\pi) - F(\pi')$$

$$e(\gamma) = \underbrace{e(\alpha) - e(\beta)}_{0} \underbrace{- e(\pi)}_{-\Delta} \underbrace{- e(\pi')}_{+\Delta'} = \Delta' - \Delta$$

Hence $(q, \Delta) \overset{\gamma}{\dashrightarrow}{}^{*} (q', \Delta')$

Θ_1 is effective $\longrightarrow$ $\boxed{\text{tutorials}}$

Θ_2 is effective — why?

What to do if Θ_1 or Θ_2 fails to hold?

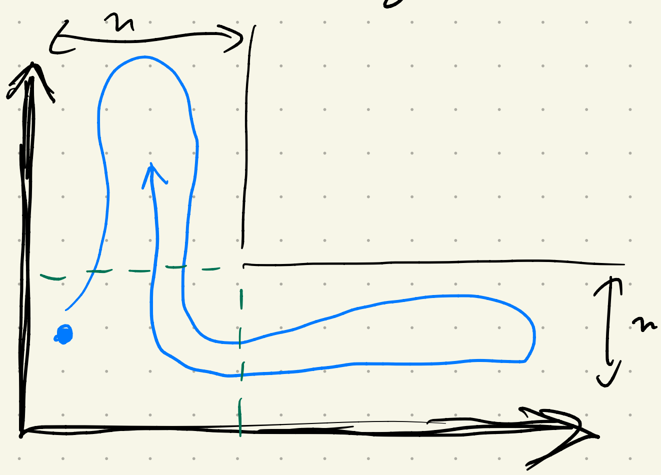
Θ_2 a) fails:

effectively computable — how? $\boxed{\text{tutorials}}$

$\exists n$. every configuration reachable from (q, v) has some coordinate $< n$

(using coverability tree) $\Downarrow$ → $\boxed{\text{tutorials}}$

$\exists n$. every run from (q, v), on some coordinate, is always $< n$

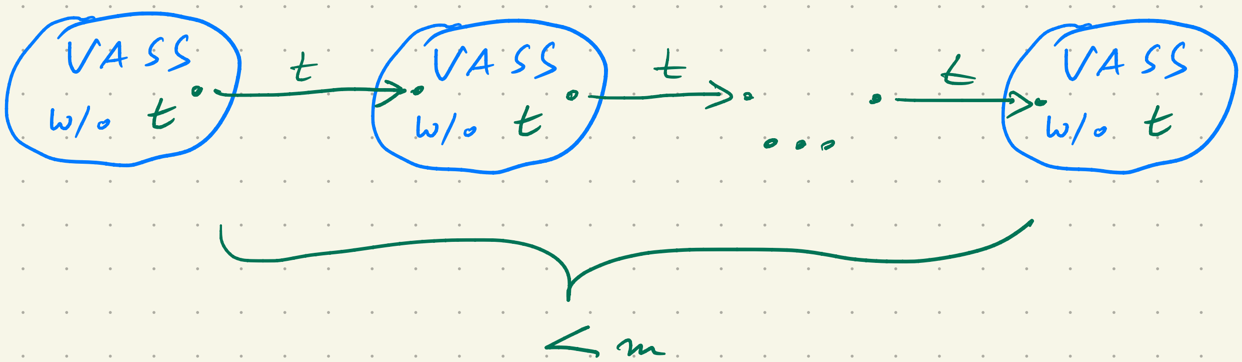


Reduction of dimension: try to bound each single dimension by n.

Θ_n fails:

$\exists m.$ every pseudorun $(q,v) \dashrightarrow^* (q',v')$ uses some transition $< m$ times

? Reduction of the number of transitions:
try to fit nr of usages of each transition t to each nr $< m$.



- We lose strong connectedness!
- Dimension is preserved, nr of transitions increases!