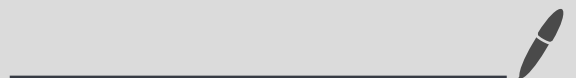


Theory of concurrency

2023/24

Lecture 13



Bisimulation and Logic

Model logic $M =$

SYNTAX:

$$\phi ::= \text{true} \mid \neg \phi \mid \phi_1 \wedge \phi_2 \mid \langle a \rangle \phi$$

SEMANTICS:

$$a \in \Sigma$$

$$P \models \text{true}$$

$$P \models \neg \phi \quad \text{iff} \quad P \not\models \phi$$

$$P \models \phi_1 \wedge \phi_2 \quad \text{iff} \quad P \models \phi_1 \text{ and } P \models \phi_2$$

$$P \models \langle a \rangle \phi \quad \text{iff} \quad P' \models \phi \text{ for some } P' \text{ s.t. } P \xrightarrow{a} P'$$

SHORTHANDS:

$$\bullet \ [a] \phi := \neg \langle a \rangle \neg \phi$$

$$P \models [a] \phi \quad \text{iff} \quad P' \models \phi \text{ for all } P' \text{ s.t. } P \xrightarrow{a} P'$$

$$\bullet \ \phi_1 \vee \phi_2$$

$$\bullet \ \langle - \rangle \phi := \bigvee_a \langle a \rangle \phi$$

$$\bullet \ \text{false} := \neg \text{true}$$

Example: $\langle a \rangle [b] \text{true} \vee [a] \langle b \rangle \text{true}$

$\langle - \rangle \text{true} \quad [-] \text{false}$

Def:

$$P \equiv_M Q \quad \text{iff} \quad \forall \phi \in M. (P \models \phi \text{ iff } Q \models \phi)$$

Lemma : $P \sim Q$ implies $P \equiv_M Q$

Proof : By structural induction over $\phi \in M$ show that $P \sim Q$ implies $(P \models \phi \iff Q \models \phi)$

• Base : $\phi = \text{true}$

$P \sim Q$

• Step : $\phi = \neg \psi$

$P \models \langle a \rangle \psi$

$Q \models \langle a \rangle \psi$

$\phi = \phi_1 \wedge \phi_2$

P

Q

$\phi = \langle a \rangle \psi$

$\downarrow a$

$\downarrow a$

$P' \models \psi$

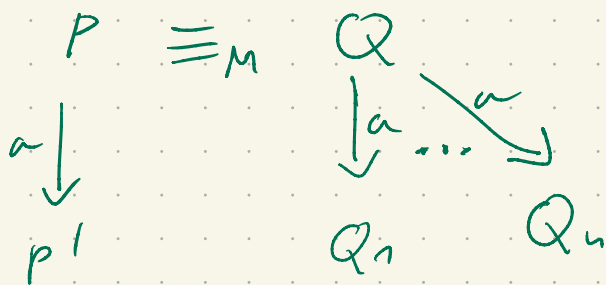
$Q' \models \psi$

$P' \sim Q'$

Lemma : Under image-finiteness assumption,

$P \equiv_M Q$ implies $P \sim Q$.

Proof : \equiv_M is a bisimulation



We need : $P' \equiv_M Q_i$ for some i

Towards contradiction, assume the contrary:

$P \models \langle a \rangle \text{true}$ $Q \not\models \langle a \rangle \text{true}$

$P' \not\equiv_M Q_i$ for all i

There are $\phi_1, \dots, \phi_n \in M$ s.t. $P' \models \phi_i$ L3

Put $\psi := \phi_1 \wedge \dots \wedge \phi_n$

$Q_i \not\models \phi_i$
for every i

$P \models \langle a \rangle \psi$ $Q \not\models \langle a \rangle \psi$ — a contradiction.

Observation: a modal formula is a witness of bisimulation non equivalence

Remark: one can drop image-finiteness assumption

Question: is negation needed in the last proof?

Variants for: • \approx

• \sim_n

• long steps $\langle a_1 \dots a_n \rangle \phi :=$

$\langle a_1 \rangle \dots \langle a_n \rangle \phi$