Theory of Concurrency

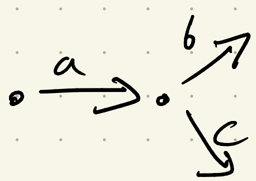
2023/24

Lecture 12

(Observational) Equality of processes

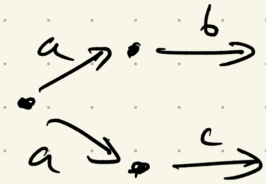
- CCS $\quad (P_1[f_1] \mid \ldots \mid P_n[f_n]) \setminus L$

- <u>strong</u> bisimulation equivalence $\sim$



$\neq$

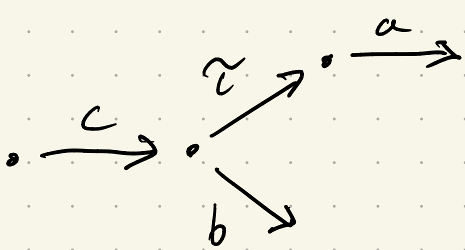
$\nsim$

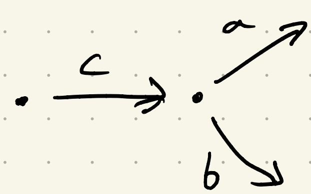


- <u>weak</u> bisimulation equivalence $\approx$

$$\tau . a \overset{?}{=} a$$

$$\tau . a + b \overset{?}{=} a + b$$





Definition: weak bisimulation S

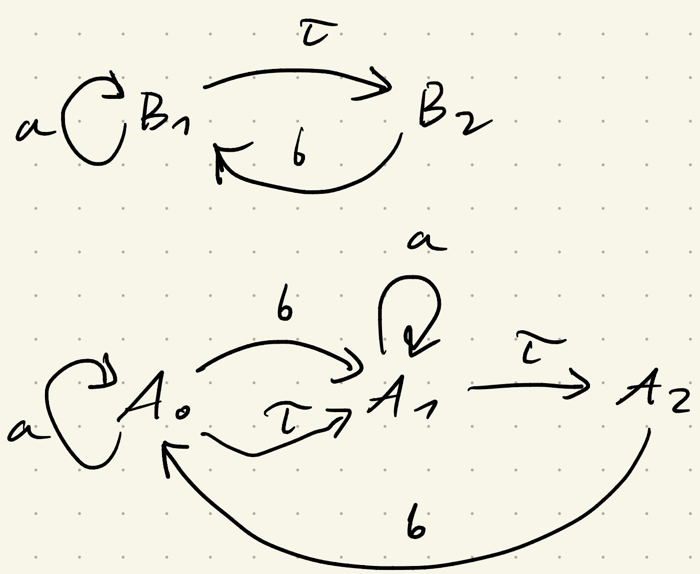
$(a \neq \tau)$
$$\begin{tikzcd} P \arrow[r, dashed, no head, "S"] \arrow[ddd, "a"'] & Q \arrow[d, "\tau^*"] \\ & \cdot \arrow[d, "a"] \\ & \cdot \arrow[d, "\tau^*"] \\ P' \arrow[r, dashed, no head, "S"] & Q' \end{tikzcd}$$

$(a = \tau)$
$$\begin{tikzcd} P \arrow[r, dashed, no head, "S"] \arrow[d, "\tau"'] & Q \arrow[d, "\tau^*"] \\ P' \arrow[r, dashed, no head, "S"] & Q' \end{tikzcd}$$

$(\text{any } a)$
$$\begin{tikzcd} P \arrow[r, dashed, no head, "S"] \arrow[d, "a"'] & Q \arrow[d, Rightarrow, "a"] \\ P' \arrow[r, dashed, no head, "S"] & Q' \end{tikzcd}$$

Example:

$$S = \{(B_1, A_0), (B_1, A_1), (B_2, A_2)\}$$



Definition: weak bisimulation equivalence

$$\approx \; = \bigcup \{ S : S \text{ weak bisimulation} \}$$

Example: $\tau.P \approx P$ $\quad \{(\tau.P, P)\} \cup \mathrm{Id}$

Question: What about:

$$\text{(any } a\text{)}\quad
\begin{tikzcd}
P \arrow[r, dashed, dash, "S"] \arrow[ddd, "a"'] & Q \arrow[d, "\tau^*"] \\
& \cdot \arrow[d, "a"] \\
& \cdot \arrow[d, "\tau^*"] \\
P' \arrow[r, dashed, dash, "S"] & Q'
\end{tikzcd}
\quad = \quad
\begin{tikzcd}
P \arrow[r, dashed, dash, "S"] \arrow[d, "\tau^*"] & Q \arrow[d, "\tau^*"] \\
\cdot \arrow[d, "a"] & \cdot \arrow[d, "a"] \\
\cdot \arrow[d, "\tau^*"] & \cdot \arrow[d, "\tau^*"] \\
P' \arrow[r, dashed, dash, "S"] & Q'
\end{tikzcd}
\quad ?$$

$$\tau.\tau.0 \not\approx \tau.0\,!$$

Question: is $\simeq$ transitive?

$$\begin{tikzcd}
P \arrow[r, dashed, dash, "S"] \arrow[d, Rightarrow, "a"'] & Q \arrow[d, Rightarrow, "a"] \\
P' \arrow[r, dashed, dash, "S"] & Q'
\end{tikzcd}$$

Question: Is $\approx$ a congruence?

$$P \approx P' \overset{?}{\Longrightarrow}$$

- $a.P \approx a.P'$
- $P+Q \approx P'+Q$
- $P \mid Q \approx P' \mid Q$
- $P \setminus L \approx P' \setminus L$
- $P[f] \approx P'[f]$

$\tau.a \approx a$ but

$\tau.a + b \not\approx a + b$

What about:

$$(a \neq \tau)\quad
\begin{tikzcd}
P \arrow[r, dashed, dash, "S"] \arrow[d, "\tau^*"] & Q \arrow[d, "\tau^*"] \\
\cdot \arrow[d, "a"] & \cdot \arrow[d, "a"] \\
\cdot \arrow[d, "\tau^*"] & \cdot \arrow[d, "\tau^*"] \\
P' \arrow[r, dashed, dash, "S"] & Q'
\end{tikzcd}
\quad ?$$

$\tau.a + \tau \approx a$ but

$b.(\tau.a+\tau) \not\approx b.a$

$b \mid (\tau.a+\tau) \not\approx b \mid a$

Definition: Observational equality $=$

$P = Q \iff \forall a \in \Sigma$

- if $P \xrightarrow{a} P'$ then, for some Q', $Q \xrightarrow{\tau^*} \xrightarrow{a} \xrightarrow{\tau^*} Q'$, $P' \approx Q'$
- if $Q \xrightarrow{a} Q'$ then, for some P', $P \xrightarrow{\tau^*} \xrightarrow{a} \xrightarrow{\tau^*} P'$, $P' \approx Q'$

$$\begin{tikzcd}
P \arrow[r, dashed, no head, "="] \arrow[ddd, "a"'] & Q \arrow[d, "\tau^*"] \\
 & \cdot \arrow[d, "a"] \\
 & \cdot \arrow[d, "\tau^*"] \\
P' \arrow[r, dashed, no head, "\approx"] & Q'
\end{tikzcd}$$

separating examples?

$$\boxed{\sim \quad \subsetneq \quad = \quad \subsetneq \quad \approx}$$

Examples: $\tau.a \neq a$

$a.\tau.P = a.P$

$P + \tau.P = \tau.P$

$a.(P + \tau.Q) + a.Q = a.(P + \tau.Q)$

Lemma:

$$P = Q \iff \forall R.\ P + R \approx Q + R$$

(the greatest congruence included in $\approx$)

Question: $=$ or $\approx$?

$$P \mid Q \approx P \mid \tau.Q$$

$$P \mid Q \approx \tau.(P \mid Q)$$

$$P \mid \tau.Q = \tau.(P \mid Q)$$

- Weak bisimulation may be used to prove $=$:

Lemma: $P \approx Q \implies a.P = a.Q$

Lemma: $P \stackrel{\tau}{\not\Rightarrow},\ Q \stackrel{\tau}{\not\Rightarrow},\ P \approx Q \implies P = Q$

- $\approx$ is very close to $=$:

Lemma: $P \approx Q \iff P = Q$ or $P = \tau.Q$ or $\tau.P = Q$