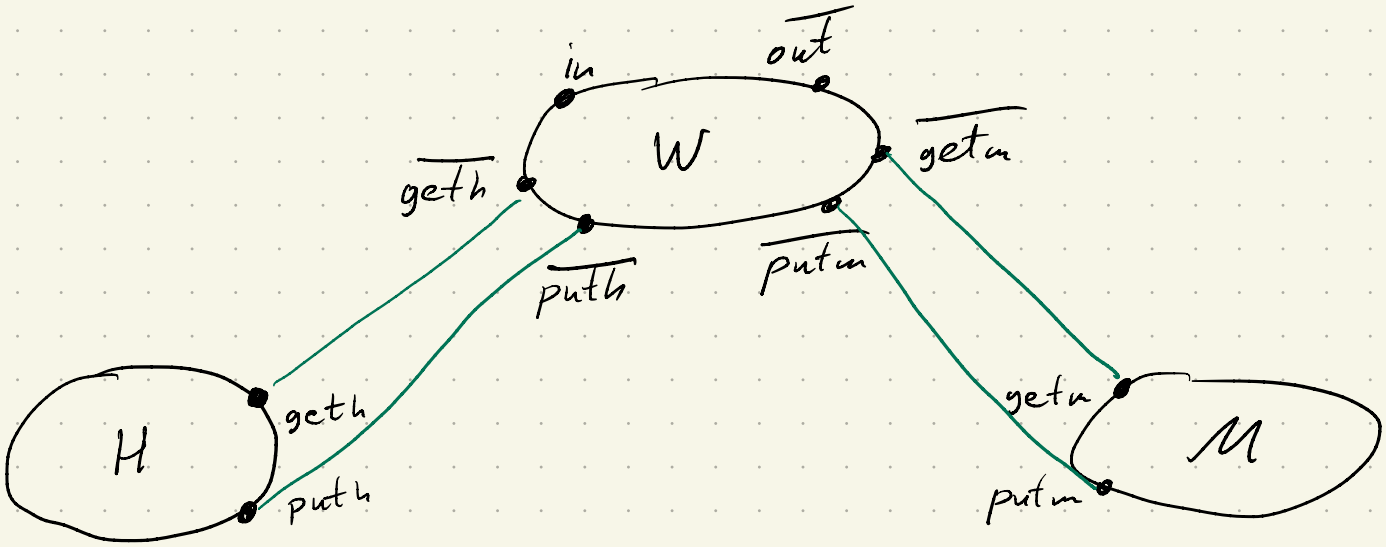
Theory of concurrency

2023/24

Lecture 10

Process algebra CCS

(Calculus of Communicating Systems)



recursive def.

prefix

$$H \overset{def}{=} geth.\ puth.\ H$$

$$M \overset{def}{=} getm.\ putm.\ M$$

$$W \overset{def}{=} in(a).\ \overline{out}(a).\ W$$
$$+$$
$$in(b).\ \overline{geth}.\ \overline{puth}.\ \overline{out}(b).\ W$$
$$+$$
$$in(c).\ (\ \overline{geth}.\ \overline{puth}.\ \overline{out}(c).\ W$$
$$+$$
$$\overline{getm}.\ \overline{putm}.\ out(c).\ W\ )$$

summation (choice)

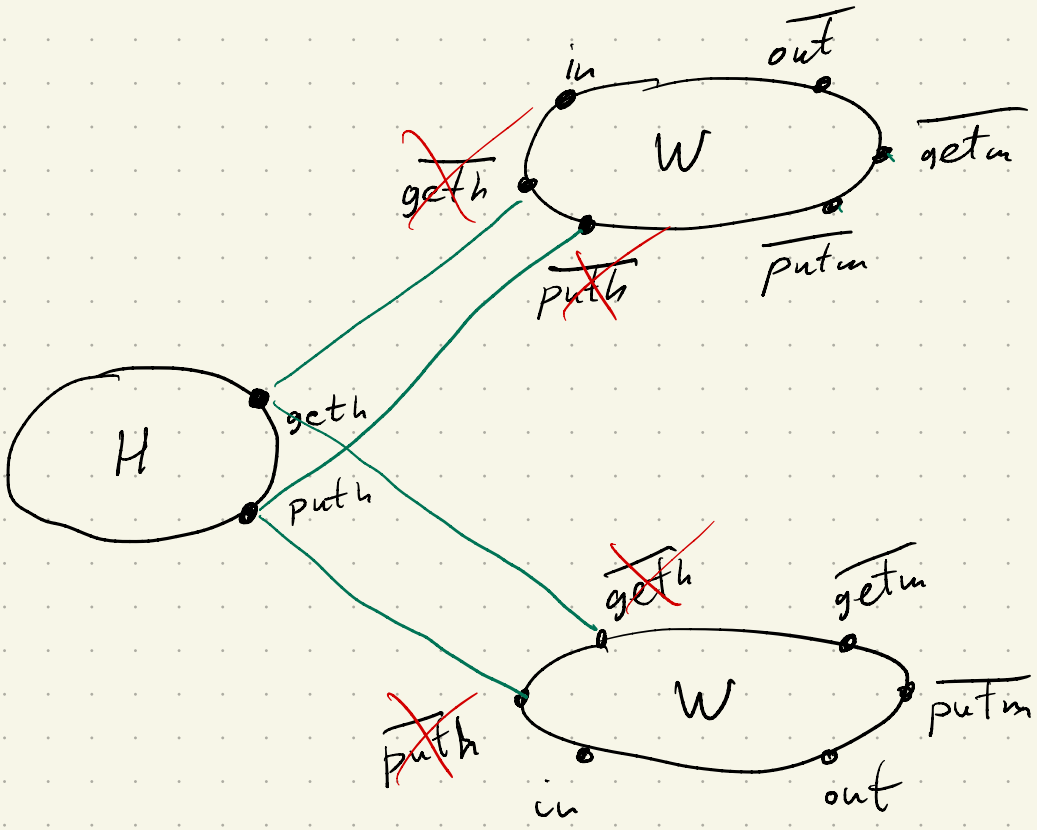
$H \mid W$

← parallel composition

$$(H \mid W) \mid W \equiv H \mid (W \mid W) \rightsquigarrow H \mid W \mid W$$

restriction ↓

$$(H \mid W \mid W) \setminus \{geth, puth\}$$



$$\left((H \mid W \mid W) \setminus \{geth, puth\} \mid M\right) \setminus \{getm, putm\}$$

$$\equiv$$

$$\equiv^{det} (H \mid W \mid W \mid M) \setminus \{geth, puth, getm, putm\}$$

$$M \overset{\text{def}}{=} H[get_m/get_h, put_m/put_h]$$

renaming

Important aspects:

- compositionality, black-box
- behaviour = communication
- equality = identity of behaviour

$$Q \overset{\text{def}}{=} in(a).\overline{out(a)}.Q + in(b).\overline{out(b)}.Q + in(c).\overline{out(c)}.Q$$

Question: $F \overset{?}{=} Q \mid Q$

$W' \overset{\text{def}}{=}$ like W but first $\overline{out}$ then $\overline{put_h}$ / $\overline{put_m}$

Question: $F \overset{?}{=} F'$

Transitions:

$(H \mid W \mid W \mid M) \setminus \{geth, puth, getw, putw\}$

$\overline{out(a)}$ ↑ , $in(a)$ ↓

$(H \mid \overline{out(a)}.W \mid W \mid M) \setminus \{geth, puth, getw, putw\}$

$in(a)$ ↙ , $in(b)$ ↓ , $in(c)$ ↘

$(H \mid \overline{out(a)}.W \mid \overline{geth}.\overline{puth}.\overline{out(b)}.W \mid M) \setminus \{\dots\}$

↓ ?

$(puth.H \mid \overline{out(a)}.W \mid \overline{puth}.\overline{out(b)}.W \mid M) \setminus \{\dots\}$

Operational semantics:

- actions $A = \{a, b, c, \dots\}$ co-actions $\overline{A} = \{\bar{a}, \bar{b}, \dots\}$

$\bar{\bar{a}} = a$

- silent action τ (internal communication)

$\Sigma = A \cup \overline{A} \cup \{\tau\}$

- renaming function $f : \Sigma \to \Sigma$ s.t.

$f(\bar{a}) = \overline{f(a)}$, $f(\tau) = \tau$

- restriction set $L \subseteq A \cup \overline{A}$
- inactive process 0 – deadlock

$$\frac{}{a.P \xrightarrow{a} P}$$

$$\frac{P \xrightarrow{a} P'}{P+Q \xrightarrow{a} P'} \qquad \frac{Q \xrightarrow{a} Q'}{P+Q \xrightarrow{a} Q'}$$

$$\frac{P \xrightarrow{a} P'}{A \xrightarrow{a} P'} \quad (A \stackrel{def}{=} P)$$

$$\frac{P \xrightarrow{a} P'}{P|Q \xrightarrow{a} P'|Q} \qquad \frac{Q \xrightarrow{a} Q'}{P|Q \xrightarrow{a} P|Q'}$$

$$\frac{P \xrightarrow{a} P' \quad Q \xrightarrow{\bar{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

$$\frac{P \xrightarrow{a} P'}{P \setminus L \xrightarrow{a} P' \setminus L} \quad (a, \bar{a} \notin L)$$

$$\frac{P \xrightarrow{a} P'}{P[f] \xrightarrow{f(a)} P'[f]}$$

$$\frac{P_i \xrightarrow{a} P'}{\sum_{i \in I} P_i \xrightarrow{a} P'}$$

$$0 \stackrel{def}{=} \sum_{i \in \phi} P_i$$

Derivation:

$$\frac{\dfrac{\dfrac{}{a.P \xrightarrow{a} P}}{a.P + b.0 \xrightarrow{a} P} \qquad \dfrac{}{\bar{a}.Q \xrightarrow{\bar{a}} Q}}{(a.P + b.0) \mid \bar{a}.Q \xrightarrow{\tau} P \mid Q}$$

$$\frac{((a.P + b.0) \mid \bar{a}.Q) \setminus a \xrightarrow{\tau} (P \mid Q) \setminus a}{}$$