Theory of concurrency

2023/24

Lecture 8

Confusion-free Petri nets

Assumptions: – $F(t,p), F(p,t) \in \{0,1\}$
$F \subseteq P \times T \cup T \times P$ (bipartite graph)
– connected

Confusion = interaction between conflict and concurrency

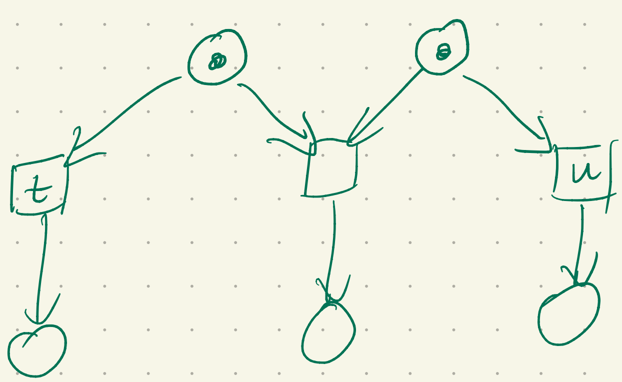
Def. $\mathrm{conf}(M,t) = \{u \in T : t$ and u are in conflict in $M\}$

$M \xrightarrow{t}$, $M \xrightarrow{u}$ $\quad {}^\bullet t \cap {}^\bullet u \neq \emptyset$

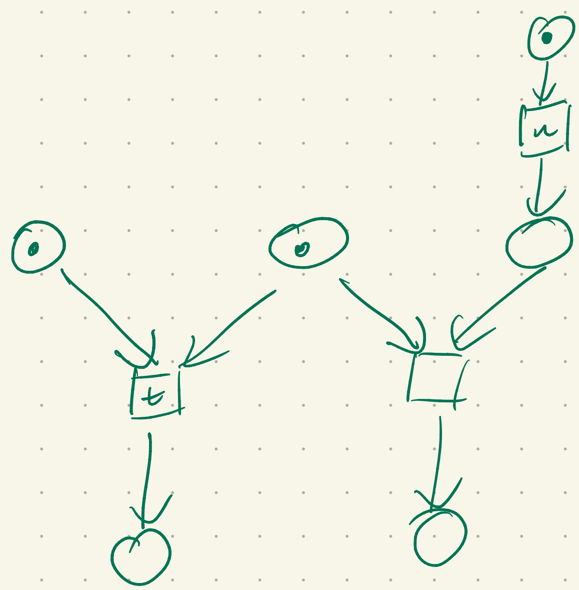
t, u are in confusion in M:

$M \xrightarrow{t}$, $M \xrightarrow{u}$ $\quad {}^\bullet t \cap {}^\bullet u = \emptyset$ $\quad$ execution of one of them changes $\mathrm{conf}(-)$ of the other

Examples:

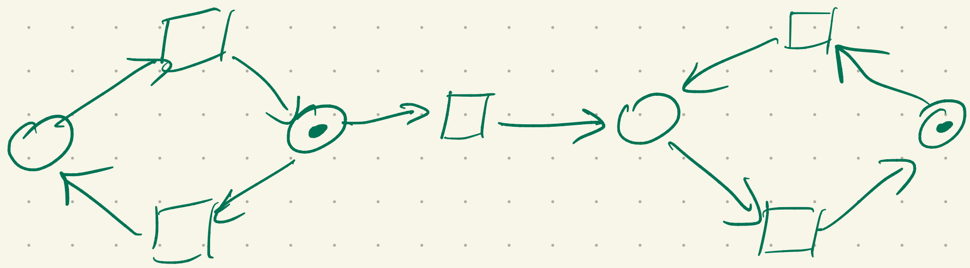


u decreases $\mathrm{conf}(t)$

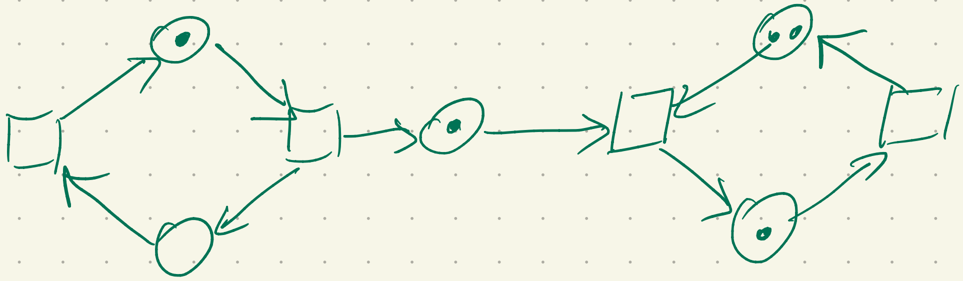


u increases $\mathrm{conf}(t)$

a) P-nets : $\forall t \in T \quad |{}^\bullet t| = |t^\bullet| = 1$

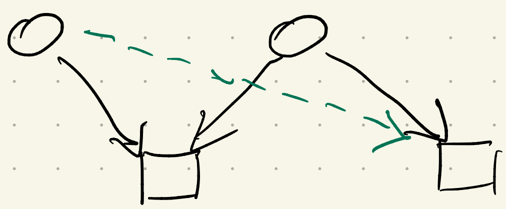


b) T-nets : $\forall p \in P \quad |{}^\bullet p| = |p^\bullet| = 1$



c) free-choice nets :

- $\forall p, q \in P. \quad p^\bullet \cap q^\bullet = \emptyset$ or $p^\bullet = q^\bullet$
- $\forall t, u \in T. \quad {}^\bullet t \cap {}^\bullet u = \emptyset$ or ${}^\bullet t = {}^\bullet u$
- $\forall p \in P, t \in T. \quad {}^\bullet t \times p^\bullet \subseteq F$



Fact : A P-net is live $\Longleftrightarrow$ strongly connected and nonempty

A T-net is live $\Longleftrightarrow$ every cycle is nonempty

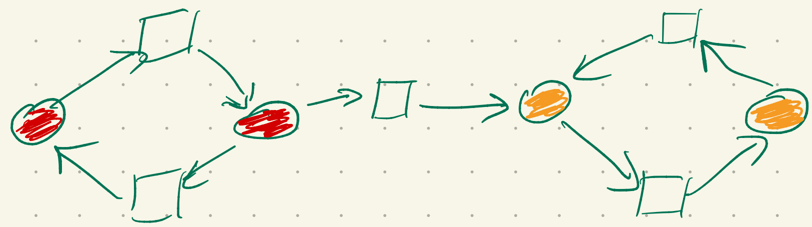
Theorem : A free-choice connected net is live $\Longleftrightarrow$ every proper siphon includes a nonempty trap

Def: $R \subseteq P$ is a *trap* if $R^\bullet \subseteq {}^\bullet R$

a *siphon* if ${}^\bullet R \subseteq R^\bullet$

R is *proper* if $R \neq \emptyset$

Example:



Fact: Empty siphon remains empty
Nonempty trap remains nonempty

Fact: Siphons and traps are closed under union.

Fact: In deadlock, the set of empty places is a siphon.

Corollary: A net is
deadlock-free $\Longleftarrow$ every proper siphon includes a nonempty trap

$t \in T$ is useful in M if $\exists M'$
$M \longrightarrow^* M' \xrightarrow{t}$

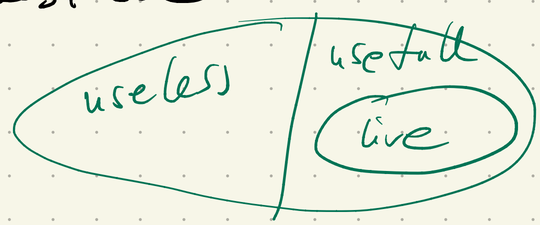
$t \in T$ is live in M if $\forall M'$
$M \longrightarrow^* M' \implies t$ useful in M'

Theorem (Commoner):
A free-choice net is live $\Longleftrightarrow$ every proper siphon includes a nonempty trap

Proof of $\Leftarrow$: Suppose a net is not live.

There is a reachable configuration M in which each transition is either useless or live.

at least one



Let $\mathcal{U} \subseteq T$ useless transitions in M. $\mathcal{U} \neq \emptyset$.
Let $L = T \setminus \mathcal{U}$ live transitions in M.

Claim: $\forall u \in \mathcal{U} \ \exists p_u \in {}^\bullet u \setminus L^\bullet$ empty in M.

Indeed, if all places in ${}^\bullet u$ are either in $L^\bullet$ or nonempty in M then, by free-choice, some configuration reachable from M enables u.

Claim: $R = \{ p_u : u \in \mathcal{U} \}$ is a proper siphon, empty in M.

In the initial configuration, R does not include a nonempty trap, as this trap would be still nonempty in M. $\square$

5

Proof of $\Rightarrow$: Suppose there is a proper siphon R s.t. the largest trap $Q \subseteq R$ is empty.

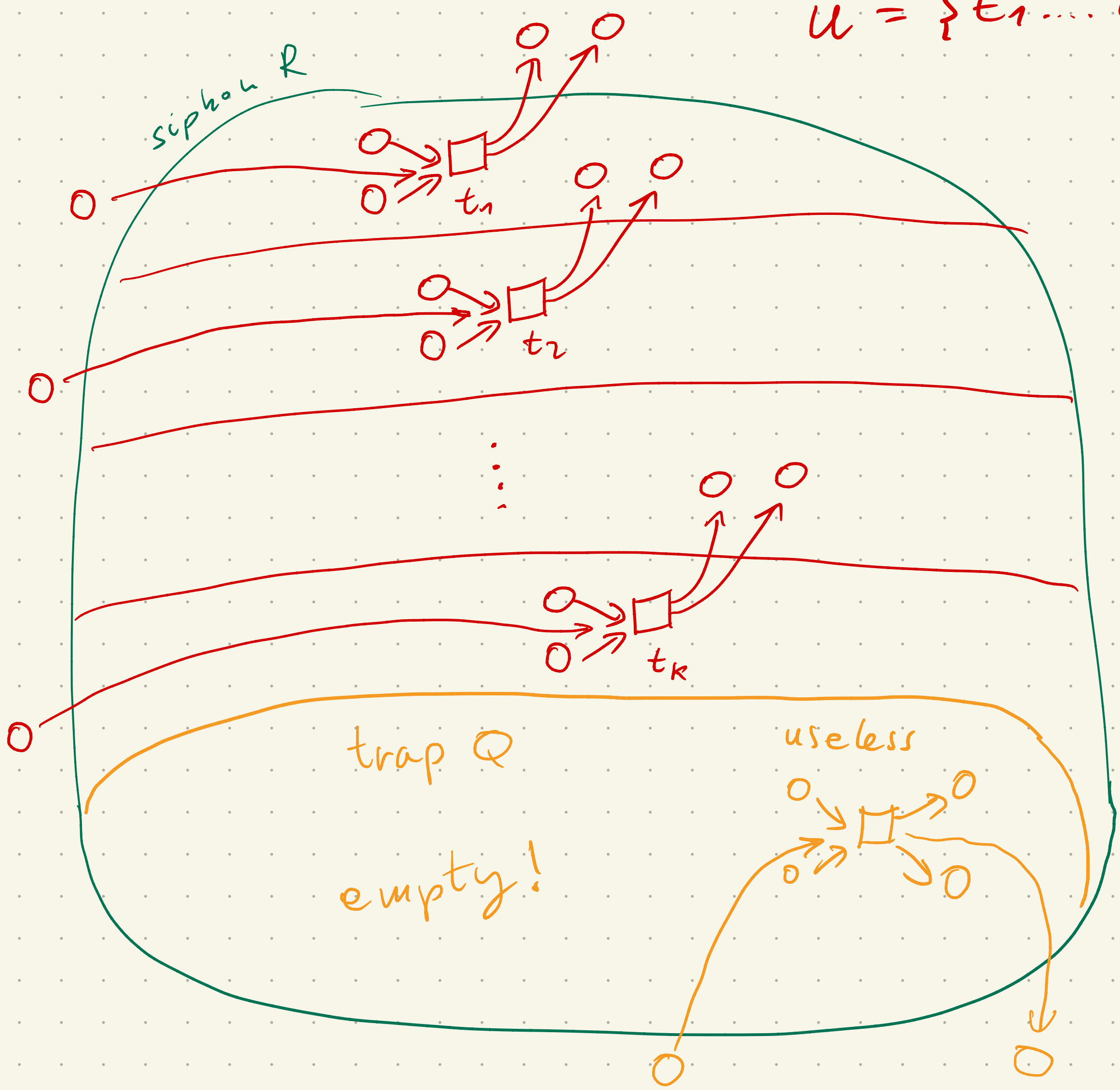
A) $Q = R \longrightarrow R^\bullet$ are useless in M

nonempty by connectedness

B) Construct $M \longrightarrow^* M'$: $\mathcal{U}$ are useless in M'.

$\mathcal{U} \subseteq R^\bullet$ nonempty

$\mathcal{U} = \{t_1 \dots t_k\}$



Repeat:

- $M \longrightarrow^* M' \xrightarrow{t} M''$
- $M := M''$

for some M' and $t \in \mathcal{U}$ s.t.

$M' =_R M$ (as ${}^\bullet R \subseteq R^\bullet$)

$\Rightarrow M'' <_R M$

<u>Corollary</u> : Liveness of free-choice nets is in co-NP.

It is co-NP-complete → $\boxed{\text{TUTORIALS}}$

<u>Question</u> : Can connectedness assumption be dropped? → $\boxed{\text{TUTORIALS}}$