Theory of concurrency

2023/24

Lecture 6

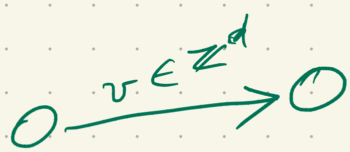
Petri nets reachability problem – – sketch of decidability proof

Input: A VASS and two configurations c, c'

Question: $c \longrightarrow^* c'$?

(q, v) (q', v')

VASS:



Brief history: .....

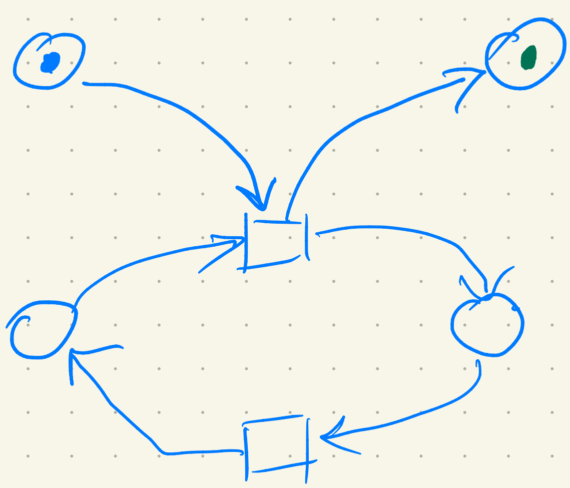
Pseudo runs: $c \dashrightarrow^* c'$

intermediate vectors may drop below 0

Question: $c \dashrightarrow^* c' \implies c \longrightarrow^* c'$?

No!

- initial
- final



Sufficient condition for $c \longrightarrow^* c'$:

Θ_1: For every $m \geq 1$, $(q,v) \dashrightarrow^* (q',v')$ using every transition $\geq m$ times. (UNBOUNDEDNESS)

implies strong connectedness

Θ_2: For some $\Delta, \Delta' \geq \vec{1}$,

a) $(q,v) \xrightarrow{\pi}{}^{*} (q, v+\Delta)$

b) $(q',v') \,{}^{*}\!\xleftarrow{\pi'} (q', v'+\Delta')$ (PUMPABILITY)

Claim: $(q',\Delta) \dashrightarrow^* (q',\Delta')$

Lemma: $\Theta_1 \wedge \Theta_2 \implies (q,v) \longrightarrow^* (q',v')$

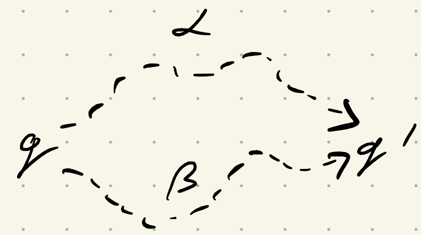
$$
\begin{tikzcd}
 & q', v'+n\Delta \arrow[dr, bend left, "\overset{\Theta_1}{\text{Claim}} \times n"] & \\
q, v+n\Delta \arrow[ur, bend left, "\Theta_1"] & & q', v'+n\Delta' \arrow[d, "\Theta_2 \times n", "*"'] \\
q, v \arrow[u, "*", "\Theta_2 \times n"'] & & q', v'
\end{tikzcd}
$$

effect of a pseudorun $e(\alpha)$: final vector − initial vector $\in \mathbb{Z}^d$

determines

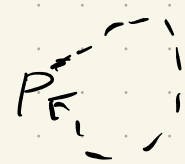
folding of a pseudorun $F(\alpha)$: $\in \mathbb{N}^T$

Observation: Two pseudoruns



$$F(\alpha) - F(\beta) \geq \vec{1}.$$

For every non-isolated control state p there is a pseudorun



γ s.t.

$$F(\gamma) = F(\alpha) - F(\beta).$$

Proof: (Euler cycle)

<u>Proof of the claim:</u> Pseudoruns (using Θ_1)

$$\begin{tikzcd}(q,v) \arrow[r, bend left, dashed, "\alpha"] \arrow[r, bend right, dashed, "\beta"'] & (q',v')\end{tikzcd}$$

such that

$$F(\alpha) - F(\beta) - F(\pi) - F(\pi') \geq \vec{1}$$

By Observation, there is a pseudorun $q' \circlearrowleft \gamma$
s.t.

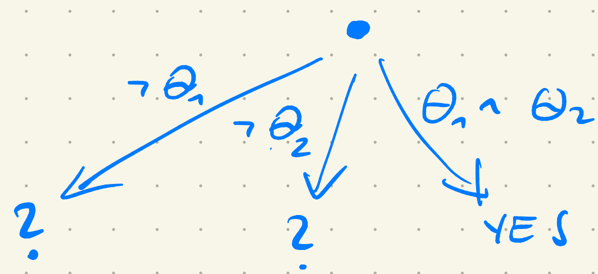
$$F(\gamma) = F(\alpha) - F(\beta) - F(\pi) - F(\pi')$$

$$e(\gamma) = \underbrace{e(\alpha) - e(\beta)}_{0} \underbrace{- e(\pi)}_{-\Delta} \underbrace{- e(\pi')}_{+\Delta'} = \Delta' - \Delta$$

Hence $(q', \Delta) \overset{\gamma}{\dashrightarrow}{}^{*} (q', \Delta')$

Θ_1 is effective $\longrightarrow$ $\boxed{\text{tutorials}}$

Θ_2 is effective — why?



What to do if Θ_1 or Θ_2 fails to hold?

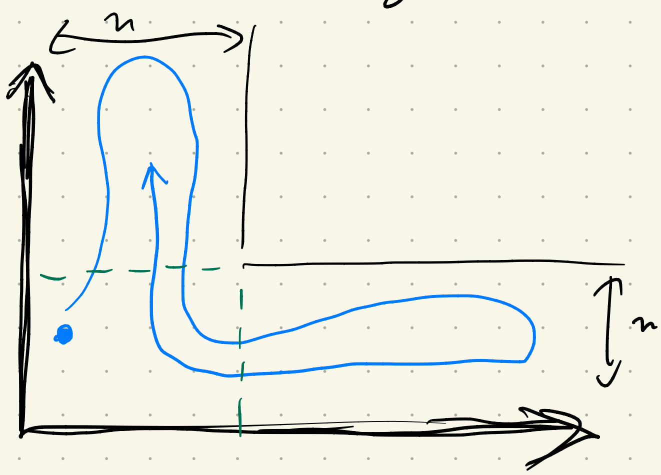
Θ_2 a) fails:

effectively computable — how?

$\exists n.$ every configuration reachable from (q, v) has some coordinate $< n$

(using coverability tree) $\Downarrow$ → $\boxed{\text{tutorials}}$

$\exists n.$ every run from (q, v), on some coordinate, is always $< n$

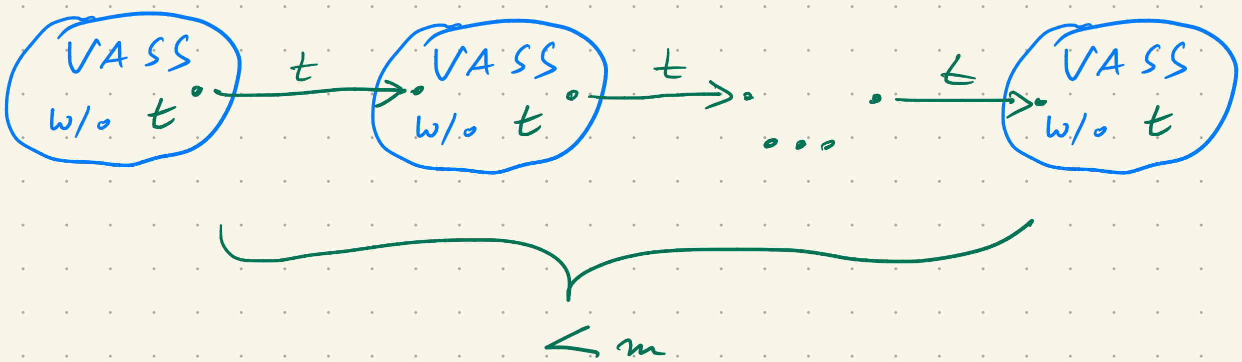


Reduction of dimension: try to bound each single dimension by n.

Θ_1 fails:

$\exists m.$ every pseudorun $(q,v) \dashrightarrow^* (q',v')$ uses some transition $< m$ times

? Reduction of the number of transitions: try to fit nr of usages of each transition t to each nr $< m$.



- We lose strong connectedness!
- Dimension is preserved, nr of transitions increases!