Teoria współbieżności

2023/24

Wykład 2

Elementary nets

$$S = (P, T, F, M_0)$$

P: conditions $\in \{0,1\}$

Configurations = subsets of places $M \subseteq P$

$$\underbrace{M \xrightarrow{t}}_{t \text{ enabled}} \iff {}^\bullet t \subseteq M,\ t^\bullet \cap M = \emptyset$$

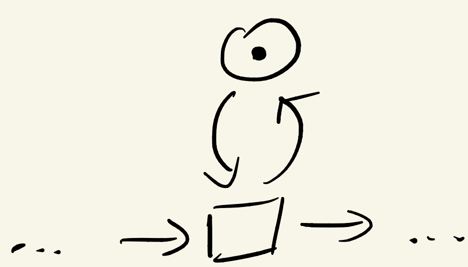
$$\left(t^\bullet \subseteq P \setminus M\right)$$

Graph of reachable configurations grct(S) : $M \xrightarrow{t} M'$ (steps)

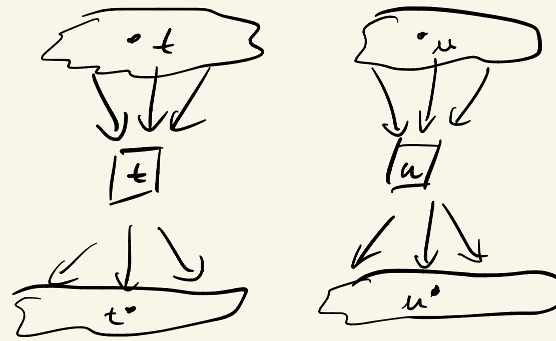
DFA without accepting states

Question: Is every DFA the configuration graph of some elementary net?

Assumption: no tight loops



Relations between transitions



- simultaneous enabledness

$$\exists\; \begin{tikzcd} & {} \\ M \arrow[ur, "t"] \arrow[dr, "u"'] & \\ & {} \end{tikzcd} \quad\Longrightarrow\quad \boxed{\begin{array}{l} {}^\bullet t \cap u^\bullet = \emptyset \\ t^\bullet \cap {}^\bullet u = \emptyset \end{array}} \qquad (J)$$

$(M = {}^\bullet t \cup {}^\bullet u)$

- sequential enabledness

$$\exists\; M \xrightarrow{tu} \quad\Longrightarrow\quad \boxed{\begin{array}{l} {}^\bullet t \cap {}^\bullet u = \emptyset \\ t^\bullet \cap u^\bullet = \emptyset \end{array}} \qquad (S)$$

$(M = {}^\bullet t \cup ({}^\bullet u \setminus t^\bullet))$

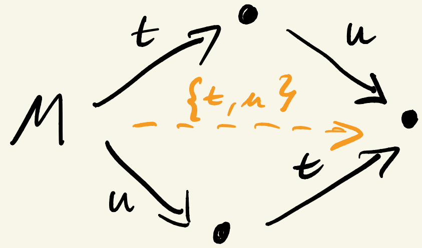
Fact:

$$\begin{tikzcd} & {} \\ M \arrow[ur, "t"] \arrow[dr, "u"'] & \\ & {} \end{tikzcd} \qquad M \xrightarrow{tu} \quad\Longrightarrow\quad M \xrightarrow{ut}$$

$$\begin{tikzcd} & \bullet \arrow[dr, "u"] & \\ M \arrow[ur, "t"] \arrow[dr, "u"'] & & \bullet \\ & \bullet \arrow[ur, dashed, "t"] & \end{tikzcd}$$

(A) concurrency: $(J) \wedge (S)$ $\quad {}^\bullet t^\bullet \cap {}^\bullet u^\bullet = \emptyset$

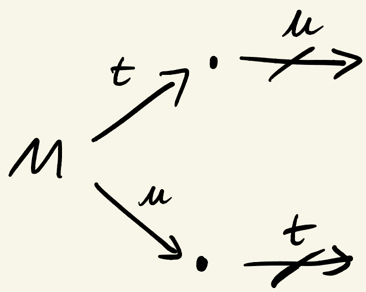
$(M = {}^\bullet t \cup {}^\bullet u)$



concurrent step

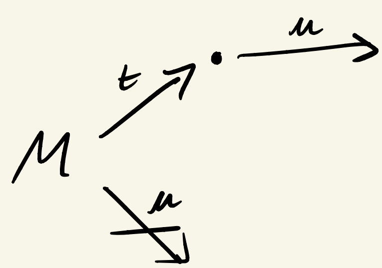
(B) conflict: $(J) \wedge \neg (S)$

$(M = {}^\bullet t \cup {}^\bullet u)$



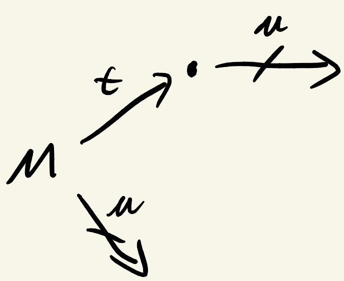
(C) causality (direct): $\neg (J) \wedge (S)$

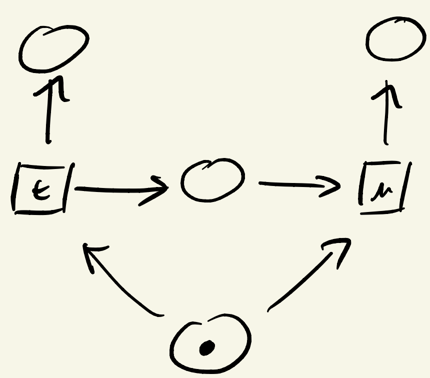
$(M = {}^\bullet t \cup {}^\bullet u \setminus {}^\bullet t)$



(D) $\neg (J) \wedge \neg (S)$

$(M = {}^\bullet t)$





Elementary nets vs 1-bounded nets

Contact : ${}^\bullet t \subseteq M \qquad t^\bullet \cap M \neq \emptyset$

$$\left(t^\bullet \not\subseteq P \setminus M \right)$$

elementary nets $\xrightarrow[\text{are introduces}]{\text{no tight loops}}$ contact-less elementary nets

$\|$

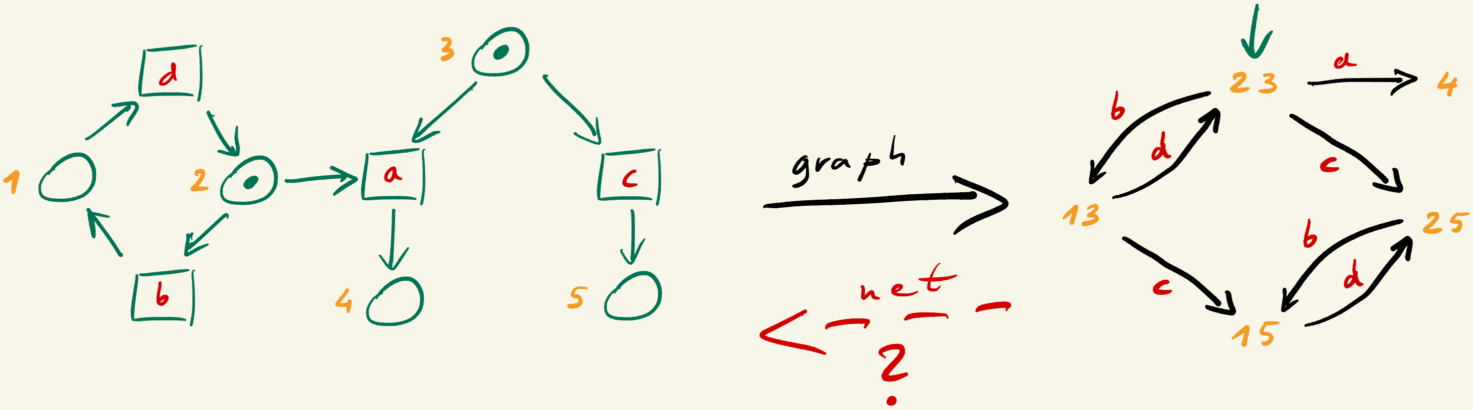
1-bounded general nets

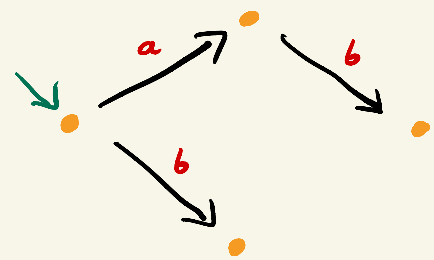
dual places

${}^\bullet \bar{p} = p^\bullet$

$\bar{p}^\bullet = {}^\bullet p$

Reconstruction (synthesis) problem





Question:

Is given NFA isomorphic to the configuration graph of some elementary net?
(no tight loops)

Answer: Exactly when

$$G \simeq \mathrm{graph}(\mathrm{net}(G))$$

Region $\approx$ all configuration containing a given place

<u>Def</u>: Graph $G = (V, \delta : V \times T \rightharpoonup V, v_0)$

$R \subseteq V$ is a <u>region</u> if $\forall a \in T$, all a-edges:

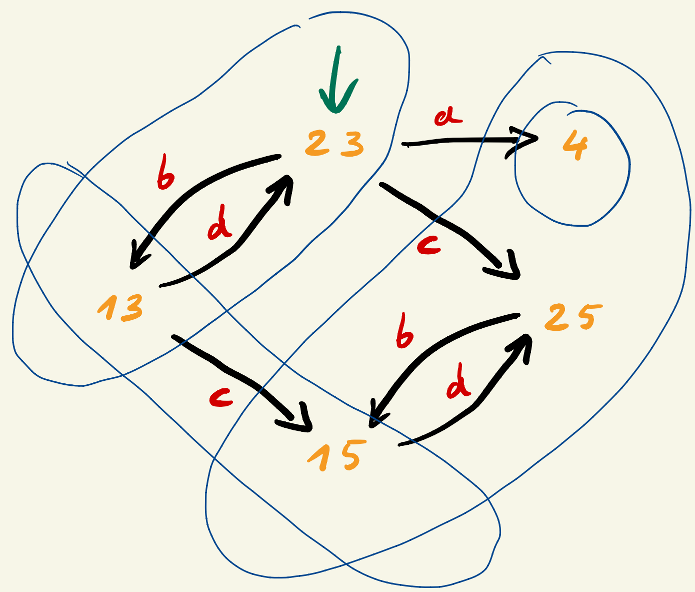
$R \neq \emptyset, R \neq V$

(1) either go from R to $V \setminus R$

(2) either go from $V \setminus R$ to R

(3) either go from R to R or from $V \setminus R$ to $V \setminus R$

Region in graph(S) is a subset of reachable configurations of S

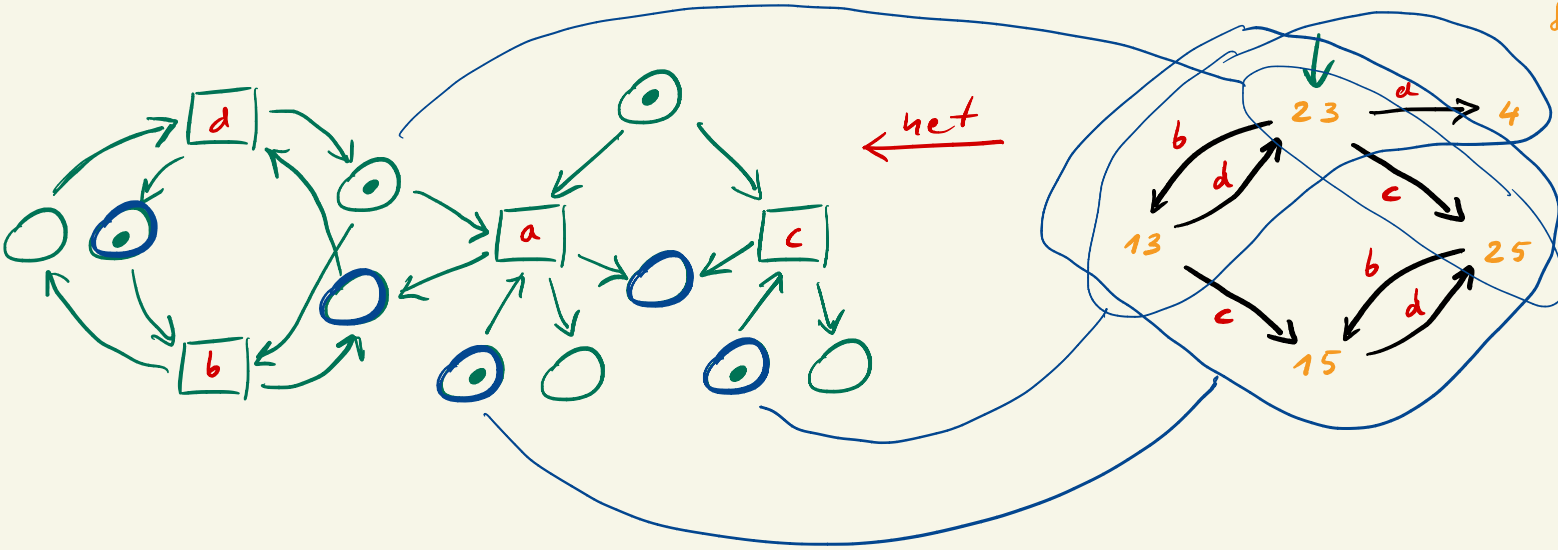


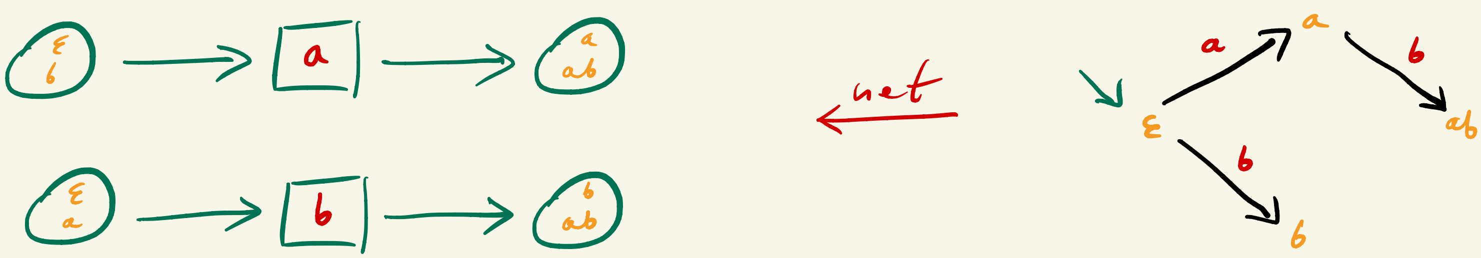
$^*a := \{ R : (1) \}$

$a^* := \{ R : (2) \}$

net (G) – region net

- places = regions
- transitions = T
- arcs : $^*a$ a^*
- initial configuration : $\{ R : v_0 \in R \}$





Lemma: $G \simeq \mathrm{graph}(S)$ for some net S

$$\Updownarrow$$

$G \simeq \mathrm{graph}(\mathrm{net}(G))$

Proof: $(\Downarrow)$

As long as some region R has no corres-ponding place in S, add R to S.

Fact: R - region in $\mathrm{graph}(S)$.

$S+R$: add place p and arcs (p,t) : $R \in {}^*t$

(t,p) : $R \in t^*$

Then $\mathrm{graph}(S) \simeq \mathrm{graph}(S+R)$

$${}^*R = {}^\circ p \qquad R^* = p^\circ$$