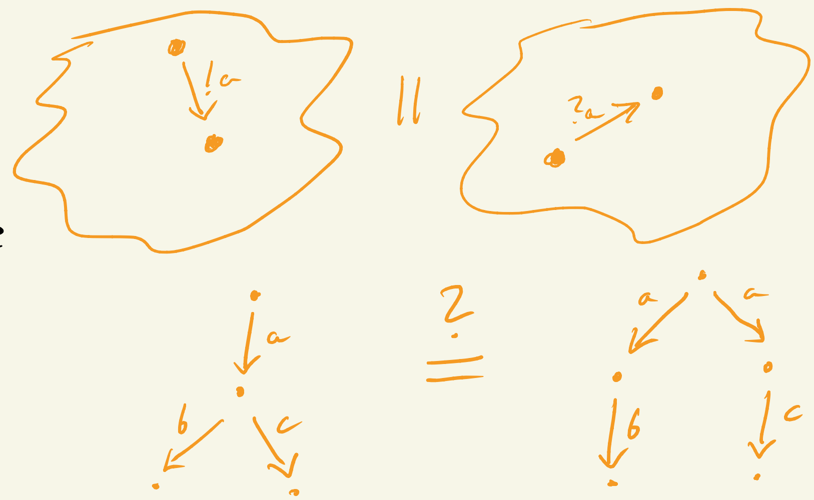
Teoria współbieżności

2023/24

Wykład 1

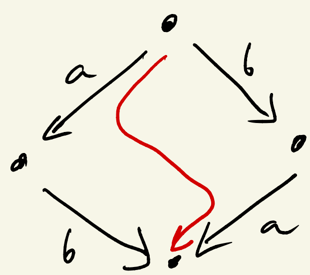
Mathematical models of Concurrent systems:

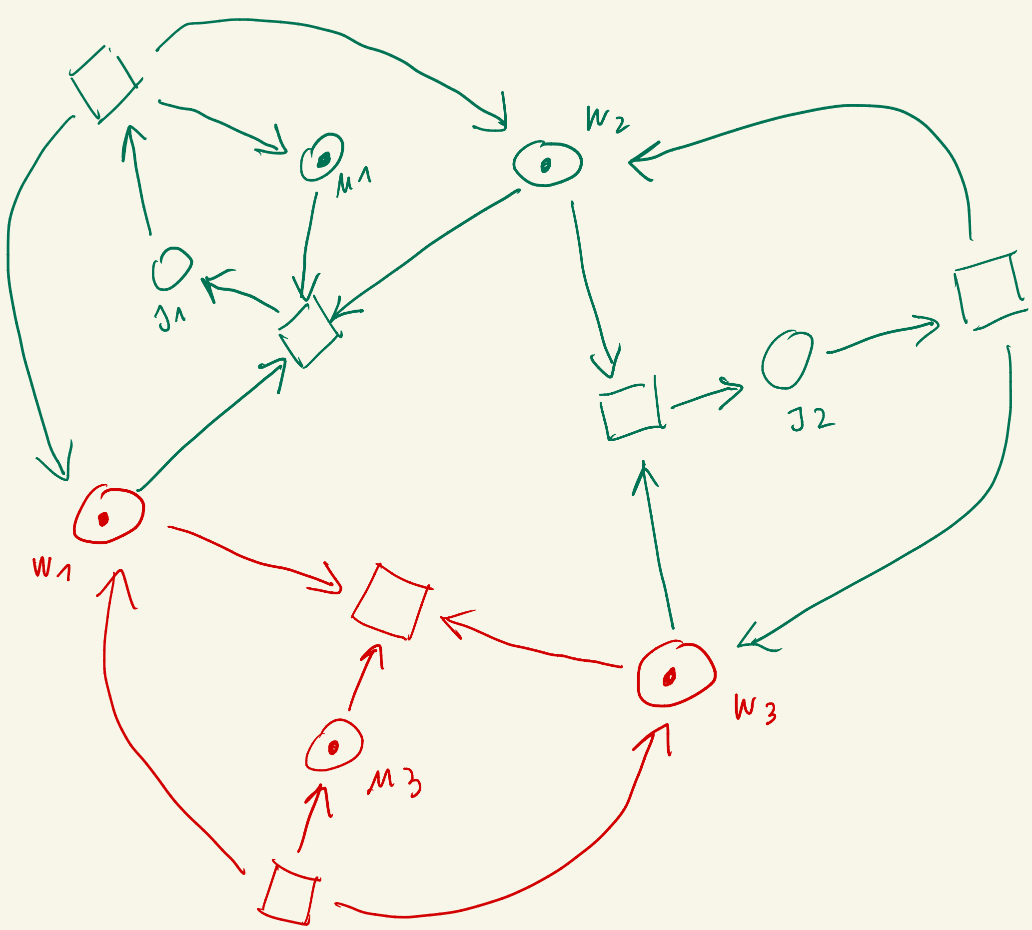
- Petri nets
- process algebra



- partially commutative languages
- geometric models

$a \parallel b$





Petri net :

- P, T – finite sets of places and transitions
- $F : P \times T \cup T \times P \longrightarrow \mathbb{N}$ arcs / weights
- $K : P \longrightarrow \{1, \omega\}$ capacity

Notation : $\bullet t = \{p : F(p,t) > 0\}$, $t^\bullet$, ${}^\bullet p$, $p^\bullet$, ${}^\bullet t^\bullet$, ${}^\bullet p^\bullet$

Configurations : $M : P \longrightarrow \mathbb{N}$ $\quad \forall p.\ M(p) \leq K(p)$

Step : $M \xrightarrow{t} M'$ o ile determined by M, t

- $\forall p \in {}^\bullet t \quad F(p,t) \leq M(p)$
- $\forall p \in {}^\bullet t^\bullet \quad M'(p) = M(p) - F(p,t) + F(t,p)$

Configuration graph $\quad M \overset{t}{\underset{u}{\rightrightarrows}} M'$

Notation : $M \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \xrightarrow{t_3} M_3$

$M \xrightarrow{t_1 t_2 t_3} M' \qquad M \xrightarrow{w} M'$ a run

$M \longrightarrow M' \qquad M \xrightarrow{t} \qquad M \longrightarrow$

blokada $M \not\longrightarrow$

$M \longrightarrow^* M'$ osiągalna z M

Elementary nets :

- $F(p,t), F(t,p) \leq 1$
- $K(p) = 1$

General nets :

- ?
- $K(p) = \omega$

Properties of Petri nets

Structure	vs	Dynamics
(net)		(net + initial conf. M_0)

- k-bounded $M(p) \leq k \quad \forall p, \forall M$ reachable
- bounded

$\Uparrow$

- terminating – only finite runs
- structurally bounded
  (for every choice of initial configuration)
- acyclic, reversible, conflict-free
- live:

$$\underbrace{\forall t \quad \forall M \quad M_0 \xrightarrow{}{}^{*} M \Rightarrow \underbrace{\exists M' \quad M \xrightarrow{}{}^{*} M' \xrightarrow{t}}_{t \text{ useful in } M}}_{t \text{ live}}$$

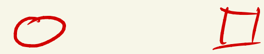
Decision problems:

given a net and a configuration M_0, M,

- $M_0 \xrightarrow{}{}^{*} M$ reachability
- $M_0 \xrightarrow{}{}^{*} M'$ for some $M' \sqsupseteq M$ coverability

Patologies :

- isolated elements

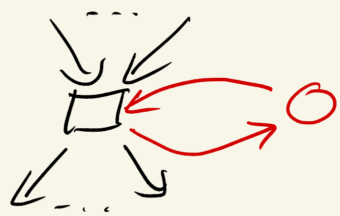


- non-connected ?
- redundant elements

- no pre/post : ${}^\bullet t = \emptyset \quad t^\bullet = \emptyset \quad {}^\bullet p = \emptyset \quad p^\bullet = \emptyset$

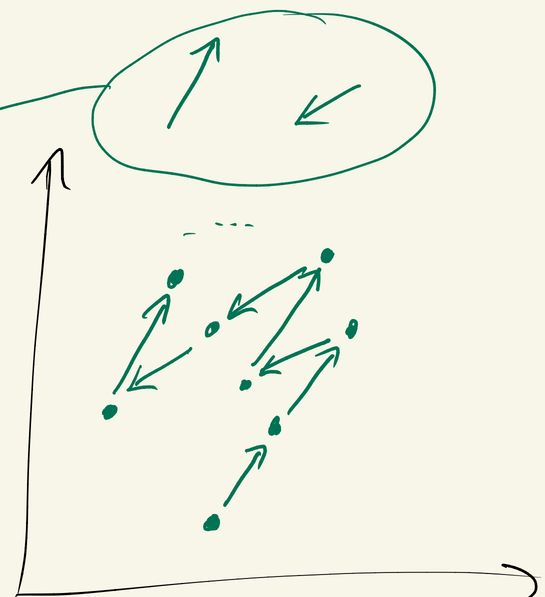
Elimination :

- tight loops



- weights > 1

Equivalent models :



- VAS $\quad T \subseteq_{fin} \mathbb{Z}^d$

  configurations : $\mathbb{N}^d$

  steps : $c \xrightarrow{t} c+t$

- VASS $\quad q \xrightarrow{t} q'$

  configurations : $Q \times \mathbb{N}^d$

  steps : $(q,c) \xrightarrow{t} (q', c+t)$

- nondeterministic automata with counters $x_1 \dots x_d$ (no 0-tests)

$q \xrightarrow{x_i{+}{+}} q' \qquad q \xrightarrow{x_i{-}{-}} q'$