Teoria współbieżności

2022/23

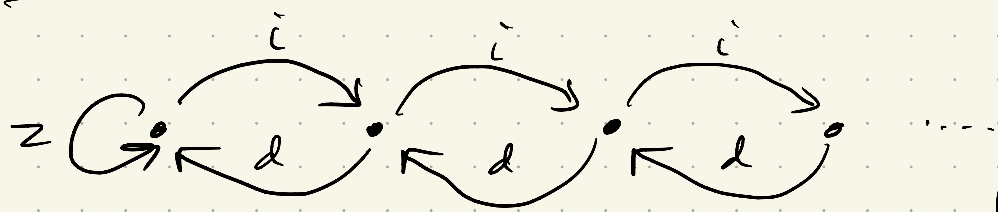
Wykład 14

Undecidability

- CCS is Turing-complete
- bisimulation equivalence for PN

labelled

Counter in CCS:



?

$$A \stackrel{\text{def}}{=} i(A \mid d.0)$$

$$Z \stackrel{\text{def}}{=} z.Z + i.(A \mid a.Z)\setminus a$$

$$A \stackrel{\text{def}}{=} d.\bar{a}.0 + i.(B \mid b.A)\setminus b$$

$$B \stackrel{\text{def}}{=} d.\bar{b}.0 + i(A \mid a.B)\setminus a$$

$$Z \xrightarrow{i} (A \mid a.Z)\setminus a \xrightarrow{i} ((B \mid b.A)\setminus b \mid a.Z)\setminus a \xrightarrow{i} (((A \mid a.B)\setminus a \mid b.A)\setminus b \mid a.Z)\setminus a$$

$$(A \mid a.Z)\setminus a \xrightarrow{d} (\bar{a}.0 \mid a.Z)\setminus a \xrightarrow{\tau} (0 \mid Z)\setminus a$$

$$((B \mid b.A)\setminus b \mid a.Z)\setminus a \xrightarrow{d} ((\bar{b}.0 \mid b.A)\setminus b \mid a.Z)\setminus a \xrightarrow{\tau} ((0 \mid A)\setminus b \mid a.Z)\setminus a$$

$$((0 \mid A)\setminus b \mid a.Z)\setminus a \xrightarrow{d} ((0 \mid \bar{a}.0)\setminus b \mid a.Z)\setminus a \xrightarrow{\tau} ((0 \mid 0)\setminus b \mid Z)\setminus a$$

Symulacja maszyny licznikowej:

i : c++ ; goto j $\qquad S_i \stackrel{def}{=} \overline{i_c} . S_j$

i : if c==0 then
    goto j
  else
    c--
    goto k

$$S_i \stackrel{def}{=} \overline{z_c} . S_j + \overline{d_c} . S_k$$

i : halt $\qquad S_i \stackrel{def}{=} 0$

$$M \stackrel{def}{=} \left(S_1 \mid Z\,[z_c/z, i_c/i, d_c/d] \mid Z\,[z_d/z, i_d/i, d_d/d] \right) \setminus \{z_c, z_d, i_c, i_d, d_c, d_d\}$$

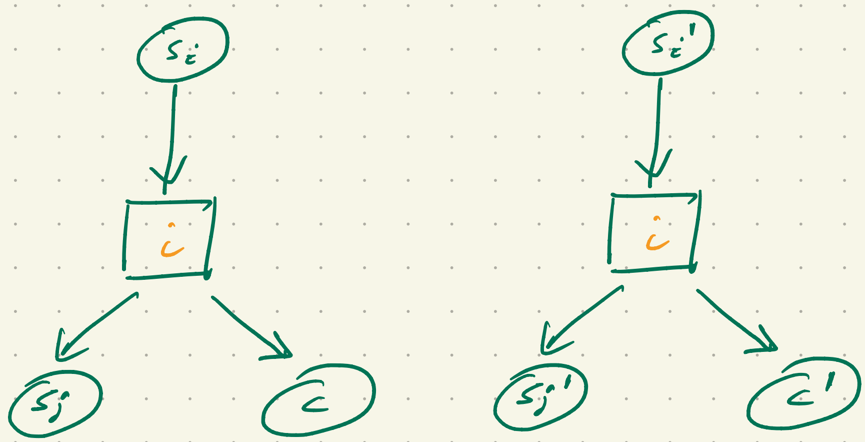
Theorem: deadlock-freeness is undecidable for CCS.

Question: what about $\sim$?

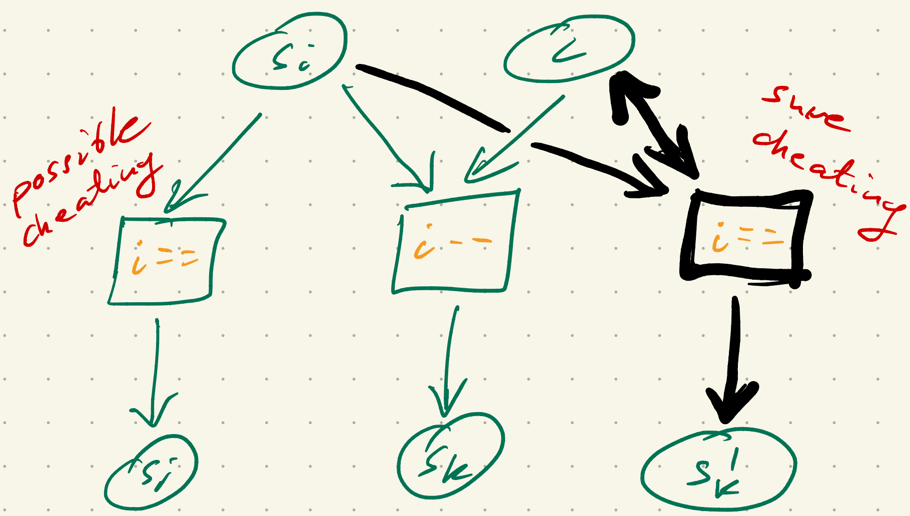
Undecidability of $\sim$ for $\boxed{\text{labelled}}$ PN

start :

i : c++ ; goto j



i : if c==0 then
goto j
else
c--
goto k



i : halt

<u>Claim</u> : The counter machine <u>does not halt</u> iff Duplicator wins.

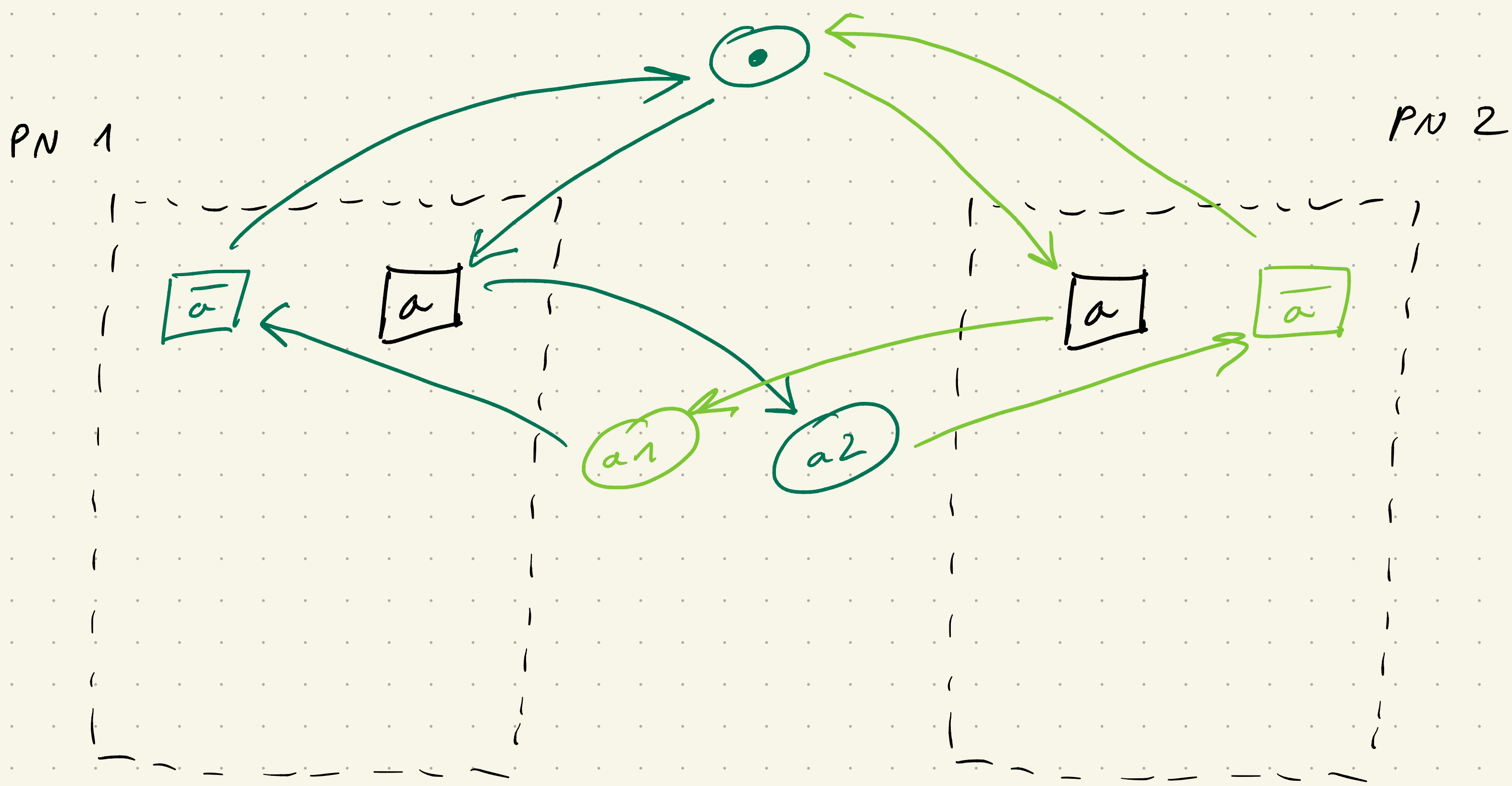
= loops

<u>Theorem</u> : $\sim$ is undecidable for PN.

Language equality also.

Decidability of $\sim$ for $\boxed{\text{unlabelled}}$ PN

↳ reduction to reachability



<u>Claim</u>: PN 1 $\sim$ PN 2

iff

no <u>bad</u> configuration is reachable

a token on some place a1 or a2
but no $\bar{a}$ transition enabled