Teoria współbieżności

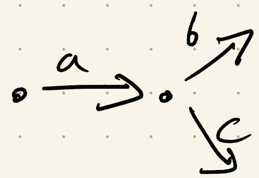
2022/23

Wykład 11

(Observational) Equality of processes

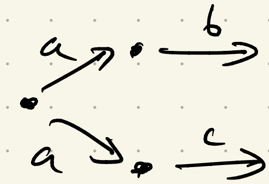
- CCS $\quad (P_1[f_1] \mid \ldots \mid P_n[f_n]) \setminus L$

- <u>strong</u> bisimulation equivalence $\sim$



$\neq$

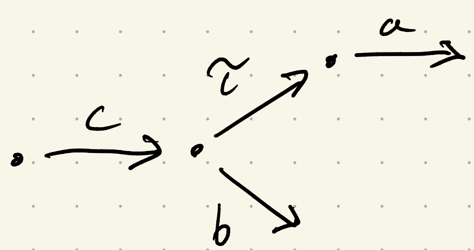
$\not\sim$

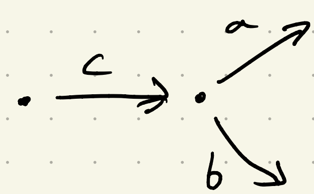


- <u>weak</u> bisimulation equivalence $\approx$

$$\tau . a \stackrel{?}{=} a$$

$$\tau . a + b \stackrel{?}{=} a + b$$





Definition: weak bisimulation S

$(a \neq \tau)$

$$\begin{tikzcd}
P \arrow[r, dash, dashed, "S"] \arrow[ddd, "a"'] & Q \arrow[d, "\tau^*"] \\
 & {} \arrow[d, "a"] \\
 & {} \arrow[d, "\tau^*"] \\
P' \arrow[r, dash, dashed, "S"] & Q'
\end{tikzcd}$$

$(a = \tau)$

$$\begin{tikzcd}
P \arrow[r, dash, dashed, "S"] \arrow[d, "\tau"'] & Q \arrow[d, "\tau^*"] \\
P' \arrow[r, dash, dashed, "S"] & Q'
\end{tikzcd}$$

(any a)

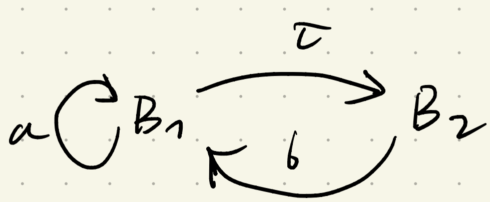
$$\begin{tikzcd}
P \arrow[r, dash, dashed, "S"] \arrow[d, "a"'] & Q \arrow[d, Rightarrow, "\hat{a}"] \\
P' \arrow[r, dash, dashed, "S"] & Q'
\end{tikzcd}$$

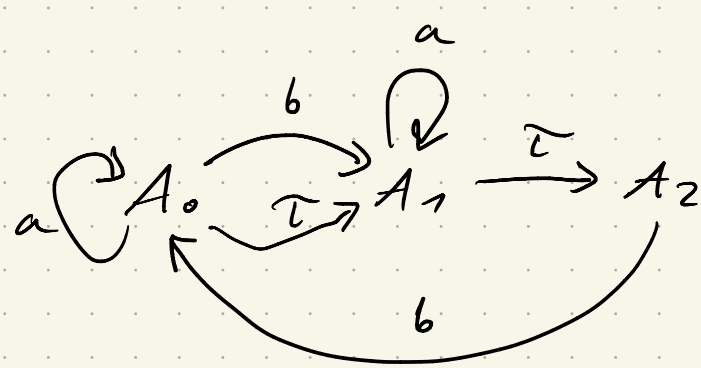
$$\xrightarrow{a_1 \cdots a_n} \;=\; \xrightarrow{a_1} \circ \dots \circ \xrightarrow{a_n}$$

$$\overset{a_1 \cdots a_n}{\Longrightarrow} \;=\; \left(\xrightarrow{\tau}\right)^* \circ \xrightarrow{a_1} \circ \dots \circ \left(\xrightarrow{\tau}\right)^* \circ \xrightarrow{a_n} \circ \left(\xrightarrow{\tau}\right)^*$$

$\Sigma^* \ni w \mapsto \hat{w}$ – removes τ's

Example:





$$S = \{(B_1, A_0),\ (B_1, A_1),\ (B_2, A_2)\}$$

Example:

$\{(\tau . P, P)\} \cup \mathrm{Id}$

Definition: weak bisimulation equivalence

$$\approx \; = \bigcup \{ S : S \text{ \underline{weak} bisimulation} \}$$

Question: is $\approx$ transitive?

$$\begin{tikzcd} P \arrow[r, dash, dashed, "S"] \arrow[d, Rightarrow, "\hat{a}"'] & Q \arrow[d, Rightarrow, "\hat{a}"] \\ P' \arrow[r, dash, dashed, "S"] & Q' \end{tikzcd}$$

optional

Question: What about

$$\begin{tikzcd} P \arrow[r, dash, dashed, "S"] \arrow[d, Rightarrow, "a"'] & Q \arrow[d, Rightarrow, "a"] \\ P' \arrow[r, dash, dashed, "S"] & Q' \end{tikzcd}$$

?

Bisimulation as a proof method

Definition: weak bisimulation up to $\approx$

$$\begin{tikzcd} & P \arrow[r, dash, dashed, "S"] \arrow[dl, "a"'] & Q \arrow[dr, Rightarrow, "\hat{a}"] & \\ P' \approx P'' \arrow[rr, dash, dashed, "S"] & & Q'' & \approx Q' \end{tikzcd}$$

Wrong definition!

Example:

$$S = \{ (\tau.a.0, \; 0) \}$$

$a \neq \tau$

Correction:

$$\begin{tikzcd} & P \arrow[r, dash, dashed, "S"] \arrow[dl, Rightarrow, "a"'] & Q \arrow[dr, Rightarrow, "\hat{a}"] & \\ P' \approx P'' \arrow[rr, dash, dashed, "S"] & & Q'' & \approx Q' \end{tikzcd}$$

$$\begin{tikzcd} & P \arrow[r, dash, dashed, "S"] \arrow[dl, "a"'] & Q \arrow[dr, Rightarrow, "\hat{a}"] & \\ P' \sim P'' \arrow[rr, dash, dashed, "S"] & & Q'' & \approx Q' \end{tikzcd}$$

Question: Is $\approx$ a congruence?

$$P \approx P' \overset{?}{\Longrightarrow}$$

- $a.P \approx a.P'$
- $P+Q \approx P'+Q$
- $P \mid Q \approx P' \mid Q$
- $P \setminus L \approx P' \setminus L$
- $P[f] \approx P'[f]$

$\tau.a \approx a$ but

$\tau.a + b \not\approx a + b$

What about

$$a \neq \tau \qquad \begin{array}{ccc} P & \overset{\approx}{\dashrightarrow} & Q \\ {\scriptstyle \hat{a}}\Downarrow & & \Downarrow{\scriptstyle \hat{a}} \\ P' & \overset{\approx}{\dashrightarrow} & Q' \end{array} \quad ?$$

$\tau.a + \tau \approx a$ but

$b.(\tau.a+\tau) \not\approx b.a$

$b \mid (\tau.a+\tau) \not\approx b \mid a$

Separating examples?

$$\boxed{\sim \quad \subsetneq \quad = \quad \subsetneq \quad \approx}$$

Definition: Observational equality $=$

$P = Q \iff \forall a \in \Sigma$

- if $P \xrightarrow{a} P'$ then, for some Q', $Q \overset{a}{\Longrightarrow} Q'$, $P' \approx Q'$
- if $Q \xrightarrow{a} Q'$ then, for some P', $P \overset{a}{\Longrightarrow} P'$, $P' \approx Q'$

Lemma:
$P = Q \iff \forall R.\ P+R \approx Q+R$

(the greatest congruence included in $\approx$)

Examples: $\tau.a \neq a$

$$a.\tau.P = a.P$$

$$P + \tau.P = \tau.P$$

$$a.(P+\tau.Q) + a.Q = a.(P+\tau.Q)$$

Question: $=$ or $\approx$?

$$P \mid Q \approx P \mid \tau.Q$$

$$\approx \quad \| $$

$$\tau.(P \mid Q)$$

- Weak bisimulation may be used to prove $=$:

Lemma: $P \approx Q \implies a.P = a.Q$

Lemma: $P \not\xrightarrow{\tau}$, $Q \not\xrightarrow{\tau}$, $P \approx Q \implies P = Q$

- $\approx$ is very close to $=$:

Lemma: $P \approx Q \iff P = Q$ or $P = \tau.Q$ or $\tau.P = Q$