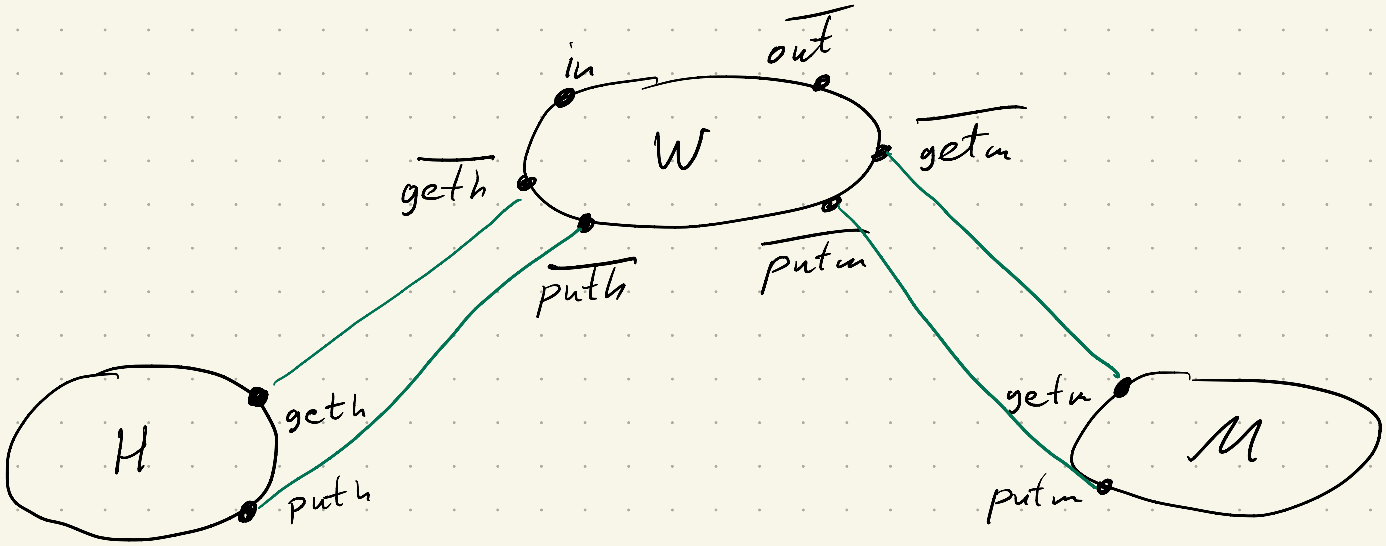
Teoria współbieżności

2022/23

Wykład 9

Process algebra CCS

(Calculus of Communicating Systems)



recursive def.

prefix

$H \stackrel{def}{=} geth \,.\, puth \,.\, H$ $\qquad$ $M \stackrel{def}{=} getm \,.\, putm \,.\, M$

summation (choice)

$$
\begin{aligned}
W \stackrel{def}{=}\; & in(a) \,.\, \overline{out}(a) \,.\, W \\
& + \\
& in(b) \,.\, \overline{geth} \,.\, \overline{puth} \,.\, \overline{out}(b) \,.\, W \\
& + \\
& in(c) \,.\, (\, \overline{geth} \,.\, \overline{puth} \,.\, \overline{out}(c) \,.\, W \\
& \qquad + \\
& \qquad \overline{getm} \,.\, \overline{putm} \,.\, out(c) \,.\, W \\
&)
\end{aligned}
$$

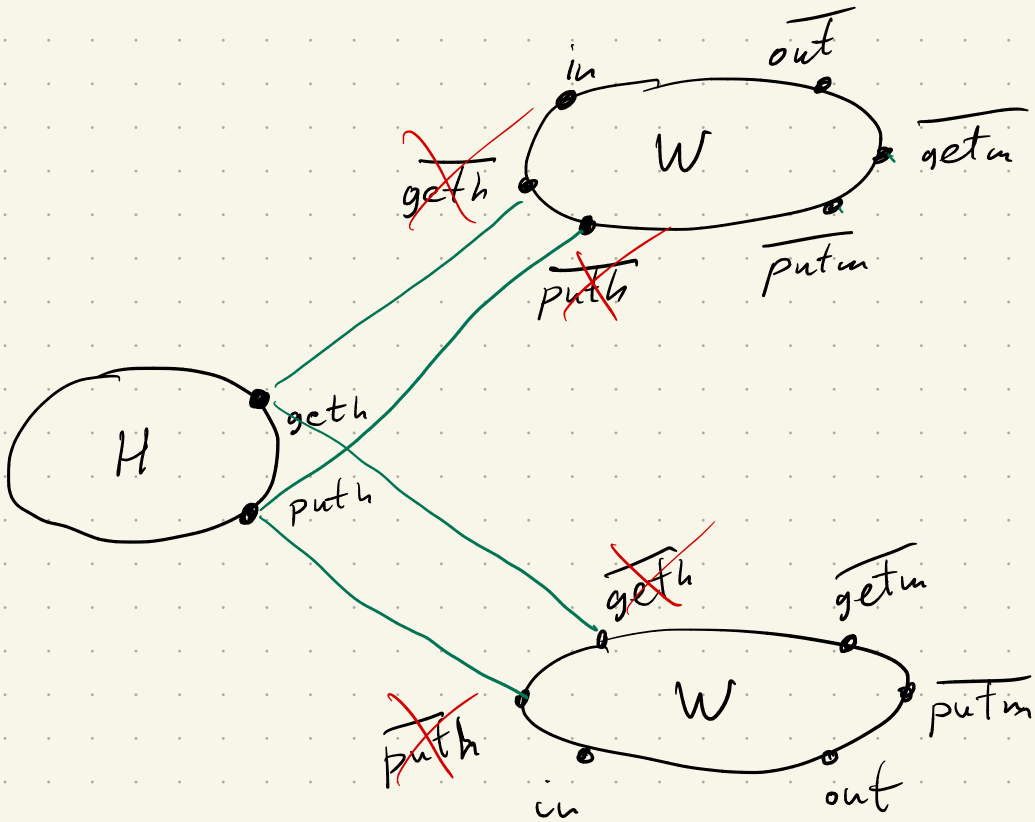
$H \mid W$

parallel composition

$$(H \mid W) \mid W = H \mid (W \mid W) \rightsquigarrow H \mid W \mid W$$

restriction

$$(H \mid W \mid W) \setminus \{geth, puth\}$$



$$((H \mid W \mid W) \setminus \{geth, puth\} \mid M) \setminus \{getm, putm\}$$

$$=$$

$$F \stackrel{def}{=} (H \mid W \mid W \mid M) \setminus \{geth, puth, getm, putm\}$$

$$M \overset{def}{=} H\left[{}^{getm}/_{geth},\ {}^{putm}/_{puth}\right]$$

renaming

Important aspects:

- compositionality, black-box
- behaviour = communication
- equality = identity of behaviour

$$Q \overset{def}{=} in(a).\overline{out}(a).Q + in(b).\overline{out}(b).Q + in(c).\overline{out}(c).Q$$

Question: $F \overset{?}{=} Q \mid Q$

$W' \overset{def}{=}$ like W but first $\overline{out}$ then $\overline{puth}$ / putm

Question: $F \overset{?}{=} F'$

Transitions:

$$(H \mid W \mid W \mid M) \setminus \{geth, puth, getm, putm\}$$

$$\xrightarrow{\overline{out}(a)} \qquad \downarrow in(a)$$

$$(H \mid \overline{out}(a).W \mid W \mid M) \setminus \{geth, puth, getm, putm\}$$

$$\swarrow in(a) \qquad \downarrow in(b) \qquad \searrow in(c)$$

$$(H \mid \overline{out}(a).W \mid geth.\overline{puth}.\overline{out}(b).W \mid M) \setminus \{ \dots \}$$

internal communication

Operational semantics:

- actions $\mathcal{A} = \{a, b, c, \dots\}$ co-actions $\overline{\mathcal{A}} = \{\overline{a}, \overline{b}, \dots\}$

  $\overline{\overline{a}} = a$
- silent action τ $\Sigma = \mathcal{A} \cup \overline{\mathcal{A}} \cup \{\tau\}$
- renaming function $f : \Sigma \to \Sigma$ s.t.

  $f(\overline{a}) = \overline{f(a)}$, $f(\tau) = \tau$
- restriction set $L \subseteq \mathcal{A} \cup \overline{\mathcal{A}}$
- inactive process 0 – deadlock

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$$\frac{}{a.P \xrightarrow{a} P}$$

+

$$\frac{P \xrightarrow{a} P'}{P+Q \xrightarrow{a} P'} \qquad \frac{Q \xrightarrow{a} Q'}{P+Q \xrightarrow{a} Q'}$$

$$\frac{P \xrightarrow{a} P'}{A \xrightarrow{a} P'} \quad (A \stackrel{def}{=} P)$$

$$\frac{P \xrightarrow{a} P'}{P|Q \xrightarrow{a} P'|Q} \qquad \frac{Q \xrightarrow{a} Q'}{P|Q \xrightarrow{a} P|Q'}$$

$$\frac{P \xrightarrow{a} P' \quad Q \xrightarrow{\bar{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

|

$$\frac{P \xrightarrow{a} P'}{P \setminus L \xrightarrow{a} P' \setminus L} \quad (a, \bar{a} \notin L)$$

$$\frac{P \xrightarrow{a} P'}{P[f] \xrightarrow{f(a)} P'[f]}$$

$$\frac{P_i \xrightarrow{a} P'}{\sum_{i \in I} P_i \xrightarrow{a} P'} \qquad \qquad 0 \stackrel{def}{=} \sum_{i \in \emptyset} P_i$$

Derivation:

$$\dfrac{\dfrac{\dfrac{\dfrac{}{a.P \xrightarrow{a} P}}{a.P + b.0 \xrightarrow{a} P} \qquad \dfrac{}{\bar{a}.Q \xrightarrow{\bar{a}} Q}}{(a.P + b.0) \mid \bar{a}.Q \xrightarrow{\tau} P \mid Q}}{((a.P + b.0) \mid \bar{a}.Q) \setminus a \xrightarrow{\tau} (P \mid Q) \setminus a}$$