Automaty nieskończone

2022/23

Wykład 7

Petri nets reachability problem – – sketch of decidability proof

<u>Input</u> : A VASS and two configurations c, c'

<u>Question</u> : $c \longrightarrow^* c'$?

(q, v) (q', v')

VASS:

$\circ \xrightarrow{v \in \mathbb{Z}^d} \circ$

Brief history : . . . . .

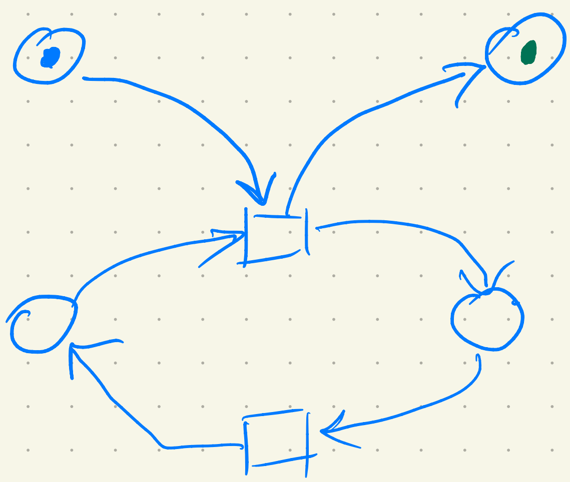
Pseudo runs : $c \dashrightarrow^* c'$

intermediate vectors may drop below 0 on some coordinates

<u>Question</u> : $c \dashrightarrow^* c' \implies c \longrightarrow^* c'$?

No!

- initial
- final



Sufficient condition for $c \longrightarrow^* c'$:

Θ_1 : For every $n \geq 1$, $(q,v) \dashrightarrow^* (q',v')$ using every transition $\geq n$ times

Implies strong connectedness

Θ_2 : For some $\Delta, \Delta' \geq \vec{1}$,

a) $(q,v) \xrightarrow{\pi}^* (q, v+\Delta)$

b) $(q',v') \xleftarrow{\pi'}^* (q', v'+\Delta')$

Claim : $(q',\Delta) \dashrightarrow^* (q',\Delta')$

$\Theta_1 \rightarrow$ Claim

$$\begin{tikzcd}
& q', v'+n\Delta \arrow[dr, bend left, "\text{Claim}"] & \\
q, v+n\Delta \arrow[ur, bend left, "\Theta_1"] & & q', v'+n\Delta \arrow[d, "\Theta_2", "*"'] \\
q, v \arrow[u, "\Theta_2"', "*"] & & q', v'
\end{tikzcd}$$

<u>effect</u> of a pseudorun $e(\alpha)$: final vector − initial vector $\in \mathbb{Z}^d$

determines

<u>folding</u> of a pseudorun $F(\alpha)$: $\in \mathbb{N}^T$

<u>Observation</u>: Two pseudoruns $\begin{tikzcd} q \arrow[r, bend left, squiggly, "\alpha"] \arrow[r, bend right, squiggly, "\beta"'] & q' \end{tikzcd}$

$$F(\alpha) - F(\beta) \geq \vec{1}.$$

For every non-isolated control state p there is a pseudorun $\begin{tikzcd} p \arrow[loop right, squiggly, "\gamma"] \end{tikzcd}$ s.t.

$$F(\gamma) = F(\alpha) - F(\beta).$$

<u>Proof</u>:

Proof of the claim: Pseudoruns (using Q_1)

$$\begin{tikzcd}(q,v) \arrow[r, bend left, "\alpha"] \arrow[r, bend right, "\beta"'] & (q',v')\end{tikzcd}$$

such that

$$F(\alpha) - F(\beta) - F(\pi) - F(\pi') \geqslant \vec{1}$$

By Observation, there is a pseudorun $\begin{tikzcd} q' \arrow[loop right, "\gamma"] \end{tikzcd}$

s.t.

$$F(\gamma) = F(\alpha) - F(\beta) - F(\pi) - F(\pi')$$

$$e(\gamma) = \underbrace{e(\alpha) - e(\beta)}_{0} \underbrace{- e(\pi)}_{-\Delta} \underbrace{- e(\pi')}_{+\Delta'} = \Delta' - \Delta$$

Hence $(q,\Delta) \overset{\gamma}{\dashrightarrow}{}^{*} (q',\Delta')$

Q_1 is effective $\longrightarrow$ $\boxed{\text{tutorials}}$

Q_2 is effective — why?

What to do if Θ_1 or Θ_2 fails to hold?

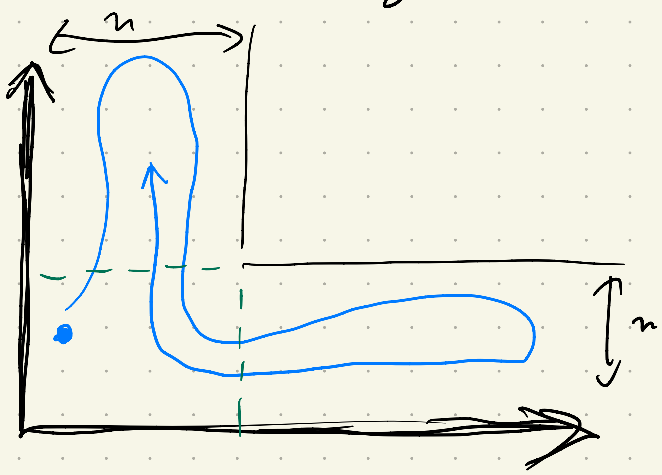
Θ_2 a) fails:

effectively computable — how?

$\exists n$. every configuration reachable from (q, v) has some coordinate $< n$

(using coverability tree) $\Downarrow$ → $\boxed{\text{tutorials}}$

$\exists n$. every run from (q, v), on some coordinate, is always $< n$

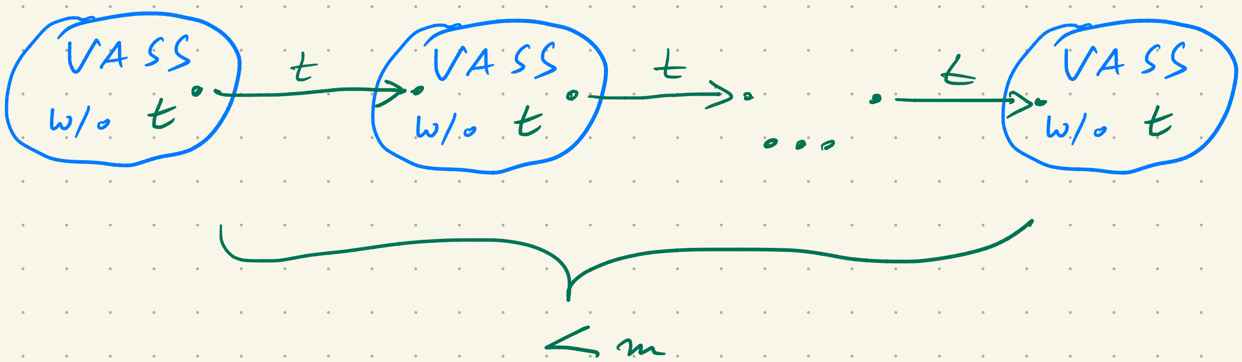


Reduction of dimension: try to bounds each single dimension by n.

Θ_n fails:

$\exists m$. every pseudorun $(q, v) \dashrightarrow^* (q', v')$ uses some transition $< m$ times

? Reduction of the number of transitions: try to fix nr of usages of each transition t to each nr $< m$.



– We lose strong connectedness!

– Dimension is preserved, nr of transitions increases!