Star problems - the second series (deadline 30.01.2022)

1. Consider vector addition systems with states (VASS) as language recognisers. To this aim we assume that each transition is additionally labeled by a letter of a finite alphabet Σ, there are distinguished subsets of initial I and accepting states F, and accepting runs are those which start in $q(\vec{0})$ for some $q \in I$ and end in $p(c)$ for some $p \in F$ and $c \in \mathbb{N}^d$, where d is the dimension of the VASS.

   Show that every two languages L_1, L_2 of VASS which are complements, i.e. $L_1 \cap L_2 = \emptyset$ and $L_1 \cup L_2 = \Sigma^*$, are actually regular languages.

   Hint: WQO.

2. Show decidability of the following problem.

Input:	A VASS and two subsets I, F of its states.
Question:	Is there a run starting in some state from I that visits infinitely often states from F?

   Thus the run has to start from a configuration $q(c)$, where $q \in I$ and $c \in \mathbb{N}^d$ are chosen arbitrarily, and d is the dimension of the VASS.

2.$^+$ The previous problem, but with an additional requirement that the run starts in $q(\vec{0})$, for some $q \in I$.

3. A language $L \subseteq \Sigma^*$ we call *weighted-recognizable* if there is a weighted automaton $\mathcal{A}$ over the rational field with the property that $\mathcal{A}(w) = 1$ for $w \in L$ and $\mathcal{A}(w) = 0$ for $w \notin L$. Show that each weighted-recognizable language is actually regular.