On coincidence of distributed and performance equivalence for Basic Parallel Processes

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Abstract. We prove that *distributed bisimilarity* and a timed equivalence called *performance equivalence* coincide on Basic Parallel Processes.

1 Definitions

Let Act be a finite set of *actions*, ranged over by a, b, etc. and let $Const$ be a finite set of *process constants*, ranged over by X, Y, etc. The set of TBPP process expressions over Act and $Const$ is given by:

$$P \ ::= \ \mathbf{0} \ | \ X \ | \ P \| P \ | \ P + P \ | \ a.P \ | \ 1 \triangleright P$$

where **0** denotes the *empty* process, $\_\|\_$ stands for a parallel composition, $\_ + \_$ means a nondeterministic choice, $a.\_$ is an action prefix and $1 \triangleright P$ denotes a process which behaves like P but delayed by one time unit. To avoid brackets, we assume that $a.\_$ and $1 \triangleright \_$ bind stronger than $\_\|\_$ and $\_\|\_$ binds stronger than $\_ + \_$. Process expressions not involving the delay operator $1 \triangleright \_$ are called *untimed* or BPP process expressions [Chr93]. For $t \geq 0$, we write $t \triangleright P$ as a shorthand for $\underbrace{1 \triangleright (1 \triangleright (\ldots (1 \triangleright P) \ldots))}_{t \text{ times}}$.

A TBPP *process definition* Δ consists of a finite $Act(\Delta)$ of actions, a finite set $Const(\Delta)$ of constants and a finite number of recursive process equations

$$X \stackrel{\text{def}}{=} P,$$

one for each constant $X \in Const(\Delta)$, where P is a process expression over $Act(\Delta)$ and $Const(\Delta)$. Sets $Const(\Delta)$ and $Act(\Delta)$ are often even not mentioned explicitly, as they can be deduced from process equations. Δ is called

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a BPP process definition if each right-hand side is a BPP process expression. In the sequel we shall only consider *guarded* process definitions, i.e., such that each occurrence of a constant on the right-hand side of a defining equation is within the scope of action prefix. By a TBPP (resp. BPP) *process* we mean a pair (P, Δ), where Δ is a TBPP (resp. BPP) process definition and P is a TBPP (resp. BPP) process expression over $Act(\Delta)$ and $Const(\Delta)$.

From now on, we do not distinguish between process expressions related by *structural congruence*, i.e., the smallest congruence induced by:

- associativity and commutativity of $\_\|\_$ and $\_ + \_$,
- nilpotency of $\mathbf{0}$: $\mathbf{0}\|P = P$, $\mathbf{0} + P = P$,
- time cancellation for $\mathbf{0}$: $1 \triangleright \mathbf{0} = \mathbf{0}$,
- clock distribution for $\_\|\_$: $1 \triangleright (P_1 \| P_2) = 1 \triangleright P_1 \| 1 \triangleright P_2$,
- clock distribution for $\_ + \_$: $1 \triangleright (P_1 + P_2) = 1 \triangleright P_1 + 1 \triangleright P_2$.

In particular, equality of process expressions is understood up to the structural congruence.

Performance equivalence ([GRS95]) Let $f : Act \to \mathbb{N} \setminus \{0\}$ be a function assigning a positive *duration* to each action. Each TBPP process definition Δ together with a duration function f induces a labelled transition system, whose states are TBPP process expressions, and whose transitions are labelled by pairs (t, a), with t a natural number and $a \in Act$. A transition $P \xrightarrow{t,a} P'$ denotes "P does a at time t and becomes P'". The transitions are given by the following SOS transition rules (recall that $\_\|\_$ and $\_ + \_$ are commutative):

$$
\begin{array}{cc}
\dfrac{}{a.P \xrightarrow{0,a} f(a) \triangleright P} & \dfrac{P \xrightarrow{t,a} P'}{1 \triangleright P \xrightarrow{t+1,a} 1 \triangleright P'} \\[2ex]
\dfrac{P \xrightarrow{t,a} P'}{P \| Q \xrightarrow{t,a} P' \| Q} & \dfrac{P \xrightarrow{t,a} P'}{P + Q \xrightarrow{t,a} P'} \\[2ex]
\multicolumn{2}{c}{\dfrac{(X \stackrel{\mathrm{def}}{=} P) \in \Delta \qquad P \xrightarrow{t,a} P'}{X \xrightarrow{t,a} P'}}
\end{array}
\tag{1}
$$

The transition system obtained is finitely-branching despite that it has infinite labelling set. Following [GRS95,CGR97] and [BLS00], we allow for *ill-timed runs*, i.e. we do not exclude $P \xrightarrow{t_1,a_1} P' \xrightarrow{t_2,a_2} P''$ when $t_1 > t_2$. It was argued convincingly in [AM96] that ill-timed semantics brings no semantical problems since the ill-timed runs are well-caused but on the other hand it greatly simplifies the technical treatment.

Given a duration function f, two TBPP processes (P_1, Δ_1) and (P_2, Δ_2) are *performance equivalent* wrt. f, denoted $(P_1, \Delta_1) \sim^{\mathrm{p}}_f (P_2, \Delta_2)$ (or $(P_1, \Delta_1) \sim^{\mathrm{p}} (P_2, \Delta_2)$ when f is understood in a context) if they are related by some *performance bisimulation* R, i.e., a binary relation over process expressions such that whenever $(P, Q) \in R$, for each $a \in Act$ and $t \geq 0$,

- if $P \xrightarrow{t,a} P'$ then $Q \xrightarrow{t,a} Q'$ for some Q' such that $(P', Q') \in R$
- if $Q \xrightarrow{t,a} Q'$ then $P \xrightarrow{t,a} P'$ for some P' such that $(P', Q') \in R$.

In [BLS00] and [CGR97], such relations were called *performance relations* and *R-bisimulations*, respectively. By definition, performance equivalence is just the strong bisimilarity for TBPP; we prefer to use a different name in order to avoid confusion, when in the sequel we investigate performance equivalence on BPP.

Due to associativity and commutativity of choice we could consider a finite choice $\sum_{1 \leq i \leq n} P_i$, $n \geq 1$ instead of the binary one, with the transition rule

$$\frac{P_j \xrightarrow{t,a} P' \quad \text{for some } \ 1 \leq j \leq n}{\sum_{1 \leq i \leq n} P_i \ \xrightarrow{t,a} P'} \ .$$

Following approach of many papers based on process rewriting systems, in [BLS00] the performance equivalence was studied only for processes in *normal form*, i.e., without choice and action prefix. In our setting this corresponds precisely to process equations of the restricted form

$$X \stackrel{\text{def}}{=} \sum_{1 \leq i \leq n} a_i.P_i, \tag{2}$$

where all P_i involve no choice and action prefix. This fragment of TBPP is called TBPP^g in the sequel. It is well known that each BPP process can be effectively transformed into normal form (2) preserving strong bisimilarity [Chr93]. This is no more the case for performance equivalence on TBPP and BPP, and for instance a process $a.\mathbf{0} + b.\mathbf{0} \| c.\mathbf{0}$ is not in TBPP^g, i.e., can not be presented by process equations (2) up to performance equivalence.

Distributed equivalence ([Cas88]) Given a BPP process definition Δ, consider the following transition rules for BPP processes borrowed from [Chr93] (recall that _‖_ and _ + _ are commutative again):

$$\boxed{\begin{array}{cc} \dfrac{P \xrightarrow{a} [P_1, P_2]}{P \| Q \xrightarrow{a} [P_1, P_2 \| Q]} & \dfrac{P \xrightarrow{a} [P_1, P_2]}{P + Q \xrightarrow{a} [P_1, P_2]} \\[2ex] \dfrac{}{a.P \xrightarrow{a} [P, \mathbf{0}]} & \dfrac{(X \stackrel{\text{def}}{=} P) \in \Delta \qquad P \xrightarrow{a} [P_1, P_2]}{X \xrightarrow{a} [P_1, P_2]} \end{array}} \tag{3}$$

These rules reflect a view on a process as distributed in space. A transition $P \xrightarrow{a} [P_1, P_2]$ gives rise to a *local* component P_1, which intuitively records a *location* at which the action is observed, and a *concurrent* component P_2, recording the part of the process separated from the local component.

Two BPP processes (P_1, Δ_1) and (P_2, Δ_2) are *distributed equivalent*, denoted $(P_1, \Delta_1) \sim^{\mathrm{d}} (P_2, \Delta_2)$ if they are related by some *distributed bisimulation* R, that is a binary relation over BPP process expressions such that whenever $(P, Q) \in R$, for each $a \in Act$ and $t \geq 0$,

- if $P \xrightarrow{a} [P_1, P_2]$ then $Q \xrightarrow{a} [Q_1, Q_2]$ for some Q_1, Q_2 such that $(P_1, Q_1) \in R$ and $(P_2, Q_2) \in R$,
- if $Q \xrightarrow{a} [Q_1, Q_2]$ then $P \xrightarrow{a} [P_1, P_2]$ for some P_1, P_2 such that $(P_1, Q_1) \in R$ and $(P_2, Q_2) \in R$,

The distributed equivalence was also called distributed bisimulation equivalence or distributed bisimilarity [Cas88,Chr92,Chr93,Cas01].

In the next section we show that the two equivalences coincide on BPP.

2 Distributed and performance equivalences coincide on BPP

Theorem 1. *For any* BPP *processes* (P_1, Δ), (P_2, Δ) *and any duration function* f, $(P_1, \Delta) \sim^{p}_{f} (P_2, \Delta)$ *iff* $(P_1, \Delta) \sim^{d} (P_2, \Delta)$.

In particular, $\sim^{\mathrm{p}}_{\Delta, f}$ does not really depend on f. This extends the same observation made in [BLS00] for TBPP^g processes. In this section we assume that f is a fixed duration function f and Δ is an arbitrary but fixed BPP process definition. Hence we write $P_1 \sim^{\mathrm{p}} P_2$ in place of $(P_1, \Delta) \sim^{\mathrm{p}}_{f} (P_2, \Delta)$; analogously $P_1 \sim^{\mathrm{d}} P_2$. Before embarking on the proof of Theorem 1, we need few lemmas.

Lemma 1. *Let* P *be a* BPP *process expression and* $a \in Act$. *If* $P \xrightarrow{0,a} P'$ *then* $P' = f(a) \triangleright P_1 \| P_2$ *for some* BPP *processes expressions* P_1, P_2.

Proof. Routine, by structural induction on P. □

Lemma 2. *Let* P *be a* BPP *process expression and* $a \in Act$. *Then* $P \xrightarrow{a} [P_1, P_2]$ *iff* $P \xrightarrow{0,a} f(a) \triangleright P_1 \| P_2$.

Proof. By structural induction over a BPP process expression P.
– Case $P = \mathbf{0}$: Obvious.
– Case $P = a.Q$: Immediate, since

$$P \xrightarrow{a} [Q, \mathbf{0}] \quad \text{and} \quad P \xrightarrow{0,a} f(a) \triangleright Q = f(a) \triangleright Q \| \mathbf{0}.$$

– Case $P = P' + P''$: If lemma holds for P' and P'', directly from the SOS rules (1) and (3) we conclude that it also holds for $P' + P''$.
– Case $P = P' \| P''$: Assume lemma holds for P' and P''. Action a of $P' \| P''$ is performed in one of components, say P', and we have by induction assumption:

$$P' \xrightarrow{a} [P'_1, P'_2] \qquad \iff \qquad P' \xrightarrow{0,a} f(a) \triangleright P'_1 \| P'_2$$

hence by the SOS rules:

$$P' \| P'' \xrightarrow{a} [P'_1, P'_2 \| P''] \qquad \iff \qquad P' \| P'' \xrightarrow{0,a} t \triangleright P'_1 \| P'_2 \| P''.$$

– Case $P = X$: Immediate. Note that the induction is correct since Δ is assumed to be guarded. This completes the proof. □

Lemma 5 below extends the decomposition result of [BLS00] to the whole TBPP. We propose a simpler proof method, based on the auxiliary Lemma 4.

Lemma 3 (time progress). *For each* TBPP *process expression P and $t \geq 0$, there is no infinite sequence of transitions labelled by t starting in P:*

$$P \xrightarrow{t,a_1} P' \xrightarrow{t,a_2} P'' \xrightarrow{t,a_3} \dots$$

Proof. By structural induction over P, we give a finite upper bound $|P|$ on the number of consecutive transitions labelled by t. When $P = a.P'$, this bound is equal 1, i.e., P can have *at most* 1 transition labelled by t in sequence, $P \xrightarrow{t,a} f(a) \triangleright P'$, since $f(a) > 0$. When $P = 1 \triangleright P'$, we put $|1 \triangleright P'| = |P'|$. Furthermore, $|P_1 + P_2| = \max\{|P_1|, |P_2|\}$ and $|P_1 \| P_2| = |P_1| + |P_2|$. Finally, when $P = X$, $|X| = |E|$, where $(X \stackrel{\text{def}}{=} E) \in \Delta$. □

Lemma 4. *Let P_1, P_2, Q_1 and Q_2 be arbitrary* TBPP *process expressions. Then $P_1 \| Q_1 \sim^p P_2 \| Q_2$ and $Q_1 \sim^p Q_2$ implies $P_1 \sim^p P_2$.*

Proof. Assume $P_1 \| Q_1 \sim^{\mathrm{p}} P_2 \| Q_2$ and $Q_1 \sim^{\mathrm{p}} Q_2$. We can substitute Q_1 for Q_2 to obtain: $P_1 \| Q_1 \sim^{\mathrm{p}} P_2 \| Q_1$. To show $P_1 \sim^{\mathrm{p}} P_2$, we shall demonstrate that the relation

$$R = \{(P_1, P_2) : P_1 \| Q \sim^{\mathrm{p}} P_2 \| Q \text{ for some } Q\}$$

is a performance bisimulation. For each $(P_1, P_2) \in R$ we need to show that each transition of P_1 can be matched by a transition of P_2, and vice versa. So, assume $P_1 \xrightarrow{t,a} P'_1$. Hence $P_1 \| Q \xrightarrow{t,a} P'_1 \| Q$, where Q is chosen so that $P_1 \| Q \sim^{\mathrm{p}} P_2 \| Q$. Therefore, $P_2 \| Q$ has a matching transition. If the matching transition is performed in P_2, i.e., $P_2 \xrightarrow{t,a} P'_2$ and $P'_1 \| Q \sim^{\mathrm{p}} P'_2 \| Q$, we are done. Otherwise, assume that the matching transition is performed in Q, i.e.,

$$Q \xrightarrow{t,a} Q' \tag{4}$$

and $P_1'\|Q \sim^{\mathrm{p}} P_2\|Q'$. But due to (4), process $P_1'\|Q$ has again a transition labelled by t and a, $P_1'\|Q \xrightarrow{t,a} P_1'\|Q'$. And again, if the response of $P_2\|Q'$ is performed by P_2, we are done. Otherwise,

$$Q' \xrightarrow{t,a} Q'' \tag{5}$$

and $P_1'\|Q' \sim^{\mathrm{p}} P_2\|Q''$. Now a crucial observation is that we can not continue forever in this way, with transitions (4), (5), and so on, since this would yield an infinite sequence of transitions labelled by t and a,

$$Q \xrightarrow{t,a} Q' \xrightarrow{t,a} Q'' \xrightarrow{t,a} \ldots$$

which is excluded by Lemma 3. Hence after some finite number of iterations, a response must be eventually performed in P_2. This completes the proof. □

Lemma 5. *For arbitrary* BPP *process expressions P_1, P_2, Q_1, Q_2 and $t > 0$, whenever $P_1\|t \triangleright Q_1 \sim^p P_2\|t \triangleright Q_2$ then $P_1 \sim^p P_2$ and $Q_1 \sim^p Q_2$.*

Proof. To prove $Q_1 \sim^{\mathrm{p}} Q_2$, it is enough to observe that for fixed P_1 and P_2, the relation $\{(Q_1, Q_2) : P_1\|t \triangleright Q_1 \sim^{\mathrm{p}} P_2\|t \triangleright Q_2 \text{ for some } t \geq 1\}$ is a performance bisimulation. Then $P_1 \sim^{\mathrm{p}} P_2$ follows immediately by Lemma 4. □

Proof of Theorem 1. We will show $\sim^{\mathrm{d}} \subseteq \sim^{\mathrm{p}}$ and $\sim^{\mathrm{p}} \subseteq \sim^{\mathrm{d}}$. For $\sim^{\mathrm{d}} \subseteq \sim^{\mathrm{p}}$, we shall prove that the relation R defined as:

$$\{(t_1 \triangleright P_1\|\ldots\|t_k \triangleright P_k,\ t_1 \triangleright Q_1\|\ldots\|t_k \triangleright Q_k) : k \geq 0,\ \forall_{1 \leq i \leq k}\ P_i \sim^{\mathrm{d}} Q_i\}$$

is a performance bisimulation. Consider $(t_1 \triangleright P_1\|\ldots\|t_k \triangleright P_k,\ t_1 \triangleright Q_1\|\ldots\|t_k \triangleright Q_k)$ in R and let $t_1 \triangleright P_1\|\ldots\|t_k \triangleright P_k \xrightarrow{t,a} \bar{P}$. As all P_i and Q_i are assumed to be BPP process expressions, there is some i such that $t = t_i$ and this transition is performed in P_i, $P_i \xrightarrow{0,a}$. By Lemma 1 we conclude $P_i \xrightarrow{0,a} f(a) \triangleright P'\|P''$ and

$$\bar{P} \;=\; t_1 \triangleright P_1\|\ldots\|t_{i-1} \triangleright P_{i-1}\|(t_i + f(a)) \triangleright P'\|t_i \triangleright P''\|t_{i+1} \triangleright P_{i+1}\|\ldots\|t_k \triangleright P_k.$$

By Lemma 2 we have $P_i \xrightarrow{a} [P', P'']$. Since $P_i \sim^{\mathrm{d}} Q_i$, we know that there are some Q' and Q'' such that $Q_i \xrightarrow{a} [Q', Q'']$ and $P' \sim^{\mathrm{d}} Q'$ and $P'' \sim^{\mathrm{d}} Q''$. Now we apply Lemma 2 again to get $Q_i \xrightarrow{0,a} f(a) \triangleright Q'\|Q''$. Hence

$$t_1 \triangleright Q_1\|\ldots\|t_k \triangleright Q_k \xrightarrow{t,a} \bar{Q}, \text{ and } (\bar{P}, \bar{Q}) \in R, \text{ where}$$
$$\bar{Q} \;=\; t_1 \triangleright Q_1\|\ldots\|t_{i-1} \triangleright Q_{i-1}\|(t_i + f(a)) \triangleright Q'\|t_i \triangleright Q''\|t_{i+1} \triangleright Q_{i+1}\|\ldots\|t_k \triangleright Q_k.$$

For $\sim^{\mathrm{p}} \subseteq \sim^{\mathrm{d}}$, we will show that $\sim^{\mathrm{p}}$ restricted to pairs of BPP process expressions is a distributed bisimulation. Assume $P \sim^{\mathrm{p}} Q$, P and Q are BPP process expressions, and let $P \xrightarrow{a} [P', P'']$. Then $P \xrightarrow{0,a} f(a) \triangleright P' \| P''$ for some P', P'', by Lemma 2. Since $P \sim^{\mathrm{p}} Q$, by Lemma 1 $Q \xrightarrow{0,a} f(a) \triangleright Q' \| Q''$ and $f(a) \triangleright P' \| P'' \sim^{\mathrm{p}} f(a) \triangleright Q' \| Q''$. Again by Lemma 2, $Q \xrightarrow{a} [Q', Q'']$. Moreover by Lemma 5 we have $P' \sim^{\mathrm{p}} Q'$ and $P'' \sim^{\mathrm{p}} Q''$. This completes the proof. $\square$

3 Communication

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