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A Rough Set Approach to Measuring Information Granules

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Abstract

This article introduces an approach to measures of information granules based on rough set theory. Informally, an information granule is a representation of a multiset (or bag) of real-world objects that are somehow indistinguishable (e.g., water samples taken from the same source at approximately the same time), or similar (e.g., various renditions of Chopin's sonatas or various series of very high, tinkling trills common in the songs of winter wrens), or which cause the same functionality (e.g., unmanned helicopters, line-crawling robots). Examples of measures of information granules based on rough set theory are inclusion, closeness, size, and enclosure. All of these measures are based on rough inclusion. This paper is limited to a consideration of measures of inclusion based on a straightforward extension of classical rough membership functions and closeness based on measurement of separation of equivalence classes in a partition of the universe containing information granules. Measurement of sensor-based information granules has been motivated by recent studies of sensor signals. A sensor signal is a non-empty, finite set of sample sensor signal values temporally ordered. Classification of sensor signals requires measurements of sample signal values over subintervals of time. The contribution of this article is the introduction of a rough set approach to measuring information granule inclusion and closeness.

Keywords: Closeness, inclusion, indistinguishability, information granule, measure, rough sets, sensor.

1. Introduction

Measures inclusion and closeness of information granules based on rough set theory are introduced in this article. This research is part of rapidly growing research on information granulation, granular computing and computing with words introduced by Zadeh [26] and the calculus of granules [13], [18]. In this research, an information granule is a temporally ordered multiset (or bag) [24]-[25] of real-world objects (e.g., sample sensor signal values). A multiset is a set where duplicates are counted. Such information granules are constructed from vectors of real numbers using a variant of the traditional indiscernibility relation in rough set theory. These information granules are associated with indiscernibility classes containing sample signal values that occur in precisely defined temporal intervals. The measure of inclusion of information granules considered in this paper is based on a straightforward extension of rough membership functions [2]. The partition of a universe of objects into temporally-ordered information granules containing equivalent objects (i.e., equivalence classes) provides a basis for a measure of closeness of information granules. This partition of the universe is accomplished with an indistinguishability relation *Ing* introduced in [30]. The relation *Ing* is briefly presented in this article. Consideration of rough measures of inclusion and closeness of information granules is motivated by an interest in the problem of determining the size and number of clusters of related objects. Granule approximation in this paper is cast in the context of infinite rather than finite universes. Consideration of granule approximation is needed to classify a number of different forms of uncountable sets (e.g., analog sensor signals such as speech, electrocardiograms, electroencephalograms). This is important in the context of parameterized approximation spaces used in designing intelligent systems [15, 18-23], especially [18]. The rough measures described in this paper have a number of practical applications, e.g., data mining [28], performance maps [27], sensor fusion [8], signal analysis [9], robotics [5], and neurocomputing [5, 29]. The contribution of this article is the introduction of an approach to measuring granule inclusion and closeness of information granules based on rough set theory.

This paper is organized as follows. Section 2 presents introduces the parameterized indistinguishability relation and approximation of sets. This section also introduces a new form of rough membership set function. A natural extension of these ideas is the introduction of a rough measure space. Measurement of rough inclusion of information granules is considered in Section 3. Measurement of closeness of information granules is defined and illustrated in Section 4.

2. Indistinguishability and Information Granule Approximation

In laying the groundwork for approximate reasoning about uncountable sets in the context of rough set theory [1]-[4], we introduce an indistinguishability relation *Ing* to partition universes of reals. That is, consider a universe that is a subset of the reals, where set approximation and measurement can be carried out relative to a partition of such a universe into equivalence classes (subintervals that are sources of granules of information about the sensorial world). This partition is accomplished using *Ing*. To see this, let $S = (U, A)$ be an information system where U is a non-empty set and A is a non-empty, finite set of attributes, where $a: U \to V_a$ and $V_a \subseteq \Re$ for every $a \in A$, so that $\Re \supseteq V = \bigcup_{a\in A} V_a$. Let $a(x) \geq 0$, $\delta > 0$ and let $\lfloor a(x)/\delta \rfloor$ denote the greatest integer less than or equal to $a(x)/\delta$ ("floor" of a(x)/δ). In addition, let $\lceil a(x)/\delta \rceil$ denote the least integer bigger than or equal to $a(x)/\delta$ ("ceiling" of $a(x)/\delta$) for attribute a. If $a(x) < 0$, then $\lfloor a(x)/\delta \rfloor = -\lceil |a(x)|/\delta \rceil$, where $|\bullet|$ denotes the absolute value of $\bullet$. The parameter δ serves as a means of computing a "neighborhood" size on real-valued intervals. Reals representing sensor measurements within the same subinterval bounded by $k\delta$ and $(k+1)\delta$ for integer k are considered δ–indistinguishable. For each $B \subseteq A$, there is associated an equivalence relation $Ing_{A,\delta}(B)$ defined as follows:

$$Ing_{A,\delta}(B) = \left\{ (x, x') \in \Re^2 \mid \forall a \in B.\ \lfloor a(x)/\delta \rfloor = \lfloor a(x')/\delta \rfloor \right\}$$

Remark. In this paper, clustering is a relation defined on vectors of real numbers. If $B = \{a_1,\ldots,a_m\}$, then an indiscernibility relation is defined on objects $(x, x_1,\ldots, x_m) = (x(t), a_1(x(t)), ..., a_m(x(t))$ and the indiscernibility classes are cells. Two vectors $(x, x_1,\ldots, x_m)$ and $(y, y_1,\ldots, y_m)$ are indistinguishable if, and only if,

$\lfloor x/\delta \rfloor = \lfloor y/\delta \rfloor$ and $\lfloor a_i(x_i)/\delta \rfloor = \lfloor a_i(y_i)/\delta \rfloor$ for i = 1, …, m.

If $(x, x') \in Ing_{A,\delta}(B)$, we say that objects x and x' are indistinguishable from each other relative to attributes from B. A subscript *Id* denotes a set consisting of the identity sensor $id(x) = x$. If, for example, attribute *id* partitions U on intervals [0, 1), [1, 2), It can be shown that $Ing_{A,\delta}(B \cup Id)$ is an equivalence relation. The notation $[x]^{\delta}_{B\cup Id}$ denotes equivalence classes of $Ing_{A,\delta}(B \cup Id)$. Further, partition $U/Ing_{A,\delta}(B \cup Id)$ denotes the family of all equivalence classes of relation $Ing_{A,\delta}(B \cup Id)$ on U. For $X \subseteq U$, the set X can be approximated only from information contained in B by constructing a B-lower and a B-upper approximation denoted by $\underline{B}X$ and $\overline{B}X$, respectively. The B-lower approximation of X is the set $\underline{B}X = \left\{ x \mid [x]^{\delta}_{B\cup Id} \subseteq X \right\}$ and the B-upper approximation of X is the set $\overline{B}X = \left\{ x \mid [x]^{\delta}_{B\cup Id} \cap X \neq \varnothing \right\}$.

3. Rough Inclusion

A particular form of rough inclusion is defined with a set function form of the original rough membership function [2]. Let $S = (U, A)$ be an information system with non-empty set U and non-empty set of attributes A. Further, let $B \subseteq A$ and let $[y]^{\delta}_{B}$ be an equivalence class of any sensor reading $y \in \Re$. Let ρ be a measure of on $\wp(U)$, where $\wp(U)$ is the powerset of U. Then define $\mu_y^{B,\delta} : \wp(U) \to [0,1]$ as in (1).

$$\mu_y^{B,\delta}(X) = \frac{\rho\left(X \cap [y]^{\delta}_{B}\right)}{\rho\left([y]^{\delta}_{B}\right)} \tag{1}$$

for any $X \in \wp(U)$. Also, $\mu_y^{B,\delta}$ is a measure of rough inclusion of X in $[y]^{\delta}_{B}$. If $\rho([y]^{\delta}_{B}) = 0$, then $\rho(X \cap [y]^{\delta}_{B})$ = 0 and we define 0/0 (division by 0) to be equal to 0. For example, $\mu_y^{B,\delta}$ in (1) can be computed using (2).

$$\mu_y^{B,\delta}(X) = \frac{\rho\left(X \cap [y]^{\delta}_{B}\right)}{\rho\left([y]^{\delta}_{B}\right)} = \frac{\int_{X \cap [y]^{\delta}_{B}} 1\, d\rho}{\int_{[y]^{\delta}_{B}} 1\, d\rho} \tag{2}$$

Notice that $\mu_y^{B,\delta}$ in (2) is a non-negative, additive set function. Hence, $\mu_y^{B,\delta}$ is a measure of $X \in \wp(U)$. It can also be shown that $\mu_y^{B,\delta}$ in (1) is a measure on $\wp(U)$ for arbitrary measure ρ.

4. Measure of Closeness of Information Granules

In this section, a measure of closeness of information granules is introduced. The idea of a sensor signal as a temporally ordered multiset is discussed in this section. The notion of clustering temporally ordered data has been considered by others (see, e.g., [30]). What is new in this paper is the introduction of boxes representing indiscernibility classes and the construction of clusters of temporally ordered data extracted from boxes that are close to each other.

Proposition 3.1 Let S = (U, A) be an information system and let ρ and y be defined as in (2). The function $\mu^{\delta}_{B,y} : \wp(U) \to \Re$ in (3) is a measure.

$$\mu^{\delta}_{B,y}(X) = \sum_{[y']^{\delta}_{B} \subseteq \bar{B}X} \frac{\rho(X \cap [y']^{\delta}_{B})}{(d([y']^{\delta}_{B},[y]^{\delta}_{B})+1)\cdot\rho([y']^{\delta}_{B})} \quad (3)$$

where $d(\bullet)$ denotes a metric on the partition $U/\mathrm{Ing}_{A,\delta}(B)$ of U defined by equivalence relation $\mathrm{Ing}_{A,\delta}(B)$. The formula (3) may be written as in (4)

$$\mu^{\delta}_{B,y}(X) = \sum_{[y']^{\delta}_{B} \subseteq \bar{B}X} \frac{\rho(X \cap [y']^{\delta}_{B})}{(d([y']^{\delta}_{B},[y]^{\delta}_{B})+1)\cdot\rho([y]^{\delta}_{B})} =$$

$$\frac{\rho(X \cap [y]^{\delta}_{B})}{\rho([y]^{\delta}_{B})} + \sum_{\begin{cases}[y']^{\delta}_{B} \subseteq \bar{B}X \\ [y']^{\delta}_{B} \neq [y]^{\delta}_{B}\end{cases}} \frac{\rho(X \cap [y']^{\delta}_{B})}{\rho([y']^{\delta}_{B})} \cdot \frac{1}{d([y']^{\delta}_{B},[y]^{\delta}_{B})+1} \quad (4)$$

It is clearly seen that $\mu^{\delta}_{B,y}$ defined in (4) is measure (1) completed with a sum of analogous measures for the remaining equivalence classes weighted by the reverse of distance (plus one) between distinct class $[y]^{\delta}_{B}$ and the remaining equivalence classes. The ratio $1/d([y']^{\delta}_{B},[y]^{\delta}_{B})+1$ serves as a weight of the sum in (4). To obtain values in the interval [0, 1] for the measure (3), the normalization coefficient α(y) in (5) is introduced.

$$\alpha(y) = \frac{1}{\sum_{[y']^{\delta}_{B} \subseteq \bar{B}X} \frac{1}{d([y']^{\delta}_{B},[y]^{\delta}_{B})+1}} \quad (5)$$

As a result, the following proposition holds. If d is a metric defined on the set $U/\mathrm{Ing}_{A,\delta}(B)$ and α(y) is as in formula (5), then function $\mu^{\delta}_{B,y} : \wp(U) \to [0,1]$ defined in (6) is a measure on the set $\wp(U)$.

$$\mu^{\delta}_{B,y}(X) = \alpha(y) \sum_{[y']^{\delta}_{B} \subseteq \bar{B}X} \frac{\rho(X \cap [y']^{\delta}_{B})}{(d([y']^{\delta}_{B},[y]^{\delta}_{B})+1)\cdot\rho([y']^{\delta}_{B})} \quad (6)$$

Example. Consider set X and equivalence class $[y]^{0.1}_{B}\big|_{y=0.5}$ defined relative to y = 0.5. For $[y']^{\delta}_{B}$, $[y]^{\delta}_{B}$, let $\lfloor [y']^{\delta}_{B}/\delta \rfloor = n$ and $\lfloor [y]^{\delta}_{B}/\delta \rfloor = m$ and define the distance metric to be $d([y']^{\delta}_{B},[y]^{\delta}_{B})$ = |n-m|. Now we can compute coefficient α such that

$$\alpha(0.5) = \frac{1}{0.33+0.5+1+0.5+0.33} = 0.375 .$$

Values $\rho([y']^{\delta}_{B})$ and $\rho(X \cap [y']^{\delta}_{B})$ are shown in Table 1.

Table 1. Sample ρ Values

Equivalence classes	$\rho([y']^{\delta}_{B})$	$\rho(X \cap [y']^{\delta}_{B})$	$d([y']^{\delta}_{B}, [y]^{\delta}_{B})+1$
$[y']^{\delta}_{B}\ \vert_{y'=0.1}$	2.36	0.67	3
$[y']^{\delta}_{B}\ \vert_{y'=0.3}$	5.75	3.26	2
$[y']^{\delta}_{B}\ \vert_{y'=0.5}$	4.43	1.96	1
$[y']^{\delta}_{B}\ \vert_{y'=0.6}$	7.47	3.72	2
$[y']^{\delta}_{B}\ \vert_{y'=0.9}$	3.99	2.39	3

Then, according to formula (6), we obtain $\mu^{0.1}_{B,y}|_{y=0.5}$ (X) = 0.476.

Consider the sample universe U = [0, 0.4)×[0, 0.3), a finite sample $X \subset U$ as shown in Fig. 1. The corresponding sample sensor values are not shown in Figure 1. Only the partition of U coming from sensor values is shown. The equivalence relation $\mathrm{Ing}_{A,\delta}$ (B) partitions this sample universe as it is shown on Figure 1. Let us assume that each equivalence class consists of just 8 points. Every equivalence class (also called a mesh cell) is numbered by a pair of indices $E_{n,m}$. Notice that two sensor values are approximately equal in $E_{0,0}$, and are not considered duplicates. In addition, sensor values in $E_{0,0}$ (and every in every cell of the mesh) are time-ordered with the relation $\leq_{before}$ (this relation is reflexive, transitive and anti-symmetric). By definition, then, $E_{n,m}$ is a partially ordered multiset. For example, assume that a(x(t)) occurs before a(x(t')). Then we write a(x(t)) $\leq_{before}$ a(x(t')). In effect, $E_{0,0}$ and every other cell in the mesh in Fig. 1 constitutes a temporally ordered multiset. For such an information system and the partition $U/Ing_{A,\delta}(B)$, a well-known maximum metric is chosen, namely, $d(E_{n1,m1}, E_{n2,m2}) = max\ \{|n_1 - n_2|, |m_1 - m_2|\}$. Finding an equivalence class with the biggest measure of X in sense of (6) leads to choice of $E_{0,0}$ while applying measure (10) gives as result class $E_{3,1}$. If we are interested in finding single equivalence class with the bigger (or least) degree of overlapping with set X, then measure (5) should be chosen, but when we want to find a group of 'neighbour' (in sense of "close") equivalence classes that overlap with X in the biggest (least) degree, then measure (9) is suggested.

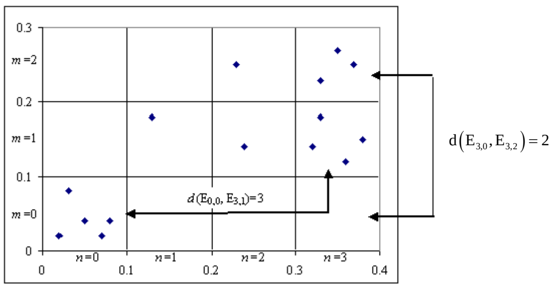

Fig. 1. Distance Measurements in a δ-mesh

Figure 1 shows how the maximum metric measures distance. For example, $d(E_{3,0},E_{3,2}) = max\{|3-3|,|2-0|\}=2$ between equivalence classes $E_{3,0}$ and $E_{3,2}$ may be of interest in cases where measurement of the separation between clusters (i.e., multiset that is the union of sensor values in a mesh cell and in neighboring cells) of sample sensor values is important (e.g., separation of cells in a mesh covering a control system performance map that contains "islands" of system response values, some normal and some verging on chaotic behavior as in [27]).

5. Conclusion

Measures of inclusion and closeness of information granules have been presented in the context of rough set theory. Measurement of the degree of inclusion of one granule in another granule is made possible by the introduction of an indistinguishability equivalence relation *Ing* and a straightforward extension of the rough membership function. The relation *Ing* has been introduced to make it possible to identify elements that are considered "indistinguishable" from each other because the elements belong to the same subinterval of reals. The partition of a universe using *Ing* results in a mesh of cells (called a δ-mesh), where each cell of the mesh represents an equivalence class. The configuration of cells in a δ-mesh yields a useful granule measure. That is, a measure of closeness of a pair of information granules contained in cells of the δ-mesh results from determining the number of cells separating members of the pair using a distance metric. The measures inclusion and closeness of information granules presented in this article have far-reaching implications. These measures provide a basis for a new approach to clustering of temporally ordered objects as well as granular derivatives and granular integrals based on rough set theory, which have a number of practical applications, namely, design of rough processors, the calibration of parameters (learning) of rough neural networks, measures of gradients in performance maps for dynamical systems, signal analysis, and digital image processing. The presentation of granular derivatives and integrals and their application is outside the scope of this paper.

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