Rough Set Theory:
A new Mathematical Approach to Data Analysis

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1 Introduction

Rough set theory is a new mathematical approach to data analysis recently known also as data mining. In recent years we witnessed a rapid grow of interest in rough set theory and its applications, worldwide.

The rough set approach seems to be of fundamental importance to AI and cognitive sciences, especially in the areas of machine learning, knowledge acquisition, decision analysis, knowledge discovery from databases, expert systems, inductive reasoning and pattern recognition.

The main advantage of rough set theory is that it does not need any preliminary or additional information about data – like probability in statistics or basic probability assignment in Dempster-Shafer theory and grade of membership or the value of possibility in fuzzy set theory. Besides, the theory allows straightforward interpretation of results.

Rough set theory has been successfully applied in many real-life problems in medicine, pharmacology, engineering, banking, financial and market analysis and others.

This paper discusses basic concepts of rough set theory. The presented approach is too simple to many real-life applications, therefore it was extended in many ways, but we will not discuss these extensions here.

More about rough sets and their applications can be found in the references.

2 Rough sets – the philosophy

Rough set theory is based on the indiscernibility relation. The indiscernibility relation identifies objects displaying the same properties, i.e., groups together elements of interest into *granules* of indiscernible (similar) objects. These granules, called *elementary sets* (*concepts*), are basic building blocks (concepts) of knowledge about the universe.

Union of elementary concepts is referred to as a *crisp* or *precise* concept (set); otherwise a concept (set) is called *rough, vague* or *imprecise*. Thus rough concepts cannot be expressed in terms of elementary concepts. However, they can be expressed approximately by means of elementary concepts using the idea of the *lower* and the *upper approximation* of a concept. The lower approximation of the concept is the union of all elementary

Table 1: An example of a database

Car	*F*	*P*	*S*	*M*
1	*med.*	*med.*	*med.*	*poor*
2	*high*	*med.*	*large*	*poor*
3	*med.*	*low*	*large*	*poor*
4	*low*	*med.*	*med.*	*good*
5	*high*	*low*	*small*	*poor*
6	*med.*	*low*	*large*	*good*

concepts which are included in it, whereas the upper approximation is the union of all elementary concepts which have nonempty intersection with the concept, i.e., the lower and the upper approximations of a concept are the union of all elementary concepts which are surely and possibly included in the concept, respectively. The difference between the lower and the upper approximation of the concept is its *boundary region*. Hence a concept is rough if it has nonempty boundary region.

Approximations are basic operations in rough set theory. They are used to deal with rough (vague) concepts, since in the rough set approach we replace rough concepts by pairs of precise concepts – the lower and the upper approximations of the rough concept. Thus approximations are used to express precisely our knowledge about imprecise concepts.

The problem of expressing vague concepts in terms of precise concepts in rough set theory can be also formulated differently, by employing the idea of dependency (partial) between concepts. We say that a concept (set) depends totally on a set of concepts if it is the union of all those concepts; if it is the union of some concepts it depends partially on these concepts. Thus partial dependency can be also used to express vague concepts in terms of precise concepts.

Both, approximations and dependencies are defined using decision rules, which are implications in the form *"if...then..."*

Approximations, dependencies and decision rules are basic tools of rough set theory and will be discussed in detail in the next sections.

3 Database

Rough set theory is mainly meant to be used to data analysis, therefore in what follows the theory will be formulated not in general terms but with refererence to data. Hence we will start our consideration from a database. Intuitively by the database we will understand a data table whose columns are labelled by attributes (e.g., color, temerature, etc.), rows are lablled by objects of interst (e.g., patients, states, processes etc.) and entries of the table are attribute values (e.g., red, high, etc.). A very simple example of a database is shown below:

The table contains data about six cars where *F, P, S* and *M* denote fuel consumption, selling price, size and marketability, respectively.

Formally the database is defined as follows.

By a *database* we will understand a pair $S = (U, A)$, where U and A, are finite, nonempty sets called the *universe*, and a set of *attributes* respectively. With every attribute $a \in A$ we associate a set V_a, of its *values*, called the *domain* of a. Any subset B of A determines a binary relation $I(B)$ on U, which will be called an *indiscernibility relation*, and is defined as follows:

$(x, y) \in I(B)$ if and only if $a(x) = a(y)$ for every $a \in A$, where $a(x)$ denotes the value of attribute a for element x.

It can easily be seen that $I(B)$ is an equivalence relation. The family of all equivalence classes of $I(B)$, i.e., partition determined by B, will be denoted by $U/I(B)$, or simple U/B; an equivalence class of $I(B)$, i.e., block of the partition U/B, containing x will be denoted by $B(x)$.

If (x, y) belongs to $I(B)$ we will say that x and y are *B-indiscernible* or indiscernible with respect to B. Equivalence classes of the relation $I(B)$ (or blocks of the partition U/B) are refereed to as *B-elementary sets* or *B-granules*.

For example, cars 1, 3 and 6 are pairwise indiscernible with respect to the attribute *F*. If $B = \{P, S\}$ and $x = 3$, then $B(x) = \{3, 6\}$.

Instead of an equivalence relation as a basis for rough set theory many authors proposed another relations, e.g., a tolerance relation, an ordering relations and others. However for the sake of simplicity in this paper we will stick to the equivalence relation.

4 Approximations

The indiscernibility relation will be used to define basic operations of rough set theory, which are defined as follows:

$$B_*(X) = \bigcup_{x \in U} \{B(x) : B(x) \subseteq X\},$$

$$B^*(X) = \bigcup_{x \in U} \{B(x) : B(x) \cap X \neq \emptyset\},$$

assigning to every $X \subseteq U$ two sets $B_*(X)$ and $B^*(X)$, called the *B-lower* and the *B-upper approximation* of X, respectively.

Hence, the *B*-lower approximation of a concept is the union of all *B*-granules that are included in the concept, whereas the *B-upper* approximation of a concept is the union of all *B*-granules that have a nonempty intersection with the concept. The set

$$BN_B(X) = B^*(X) - B_*(X)$$

will be referred to as the *B-boundary region* of X.

If the boundary region of X is the empty set, i.e., $BN_B(X) = \emptyset$, then X is *crisp* (*exact*) with respect to B; in the opposite case, i.e., if $BN_B(X) \neq \emptyset$, X is referred to as *rough* (*inexact*) with respect to B.

Rough sets can be also defined using a *rough membership function*, defined as

$$\mu_X^B(x) = \frac{card(B(x) \cap X)}{card(B(x))}.$$

Obviously we have

$$\mu_X^B(x) \in [0,1].$$

Value of the membership function $\mu_X^B(x)$ is conditional probability, and can be interpreted as a degree of *certainty* to which x belongs to X.

The rough membership function, can be used to define approximations and the boundary region of a set, as shown below:

$$B_*(X) = \{x \in U : \mu_X^B(x) = 1\},$$
$$B^*(X) = \{x \in U : \mu_X^B(x) > 0\},$$
$$BN_B(X) = \{x \in U : 0 < \mu_X^B(x) < 1\}.$$

5 Dependency of attributes

Discovering dependencies between attributes is another important issue in data analysis. Intuitively, a set of attributes D *depends totally* on the set of attributes C, denoted $C \Rightarrow D$, if all values of attributes from D are uniquely determined by values of attributes from C. In other words, D depends totally on C, if there exists a functional dependency between values of D and C.

We would need also a more general concept of dependency, called a *partial dependency* of attributes. Intuitively, the partial dependency means that only some values of D are determined by values of C.

Formally dependency can be defined in the following way. Let D and C be subsets of A.

We will say that D *depends on* C in a *degree* k $(0 \leq k \leq 1)$, denoted $C \Rightarrow_k D$, if

$$k = \gamma(C,D) = \frac{\sum_{X \in U/D} card(C_*(X))}{card\, U},$$

If $C \Rightarrow_k D$, we will call C – condition and D – decision attributes, respectively. Any database with distinguished condition and decision attributes is usually called a *decision table*.

If $k = 1$ we say that D *depends totally* on C, and if $k < 1$, we say that D *depends partially* (in a *degree* k) on C, and if $k = 0$, *does not depend on* C.

The coefficient k expresses the ratio of all elements of the universe, which can be properly classified to blocks of the partition U/D, employing attributes C and will be called the *degree of the dependency*.

For example, the degree of dependency between the set of condition attributes $\{F, P, S\}$ and the decision attribute $\{M\}$ in Table 1 is 2/3.

6 Decision rules

With every information system $S = (U, A)$ we associate a formal language $L(S)$, written L when S is understood. Expressions of the language L are logical formulas denoted by Φ, Ψ etc. built up from attribute-value pairs by means of logical connectives $\wedge$ (*and*), $\vee$ (*or*), $\sim$ (*not*) in the standard way. We will denote by $||\Phi||_S$ the set of all objects $x \in U$ satisfying Φ in S and refer to as the *meaning* of Φ in S.

The meaning of Φ in S is defined inductively as follows:

1) $||(a,v)||_S = \{v \in U : a(v) = U\}$ for all $a \in A$ and $v \in V_a$,

2) $||\Phi \vee \Psi||_S = ||\Phi||_S \cup ||\Psi||_S$,

3) $||\Phi \wedge \Psi||_S = ||\Phi||_S \cap ||\Psi||_S$,

4) $|| \sim \Phi||_S = U - ||\Phi||_S$.

A formula Φ is *true* in S if $||\Phi||_S = U$.

A *decision rule* in L is an expression $\Phi \rightarrow \Psi$, read *if* Φ *then* Ψ, where Φ and Ψ are referred to as *conditions* and *decisions* of the rule, respectively.

An example of a decision rule is given below

$(F, med.) \wedge (P, low) \wedge (S, large) \rightarrow (M, poor)$.

Obviously a decision rule $\Phi \rightarrow \Psi$ is *true* in S if $||\Phi||_S \subseteq ||\Psi||_S$.

With every decision rule $\Phi \rightarrow \Psi$ we associate a conditional probability $\pi_S(\Psi|\Phi)$ that Ψ is true in S, given Φ is true in S with the probability $\pi_S(\Phi)\frac{card(||\Phi||_S)}{card(U)}$, called the *certainty factor* and defined as follows:

$$\pi_S(\Psi|\Phi) = \frac{card(||\Phi \wedge \Psi||_S)}{card(||\Phi||_S)},$$

where $||\Phi||_S \neq 0$.

This coefficient is widely used in data mining and is called "confidence coefficient".

Obviously, $\pi_S(\Psi|\Phi) = 1$ if and only if $\Phi \rightarrow \Psi$ is true in S.

If $\pi_S(\Psi|\Phi) = 1$, then $\Phi \rightarrow \Psi$ will be called a *certain* decision rule; if $0 < \pi_S(\Psi|\Phi) < 1$ the decision rule will be referred to as a *possible* decision rule.

Besides, we will also need a *coverage factor*

$$\pi_S(\Phi|\Psi) = \frac{card(||\Phi \wedge \Psi||_S)}{card(||\Psi||_S)},$$

which is the conditional probability that Φ is true in S, given Ψ is true in S with the probability $\pi_S(\Psi)$.

Certainty and coverage factors for decision rules associated with Table 1 are given in Table 2.

Table 2: Certainty and coverage factors

Car	*F*	*P*	*S*	*M*	*Cert.*	*Cov.*
1	*med.*	*med.*	*med.*	*poor*	1	1/4
2	*high*	*med.*	*large*	*poor*	1	1/4
3	*med.*	*low*	*large*	*poor*	1/2	1/4
4	*low*	*med.*	*med.*	*good*	1	1/2
5	*high*	*low*	*small*	*poor*	1	1/4
6	*med.*	*low*	*large*	*good*	1/2	1/2

7 Decision rules and approximations

Let $\{\Phi_i \to \Psi\}_n$ be a set of decision rules such that:

$$\begin{aligned} &\text{all conditions } \Phi_i \text{ are pairwise mutally exclusive, i.e., } ||\Phi_i \wedge \Phi_j||_S = \emptyset, \text{for any} \\ &1 \leq i, j \leq n,\ i \neq j, \text{ and} \\ &\sum_{i=1}^{n} \pi_S(\Phi_i|\Psi) = 1. \end{aligned} \tag{1}$$

Let C and D be condition and decision attributes, respectively, and let $\{\Phi_i \to \Psi\}_n$ be a set of decision rules satisfying (1).

Then the following relationships are valid:

$$\text{a)}\quad C_*(||\Psi||_S) = || \bigvee_{\pi(\Psi|\Phi_i)=1} \Phi_i||_S,$$

$$\text{b)}\quad C^*(||\Psi||_S) = || \bigvee_{0<\pi(\Psi|\Phi_i)\leq 1} \Phi_i||_S,$$

$$\text{c)}\quad BN_C(||\Psi||_S) = || \bigvee_{0<\pi(\Psi|\Phi_i)<1} \Phi_i||_S = || \bigvee_{i=1}^{n} ||\Phi_i||_S.$$

The above properties enable us to introduce the following definitions:

i) If $||\Phi||_S = C_*(||\Psi||_S)$, then formula Φ will be called the *C-lower approximation* of the formula Ψ and will be denoted by $C_*(\Psi)$;

ii) If $||\Phi||_S = C^*(||\Psi||_S)$, then the formula Φ will be called the *C-upper approximation* of the formula Φ and will be denoted by $C^*(\Psi)$;

iii) If $||\Phi||_S = BN_C(||\Psi||_S)$, then Φ will be called the *C-boundary* of the formula Ψ and will be denoted by $BN_C(\Psi)$.

Let us consider the following example.
The *C*-lower approximation of (*M, poor*) is the formula

$$\begin{aligned} C_*(M, poor) \;=\; &((F, med.) \wedge (P, med.) \wedge (S, med.)) \vee \\ &((F, high) \wedge (P, med.) \wedge (S, large)) \vee \\ &((F, high) \wedge (P, low) \wedge (S, small)). \end{aligned}$$

The C-upper approximation of (M, *poor*) is the formula

$$\begin{aligned} C^*(M, poor) &= ((F, med.) \wedge (P, med.) \wedge (S, med.)) \vee \\ &\quad ((F, high) \wedge (P, med.) \wedge (S, large)) \vee \\ &\quad ((F, med.) \wedge (P, low) \wedge (S, large)) \vee \\ &\quad ((F, high) \wedge (P, low) \wedge (S, small)). \end{aligned}$$

The C-boundary of (M, *poor*) is the formula

$$BN_C(M, poor) = ((F, med.) \wedge (P, low) \vee (S, large)).$$

After simplification we get the following approximations

$$\begin{aligned} C_*(M, poor) &= ((F, med.) \wedge (P, med.)) \vee (F, high), \\ C^*(M, poor) &= (F, med.) \vee (F, high). \end{aligned}$$

The concepts of the lower and upper approximation of a decision allow us to define the following decision rules:

$$C_*(\Psi) \rightarrow \Psi,$$
$$C^*(\Psi) \rightarrow \Psi,$$
$$BN_C(\Psi) \rightarrow \Psi.$$

For example, from the approximations given in the example above we get the following decision rules:

((*F,med.*) $\wedge$ (*P,med.*)) $\vee$ (*F,high*) $\rightarrow$ (*M, poor*),
(*F,med.*) $\vee$ (*F,high*) $\rightarrow$ (*M, poor*),
((*F,med.*) $\wedge$ (*P,low*) $\wedge$ (*S,large*)) $\rightarrow$ (*M,poor*).

From these definitions it follows that any decision Ψ can be uniquely described by the following two decision rules:

$$C_*(\Psi) \rightarrow \Psi,$$
$$BN_C(\Psi) \rightarrow \Psi.$$

From the above calculations we can get two decision rules

((*F,med.*) $\wedge$ (*P,med.*)) $\vee$ (*F, high*) $\rightarrow$ (*M,poor*),
((*F,med.*) $\wedge$ (*P,low.*) $\wedge$ (*S,large*)) $\rightarrow$ (*M,poor*),

which are associated with the lower approximation and the boudary region of the decision (*M, poor*), respectively and describe decision (*M, poor*).

Similarly for the decision (*M, good*) we get:

(*F,low*) $\rightarrow$ (*M,good*),
((*F,med.*) $\wedge$ (*P,low.*) $\wedge$ (*S,large*)) $\rightarrow$ (*M,good*).

Table 3: Initial decision rules

Rule	*Decision*	*Certainty*	*Coverage*
certain	*poor*	1	3/4
boundary	*poor*	1/2	1/4
certain	*good*	1	1/2
boundary	*good*	1/2	1/2

8 Decision rules and the Bayes' formula

If $\{\Phi_i \rightarrow \Psi\}_n$ is a set of decision rules satisfying condition (1), then the well known formula for total probability holds:

$$\pi_S(\Psi) = \sum_{i=1}^{n} \pi_S(\Psi|\Phi_i) \cdot \pi_S(\Phi_i). \tag{2}$$

Moreover for any decision rule $\Phi \rightarrow \Psi$ the following Bayes' formula is valid:

$$\pi_S(\Phi_j|\Psi) = \frac{\pi_S(\Psi|\Phi_j) \cdot \pi_S(\Phi_j)}{\sum_{i=1}^{n} \pi_S(\Psi|\Phi_i) \cdot \pi_S(\Phi_i)}. \tag{3}$$

That is, any decision table or any set of implications satisfying condition (1) satisfies the Bayes' formula, without referring to prior and posterior probablities – fundamental in Bayesian data analysis philosophy. Bayes' formula in our case says that: if an implication $\Phi \rightarrow \Psi$ is true to the degree $\pi_S(\Psi|\Phi)$ then the implication $\Psi \rightarrow \Phi$ is true to the degree $\pi_S(\Phi|\Psi)$.

This idea can be seen as a generalization of a *modus tollens* inference rule, which says that if the implication $\Phi \rightarrow \Psi$ is true so is the implication $\sim \Psi \rightarrow \sim \Phi$.

For example, for the set of decision rules

$((F,med.) \wedge (P,med.)) \vee (F,\ high) \rightarrow (M,poor)$,
$((F,med.) \wedge (P,low) \wedge (S,large)) \rightarrow (M,poor)$,
$(F,low) \rightarrow (M,good)$,
$((F,med.) \wedge (P,low) \wedge (S,large)) \rightarrow (M,good)$,

we get the values of ceratinty and coverage factors shown in Table 3.

The above set of decision rules can be "inverted" as

$(M,poor) \rightarrow ((F,med.) \wedge (P,med.)) \vee (F,\ high)$,
$(M,poor) \rightarrow ((F,med.) \wedge (P,low) \wedge (S,large))$,
$(M,good) \rightarrow (F,low)$,
$(M,good) \rightarrow ((F,med.) \wedge (P,low) \wedge (S,large))$.

Due to Bayes' formula the certainty and coverage factors for inverse decision rules are mutually exchanged as shown in Table 4 below.

Table 4: Reversed decision rules

Rule	*Decision*	*Certainty*	*Coverage*
certain	*poor*	3/4	1
boundary	*poor*	1/4	1/2
certain	*good*	1/2	1
boundary	*good*	1/2	1/2

The Bayes' formula together with the formula for total probability allow us to compute probability of reasons for decisions, i.e., to "explain" decisions in terms of conditions.

9 Conclusions

Rough set theory is a new mathematical approach to data analysis. It has an overlap with many other disciplines, e.g. fuzzy set theory, evidence theory, statistics and others yet it can be viewed as an independent theory in its own rights.

The theory has found many applications in medicine, finances, banking, engineering, and others.

More about rough set theory, its extensions and applications can be found in the references.

Rough set theory has many important advantages in data analysis. Some of them are listed below:

- provides efficient algorithms for finding hidden patterns in data,
- finds minimal sets of data (data reduction),
- evaluates significance of data,
- generates minimal sets of decision rules from data,
- it is easy to understand and offers straightforward interpretation of results.

The method is particularly suited for parallel processing, but in order to exploit this feature fully a new hardware solutions are necessary.

Despite many important theoretical contributions and extensions of the original model some essential research problems still require due attention, however they were not discussed in this paper.

References

[1] Adams E. W. (1975) The Logic of Conditionals, an Application of Probability to Deductive Logic. D. Reidel Publishing Company, Dordrecht, Boston

[2] Bandler W., Kohout L. (1980) Fuzzy Power Sets and Fuzzy Implication Operators. Fuzzy Sets and Systems 4, 183–190

[3] Cercone N., Ziarko W. (1997) GRS: a generalized rough set model for data mining applications. In: P.P. Wang (ed.), Joint Conference of Information Sciences, March 1-5, Duke University 3, 181–186

[4] Deogun J.S., Raghavan V.V., Sarkar A. and Sever H. (1997) Data Mining: Trends in Research and Development. In: Rough Sets and Data Mining, T.Y. Lin and N. Cercone, (eds.), Kluwer Academic Publishers, 9–45

[5] Fernandes-Baizan M. C., Menasalvas Ruiz E. and Castano M. (1996) Integrating RDMS and data mining capabilities using rough sets. In: Proceedings of the Sixth International Conference, Information Processing and Management of Uncertainty in Knowledge Based Systems (IPMU'96), July 1-5, Granada, Spain, 2, 14–39–1445

[6] Greco S., Matarazzo B., Słowiński R. (1998) Fuzzy Similarity Relation as a Basis for Rough Approximations. In: Polkowski L., Skowron A. (Eds.) Rough Sets and Current Trends in Computing, Lecture Notes in Artificial Intelligence, 1424 Springer, First International Conference, RSCTC'98, Warsaw, Poland, June, Proceedings, 283–297

[7] Grzymała-Busse, J. (1986) On the Reduction of Knowledge Representation Systems. In: Proc. of the 6th International Workshop on Expert Systems and their Applications 1, Avignon, France, April 28–30, 1986, 463–478

[8] Hu X., Cercone N. (1995) Mining Knowledge Rules from Databases: A Rough Set Approach. In: Proceedings of the 12th International Conference on Data Engineering, New Orleans, 96–105

[9] Hu X., Cercone N., Ziarko W. (1997) Generation of Multiple Knowledge from Databases Based on Rough Set Theory. In: Lin T.Y., Cercone N. (Eds.) Rough Sets and Data Mining. Analysis of Imprecise Data. Kluwer Academic Publishers, Boston, Dordrecht, 109–121

[10] Hu X., Shan N., Cercone N., Ziarko W. (1994) DBROUGH: A Rough Set Based Knowledge Discovery System. In: Ras Z. W., Zemankova M. (Eds.) Proceedings of the Eighth International Symposium on Methodologies for Intelligent Systems (ISMIS'94), Charlotte, NC, October 16–19, 1994, Lecture Notes in Artificial Intelligence 869, Springer-Verlag, 386–395

[11] Kretowski M., Stepaniuk J.(1996) Selection of Objects and Attributes, a Tolerance Rough Set Approach. In: Preceedings of the Poster Session of Ninth International Symposium on Methodologies for Intelligent Systems, June 10–13, Zakopane, Poland, 169–180

[12] Lin T. Y. (Ed.) (1994) Proceedings of the Third International Workshop on Rough Sets and Soft Computing (RSSC'94). San Jose State University, San Jose, California, USA, November 10–12, 1994

[13] Lin T. Y. (Ed.) (1995) Proceedings of the Workshop on Rough Sets and Data Mining at 23rd Annual Computer Science Conference. Nashville, Tennessee, March 2, 1995

[14] Lin T. Y. (Ed.) (1996) Journal of the Intelligent Automation and Soft Computing 2/2, (special issue)

[15] Lin T. Y. (Ed.) (1996) International Journal of Approximate Reasoning 15/4, (special issue)

[16] Lin T.Y. (1996) Rough Set Theory in very large Databases. In: P. Borne, G.Dauphin-Tanguy, C. Suer, and S. El. Khattabi (eds.), Proceedings of IMACS Multiconference: Computational Engineering in Systems Applications (CESA'96) July 9-12. Lille, France, Gerf EC Lille-Cite Scientifique, 3/4, 936–941

[17] Lin T. Y., Cercone N. (Eds.) (1997) Rough Sets and Data Mining. Analysis of Imprecise Data. Kluwer Academic Publishers, Boston, Dordrecht

[18] Lin T. Y., Wildberger A. M. (Eds.) (1995) Soft Computing: Rough Sets, Fuzzy Logic, Neural Networks, Uncertainty Management, Knowledge Discovery. Simulation Councils, Inc., San Diego, CA

[19] Łukasiewicz J. (1913) Die logishen Grundlagen der Wahrscheinilchkeitsrechnung. The english translation, in: Borkowski L. (Ed.) (1970) Jan Łukasiewicz – Selected Works, North Holland Publishing Company, Amsterdam, London, Polish Scientific Publishers, Warszawa

[20] Handbook of Philosophical Logic, Vol.1, Elements of Classical Logic, Gabbay D. and Guenthner F., Kluwer Academic Publishers, Dordrecht, Boston, London, 1983.

[21] Magrez P., Smets P. (1975) Fuzzy Modus Ponens: A new Model Suitable for Applications in Knowledge-Based Systems. Information Journal of Intelligent Systems 4, 181-200

[22] Millan M., Machuca F. (1997) Using Rough Set Theory to Exploit the Data Mining Potential in Relational Databases Systems. In: P.P. Wang (ed.) Joint Conference of Information Sciences, March 1-5, Duke University, 3, 344–347

[23] Orłowska E. (Ed.) (1997) Incomplete Information: Rough Set Analysis. Physica-Verlag, Heidelberg

[24] Pal S. K., Skowron A. (Eds.) (1999) Fuzzy Sets, Rough Sets and Decision Making Processes. Springer–Verlag, Singapore

[25] Pawlak, Z. (1991) Rough Sets – Theoretical Aspects of Reasoning about Data. Kluwer Academic Publishers, Boston, Dordrecht

[26] Pawlak Z. (1998) Reasoning about Data – a Rough Set Perspective. In: Polkowski L., Skowron A. (Eds.) Rough Sets and Current Trends in Computing, Lecture Notes in Artificial Intelligence, 1424 Springer, First International Conference, RSCTC'98, Warsaw, Poland, June, Proceedings, 25–34

[27] Pawlak Z. Logic, Probability and Rough Sets (to appear)

[28] Pawlak Z. Bayes' theorem and Rough Sets (to appear)

[29] Pawlak Z. (1998) Sets, Fuzzy Sets and Rough Sets. In: Proceedings of the 5. International Wokrshop Fuzzy-Neuro Systems'98 (FNS'98), Munich, Germany, March 19–20, 1998, 1–9

[30] Pawlak Z. Skowron, A. (1994) Rough Membership Functions. In: Yaeger R. R., Fedrizzi M., and Kacprzyk J. (Eds.) Advances in the Dempster Shafer Theory of Evidence, John Wiley & Sons, Inc., New York, 251–271

[31] Polkowski L., Skowron A. (1994) Rough Mereology. In: Proceedings of the Symposium on Methodologies for Intelligent Systems, Charlotte, NC, October 16–19, Lecture Notes in Artificial Intelligence 869, Springer–Verlag, Berlin 1994, 85–94; see also: Institute of Computer Science, Warsaw University of Technology, ICS Research Report 44/94

[32] Polkowski L., Skowron A. (Eds.) (1998) Rough Sets and Current Trends in Computing, Lecture Notes in Artificial Intelligence, 1424 Springer, First International Conference, RSCTC'98, Warsaw, Poland, June, Proceedings

[33] Polkowski L., Skowron A. (Eds.) (1998) Rough Sets in Knowledge Discovery. Physica-Verlag, Vol. 1, 2

[34] Skowron A. (1994) Menagement of Uncertainty in AI: A Rough Set Approach. In: Proceedings of the conference SOFTEKS, Springer Verlag and British Computer Society, 69–86

[35] Słowiński R. (Ed.) (1992) Intelligent Decision Support – Handbook of Applications and Advances of the Rough Sets Theory. Kluwer Academic Publishers, Boston, Dordrecht

[36] Skowron A., Stepaniuk J. (1994) Generalized Approximations Spaces. In: Proceedings of the Third International Workshop on Rough Sets and Soft Computing, San Jose, November 10–12, 1994, 156–163

[37] Słowiński R. (1992) A Generalization of the Indiscernibility Relation for Rough Sets Analysis. In: Słowiński R., Stefanowski J. (Eds.) Proceedings of the First International Workshop on Rough Sets: State of the Art and Perspectives. Kiekrz – Poznań, Poland September 2–4, 1992, 68–70

[38] Słowiński R. (1992) A Generalization of the Indiscernibility Relation for Rough Set Analysis of Quantitative Information. Rivista di Matematica per le Scienze Economiche e Sociali 15/1, 65–78

[39] Słowiński R. (1992) Rough Sets with Strict and Weak Indiscernibility Relations. In: Proceedings of IEEE International Conference on Fuzzy Systems, IEEE 92CH3073-4, San Diego, California, 695–702

[40] Słowiński R., Stefanowski J. (Eds.) (1993) Foundations of Computing and Decision Sciences 18/3–4, 155–396 (special issue)

[41] Słowiński R., Stefanowski J. (1994) Rough Classification with Valued Closeness Relation. In: Diday E., Lechevallier Y., Schrader M., Bertrand P., and Burtschy B. (Eds.) New Approaches in Classification and Data Analysis, Springer–Verlag, Berlin, 482–488

[42] Słowiński R., Vanderpooten D. (1997) Similarity Relation as a Basis for Rough Approximations. Institute of Computer Science, Warsaw University of Technology, ICS Research Report **53/95** (1995); see also: Wang P. P. (Ed.) Advances in Machine Intelligence & Soft–Computing, Bookwrights, Raleigh, NC, 17–33

[43] Słowiński R., Vanderpooten D. A Generalized Definition of Rough Approximations Based on Similarity. IEEE Transactions on Data and Knowledge Engineering (to appear)

[44] Stepaniuk J. (1998) Approximations Spaces in Extensions of Rough Set Theory. In: Polkowski L., Skowron A. (Eds.) Rough Sets and Current Trends in Computing, Lecture Notes in Artificial Intelligence, 1424 Springer, First International Conference, RSCTC'98, Warsaw, Poland, June, Proceedings, 290–297

[45] Stepaniuk J., Kretowski M. (1995) Decision system based on tolerance rough sets. In: Proceedings of the Fourth International Workshop on Intelligent Information Systems, Augustów, Poland, June 5–9, 1995, 62–73

[46] Tsumoto S. Kobayashi S., Yokomori T., Tanaka H., and Nakamura A. (Eds.) (1996) Proceedings of the Fourth International Workshop on Rough Sets, Fuzzy Sets, and Machine Discovery (RSFD'96). The University of Tokyo, November 6–8, 1996

[47] Tsumoto S. (1998) Modelling Medical Diagnostic Rules Based on Rough Sets. In: Polkowski L., Skowron A. (Eds.), Rough Sets and Current Trends in Computing, Lecture Notes in Artificial Intelligence, 1424 Springer, First International Conference, RSCTC'98, Warsaw, Poland, June, Proceedings, 475–482

[48] Wang P. P. (Ed.) (1995) Proceedings of the International Workshop on Rough Sets and Soft Computing at Second Annual Joint Conference on Information Sciences (JCIS'95), Wrightsville Beach, North Carolina, 28 September - 1 October 1995

[49] Wang P. P. (Ed.) (1997) Proceedings of the Fifth International Workshop on Rough Sets and Soft Computing (RSSC'97) at Third Annual Joint Conference on Information Sciences (JCIS'97). Duke University, Durham, NC, USA, Rough Set & Computer Science 3, March 1–5, 1997

[50] Zadeh L. (1977) Fuzzy Sets as a Basis for a Theory of Possibility. Fuzzy Sets and Systems 1, 3–28

[51] Zadeh L. (1983) The Role of Fuzzy Logic in in the Management of Uncertainty in Expert Systems. Fuzzy Sets and Systems 11, 199–277

[52] Ziarko W. (1987) On Reduction of Knowledge Representation. In: Proc. 2nd International Symp. on Methodologies of Intelligent Systems, Charlotte, NC, North Holland, 99–113

[53] Ziarko W. (1988) Acquisition of Design Knowledge from Examples. Math. Comput. Modeling 10, 551–554

[54] Ziarko W. (1994) Rough Sets and Knowledge Discovery: An Overview. In: Ziarko W. (Ed.) Rough Sets, Fuzzy Sets and Knowledge Discovery (RSKD'93). Workshops in Computing, Springer-Verlag & British Computer Society, London, Berlin, 11–15

[55] Ziarko W. (Ed.) (1993) Proceedings of the Second International Workshop on Rough Sets and Knowledge Discovery (RSKD'93). Banff, Alberta, Canada, October 12–15, 1993

[56] Ziarko W. (1993) Variable Precision Rough Set Model. Journal of Computer and System Sciences 46/1, 39–59

[57] Ziarko W. (Ed.) (1994) Rough Sets, Fuzzy Sets and Knowledge Discovery (RSKD'93). Workshops in Computing, Springer-Verlag & British Computer Society, London, Berlin

[58] Ziarko W. (1995) Introduction to the Special Issue on Rough Sets and Knowledge Discovery. In: Ziarko W. (Ed.) Computational Intelligence: An International Journal 11/2, 223–226 (special issue)

[59] Ziarko W. (1996) Review of the Basics of Rough Sets in the Context of Data Mining. The Fourth Intl. Workshop on Rough Sets, Fuzzy Sets and Machine Discovery, Tokyo, 447–457