Rough Sets and Decision Analysis

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Abstract

Rough set theory is a new mathematical approach to vagueness and uncertainty. The theory has found many real life applications world wide. It is also considered as a very well suited new mathematical tool to deal with various decision problems and many papers on rough set theory and decision support have been published recently. Rough set theory gives new insight into the decision process and offers new efficient algorithms. Several real life decision problems have been successfully solved using this approach.

In this paper basic concepts of rough set theory will be given and its significance for decision analysis will be briefly discussed.

Key words: rough sets, decision analysis, vagueness, uncertainty

1 Introduction

Rough set theory seems to be a very well suited new mathematical tool to deal with various decision problems. Many papers on rough set theory and decision support have been published in recent years, "see Greco et al (1996, 1997, 1998, 1999), Kowalczyk (1998), Pal (1998), Pawlak (1994, 1997), Pawlak and Słowiński (1994), Słowiński (1992, 1993, 1994, 1995), Słowiński and Stefanowski (1993, 1995), Słowiński and Zopounidis (1995, 1994), Słowiński et al (1997)". It turned out however, that the original rough set approach does not cover some important aspect of decision analysis. As pointed out recently by Greco, Matarazzo and Słowiński "see Greco et al (1996, 1997, 1998, 1999)", decision problems usually involve attributes whose domains are preference ordered, e.g., product

quality, market share, indebtement ratio, etc. Such attributes are called criteria and may appear together with attributes, like symptoms, colors, textural features, traditionally considered in the rough set methodology. The main problem with consideration of criteria in rough approximation of ordered decision classes concerns situations where an object a is indiscernible with object b with respect to attributes and a is not worse or better than b on all criteria, however, a has been assigned to a worse decision class than b. These situations are inconsistent with respect to the dominance principle and they cannot be captured by indiscernibility or similarity relation originally used for rough approximations. The innovation proposed by Greco, Matarazzo and Słowiński consists in substitution of the indiscernibility or similarity relation by a dominance relation in the rough approximation of ordered decision classes. An important consequence of this fact is a possibility of generating rules from these approximations having a more general syntax than before, not requiring any prior discretization of quantitative attributes and distinguishing exact rules and approximate rules depending whether they observe or not the dominance principle. The introduction of the dominance relation to the rough approximations has extended the application of rough sets to analysis of preferential information in the form of decision examples and to construction of a preference model in terms of decision rules induced from these examples. This possibility made the modeling of decision maker's preferences in terms of decision rules a strong alternative for traditional preference modeling based on functions or relations. Thus, the extension of the rough set theory handling preferentially ordered attributes has far going consequences for both knowledge discovery and multiple criteria decision making. However we will refrain from discussion of dominance relation in this paper. The interested reader is advised to consult the references.

In this paper we are going to present basic concepts of rough set theory. Various exten-

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sion of the theory, "see Greco et al (1998), Kretowski and Stepaniuk (1996), Skowron and Stepaniuk (1994), Słowiński (1992, 1993), Słowiński and Vanderpooten (1997), Stepaniuk (1998), Stepaniuk and Kretowski (1995), Ziarko (1993)", however, will be not discussed here, for we would like to present in this paper rather rudiments of the theory, then its generalizations. The reader interested in more advanced results is advised to consult the references.

2 The intuitive foundations

The starting point of rough set theory is the indiscernibility relation. The indiscernibility relation identifies objects having the same properties, i.e., objects of interest having the same properties are indiscernible and consequently are treated as identical or similar. In other words the indiscernibility relation leads to clustering of elements of interest into *granules*, of indiscernible (similar) objects. In rough set theory these granules, called *elementary sets* (*concepts*) are basic building blocks (concepts) of knowledge about the universe.

Every union of elementary concepts is referred to as a *crisp* or *precise* concept (set); otherwise a concept (set) is called *rough, vague* or *imprecise.* Thus rough concepts cannot be expressed in terms of elementary concepts. However, they can be expressed approximately by means of elementary concepts by employing the idea of the *lower* and the *upper approximation* of a concept. The lower approximation of the concept is the union of all elementary concepts which are included in the concept, whereas the upper approximation is the union of all elementary concepts which have nonempty intersection with the concept. In other words, the lower and the upper approximation of a concept are the union of all elementary concepts which are surely and possibly included in the concept, respec-

tively. The difference between the lower and the upper approximation of the concept is its *boundary region*. Now it can easily be seen that a concept is rough if it has nonempty boundary region, i.e., its lower and upper approximation are nonidentical. Obviously, if the lower and the upper approximations of the concept are the same, i.e., its boundary region is empty – the concept is crisp.

3   Database

In order to give more intuitive insight into the theory we will start our consideration from a database. A simple example of a database is presented in Table 1.

Store	E	Q	L	P
1	high	good	no	profit
2	med.	good	no	loss
3	med.	good	no	profit
4	no	avg.	no	loss
5	med.	avg.	yes	loss
6	high	avg.	yes	profit

Table 1

In the database six stores are characterized by four attributes:

E – empowerment of sales personnel,

Q – perceived quality of merchandise,

L – high traffic location,

P – store profit or loss.

Each store has different description in terms of attributes E, Q, L and P, thus all stores are discernible by employing information provided by all attributes. However, stores 2 and 3 are indiscernible in terms of attributes E, Q and L, since they have the same values of these attributes. Similarly, stores 1, 2 and 3 are indiscernible with respect to attributes Q and L, etc.

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Each subset of attributes determines a partition (classification) of all objects into classes having the same description in terms of these attributes. For example, attributes Q and L aggregate all stores into the following classes $\{1, 2, 3\}, \{4\}, \{5, 6\}$. Thus, each database determines a family of classification patterns which are used as a basis of further considerations.

We are interested in the following problem: what are the characteristic features of stores having profit (or loss) in view of information available in Table 1, i.e., we want to describe set (concept) $\{1, 3, 6\}$ (or $\{2, 4, 5\}$) in terms of attributes E, Q and L. It can be easily seen that this question cannot be answered uniquely in our case since stores 2 and 3 display the same features in terms of attributes E, Q and L, but store 2 makes a profit, whereas store 3 has a loss. Therefore, we can give partial answer to this question only. In view of information contained in Table 1, we can say for sure that stores 1 and 6 make a profit, stores 4 and 5 have a loss, whereas stores 2 and 3 cannot be classified as making a profit or having a loss. Employing attributes E, Q and L, we can say that stores 1 and 6 *surely* make a profit, i.e., *surely* belong to the set $\{1, 3, 6\}$, whereas stores 1,2,3 and 6 *possibly* make a profit, i.e., possibly belong to the set $\{1, 3, 6\}$. We will say that the set $\{1, 6\}$ is the *lower approximation* of the set (concept) $\{1, 3, 6\}$, and the set $\{1, 2, 3, 6\}$ – is the *upper approximation* of the set $\{1, 3, 6\}$. The set $\{2, 3\}$, being the difference between the upper approximation and the lower approximation is referred to as the *boundary region* of the set $\{1, 3, 6\}$.

Now let us give some formal notations and definitions.

By a *database* we will understand a pair $S = (U, A)$, where U and A, are finite, nonempty sets called the *universe*, and a set of *attributes* respectively. With every attribute $a \in A$ we associate a set V_a, of its *values*, called the *domain* of a. Any subset

B of A determines a binary relation $I(B)$ on U, which will be called an *indiscernibility relation*, and is defined as follows:

$(x, y) \in I(B)$ if and only if $a(x) = a(y)$ for every $a \in A$, where $a(x)$ denotes the value of attribute a for element x.

It can easily be seen that $I(B)$ is an equivalence relation. The family of all equivalence classes of $I(B)$, i.e., partition determined by B, will be denoted by $U/I(B)$, or simple U/B; an equivalence class of $I(B)$, i.e., block of the partition U/B, containing x will be denoted by $B(x)$.

If (x, y) belongs to $I(B)$ we will say that x and y are *B-indiscernible*. Equivalence classes of the relation $I(B)$ (or blocks of the partition U/B) are refereed to as *B-elementary sets* or *B-granules*.

An equivalence relation as a basis for rough set theory seems to be a very intuitive choice, but for many applications it is not sufficient. Therefore many authors proposed another relations as a basis for rough set theory, e.g., a tolerance relation, an ordering relations and others. But for the sake of simplicity we will stick in this paper to the equivalence relation as a basis for rough set theory.

4   Approximations of sets

Now the indiscernibility relation will be used to define basic operations in rough set theory, which are defined as follows:

$$B_*(X) = \bigcup_{x \in U} \{B(x) : B(x) \subseteq X\},$$

$$B^*(X) = \bigcup_{x \in U} \{B(x) : B(x) \cap X \neq \emptyset\},$$

assigning to every $X \subseteq U$ two sets $B_*(X)$ and $B^*(X)$ called the *B-lower* and the *B-upper* *approximation* of X, respectively.

Hence, the *B*-lower approximation of a concept is the union of all *B*-granules that are included in the concept, whereas the *B-upper* approximation of a concept is the union of all *B*-granules that have a nonempty intersection with the concept. The set

$$BN_B(X) = B^*(X) - B_*(X)$$

will be referred to as the *B-boundary region* of X.

If the boundary region of X is the empty set, i.e., $BN_B(X) = \emptyset$, then X is *crisp* (*exact*) with respect to B; in the opposite case, i.e., if $BN_B(X) \neq \emptyset$, X is referred to as *rough* (*inexact*) with respect to B.

Rough sets can be also defined using a *rough membership function* "see Pawlak and Skowron (1994)", defined as

$$\mu_X^B(x) = card(B(x) \cap X)/card(B(x)).$$

Obviously

$$0 \leq \mu_X^B(x) \leq 1.$$

Value of the membership function $\mu_X^B(x)$ is a conditional probability $\pi(X|B(x))$, and can be interpreted as a degree of *certainty* to which x belongs to X (or $1 - \mu_X^B(x)$, as a degree of *uncertainty*).

The rough membership function can be generalized as follows "see Polkowski and Skowron (1994)":

$$\mu(X, Y) = card(X \cap Y)/card X,$$

where $X, Y \subseteq U, X \neq \emptyset$.

The function $\mu(X, Y)$ expresses the *degree to which X is included in Y*. Obviously, if $\mu(X, Y) = 1$, then $X \subseteq Y$.

If X is included to a degree k in X we will write $X \subseteq_k Y$, and say that X is *roughly included* in Y.

The rough inclusion can be also used to more general definition of approximations, which are defined below:

$$B_k(X) = \bigcup_{k \leq l \leq 1} \{x \in U : B(x) \subseteq_l X\},$$

$$B^k(X) = \bigcup_{0 < l \leq 1} \{x \in U : B(x) \subseteq_l X\},$$

where k $(0 < k \leq 1)$ is a threshold – and are called *k-lower* and *k-upper B-approximation* of X, respectively.

The *k-boundary B-region* of X is defined as

$$BN_B^k(X) = B^k(X) - B_k(X) = \bigcup_{0 < l \leq k} \{x \in U : B(x) \subseteq_l X\},$$

For $k = 1$ we obtain the previous definitions.

This generalization is a variety of variable precision rough set model proposed by Ziarko "see Ziarko (1993)".

5 Dependency of attributes

Another important issue in data analysis is discovering dependencies between attributes. Suppose that the set of attributes A in a database $S = (U, A)$ is divided into two subsets C and D, called *condition* and *decision* attributes respectively, such that $C \cup D = A$ and $C \cap D = \emptyset$. Such databases are called *decision tables*.

Intuitively, a set of attributes D *depends totally* on a set of attributes C, denoted $C \Rightarrow D$, if all values of attributes from D are uniquely determined by values of attributes from C. In other words, D depends totally on C, if there exists a functional dependency between values of D and C.

We would need also a more general concept of dependency, called a *partial dependency* of attributes. Intuitively, the partial dependency means that only some values of D are determined by values of C.

Formally dependency can be defined in the following way. Let D and C be subsets of A.

We will say that D *depends on* C to a *degree* k ($0 \leq k \leq 1$), denoted $C \Rightarrow_k D$, if

$$k = \gamma(C, D) = card(POS_C(D))/card\,(U),$$

where

$$POS_C(D) = \bigcup_{X \in U/D} C_*(X),$$

called a *positive region* of the partition U/D with respect to C, is the set of all elements of U that can be uniquely classified to blocks of the partition U/D, by means of C.

Obviously

$$\gamma(C, D) = \sum_{X \in U/D} card(C_*(X))/card\,(U).$$

If $k = 1$ we say that D *depends totally* on C, and if $k < 1$, we say that D *depends partially* (to a *degree* k) on C, and if $k = 0$, D *does not depend on* C.

The coefficient k expresses the ratio of all elements of the universe, which can be properly classified to blocks of the partition U/D, employing attributes C and will be called the *degree of the dependency.*

For example in Table 1 the degree of dependency between the attribute P and the set of attributes $\{E, Q, L\}$ is 2/3.

Obviously if D depends totally on C then $I(C) \subseteq I(D)$. That means that the partition generated by C is finer than the partition generated by D.

The function $\gamma(C, D)$ can be regarded as a generalization of the rough inclusion function $\mu(X, Y)$, for it expresses to what degree partition generated by C, i.e., U/C is included in the partition generated by D, i.e., U/D.

6 Reduction of attributes

A reduct is a minimal set of condition attributes that preserves the degree of dependency. It means that a reduct is a minimal subset of condition attributes that enables to make the same decisions as the whole set of condition attributes.

Formally if $C \Rightarrow_k D$ then a minimal subset C' of C, such that $\gamma(C, D) = \gamma(C', D)$ is called a *D-reduct* of C.

For example, in Table 1 we have two reducts $\{E, Q\}$ and $\{E, L\}$ of condition attributes $\{E, Q, L\}$. This means that Table 1 can be replaced either by Table 2 or Table 3.

Store	E	Q	P
1	high	good	profit
2	med.	good	loss
3	med.	good	profit
4	no	avg.	loss
5	med.	avg.	loss
6	high	avg.	profit

Table 2

Store	E	L	P
1	high	no	profit
2	med.	no	loss
3	med.	no	profit
4	no	no	loss
5	med.	yes	loss
6	high	yes	profit

Table 3

It is easy to check that both Table 2 and Table 3 preserve degree of dependency between attributes P and E, Q, L.

Reduction of attributes is the fundamental issue in rough set theory.

7   Decision rules

Let S be a decision table and let C and D be condition and decision attributes, respectively.

By Φ, Ψ etc. we will denote logical formulas built from attributes, attribute-values and logical connectives (*and, or, not*) in a standard way. We will denote by $|\Phi|_S$ the set of all object $x \in U$ satisfying Φ and refer to as the *meaning* of Φ in S.

The expression $\pi_S(\Phi) = card(|\Phi|_S)/card(U)$ will denote the probability that the formula Φ is true in S.

A *decision rule* is an expression in the form *"if...then..."*, written $\Phi \rightarrow \Psi$; Φ and Ψ are referred to as *condition* and *decision* of the rule respectively.

A decision rule $\Phi \rightarrow \Psi$ is *admissible* in S if $|\Phi|_S$ is the union of some C-elementary sets, $|\Psi|_S$ is the union of some D-elementary sets and $|\Phi \wedge \Psi|_S \neq \emptyset$. In what follows we will consider admissible decision rules only.

Examples of decision rules admissible in Table 1 are given below:

1) (*E, med.*) *and* (*Q, good*) $\rightarrow$ (*P, loss*)

2) (*E, no*) $\rightarrow$ (*P, loss*) $\qquad\qquad\qquad$ (*)

3) (*E, med.*) *and* (*Q, avg.*) $\rightarrow$ (*P, loss*)

With every decision rule $\Phi \to \Psi$ we associate a *certainty factor*

$$\pi_S(\Psi|\Phi) = card(|\Phi \wedge \Psi|_S)/card(|\Phi|_S),$$

which is the estimate of conditional probability that Ψ is true in S given Φ is true in S with the probability $\pi_S(\Phi)$.

Besides, we will also need a *coverage factor* "see Tsumoto (1998)"

$$\pi_S(\Phi|\Psi) = card(|\Phi \wedge \Psi|_S)/card(|\Psi|_S),$$

which is the estimate of conditional probability that Φ is true in S given Ψ is true in S with the probability $\pi_S(\Psi)$.

For example, for the set at decision rules (*) we have:

$$\pi(\Psi|\Phi_1) = 1/2,\ \pi(\Psi|\Phi_2) = 1,\ \pi(\Psi|\Phi_3) = 1,$$

and

$$\pi(\Phi_1|\Psi) = 1/3,\ \pi(\Phi_2|\Psi) = 1/3,\ \pi(\Phi_3|\Psi) = 1/3,$$

where Φ_i is the condition of rule i, and $\Psi = (P, loss)$. Let $\{\Phi_i \to \Psi\}_n$ be a set of decision rules such that all conditions Φ_i are pairwise mutually exclusive, i.e., $|\Phi_i \wedge \Phi_j|_S = \emptyset$, for any $1 \le i, j \le n$, $i \ne j$, and

$$\sum_{i=1}^{n} \pi_S(\Phi_i|\Psi) = 1. \qquad\qquad (**)$$

Then the following property holds:

$$\pi_S(\Psi) = \sum_{i=1}^{n} \pi_S(\Psi|\Phi_i) \cdot \pi_S(\Phi_i). \qquad\qquad (***)$$

For any decision rule $\Phi \to \Psi$ the following property is true:

$$\pi_S(\Phi|\Psi) = \pi_S(\Psi|\Phi) \cdot \pi_S(\Phi)/\sum_{i=1}^{n} \pi_S(\Psi|\Phi_i) \cdot \pi_S(\Phi_i). \qquad\qquad (****)$$

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This relationship first was observed by Łukasiewicz "see Borkowski (1970), Łukasiewicz (1913)".

It is easy to check that the set of decision rules (*) statistics properties (**), (***) and (****), i.e.,

$$\pi(\Psi) = 1/2 \cdot 1/3 + 1 \cdot 1/6 + 1 \cdot 1/6 = 1/2, \ \pi(\Phi_1|\Psi) = 1/3, \ \pi(\Phi_2|\Psi) = 1/3, \ \pi(\Phi_3|\Psi) = 1/3.$$

It can be easily seen that the relationship between the certainty factor and the coverage factor, expressed by the formula (****) is the Bayes' Theorem. However, the meaning of Bayes' Theorem in this case differs from that postulated in statistical inference. In statistical data analysis based on Bayes' Theorem, we assume that prior probability about some parameters without knowledge about the data is given. The posterior probability is computed next, which tells us what can be said about prior probability in view of the data. In the rough set approach the meaning of Bayes' Theorem is unlike. It reveals some relationships in the database, without referring to prior and posterior probabilities, and it can be used to reason about data in terms of approximate (rough) implications. Thus, the proposed approach can be seen as a new model for Bayes' Theorem, and offers a new approach to data analysis.

8 Conclusions

Basic concepts of rough set theory are, approximations, dependencies and decision rules. They can be used to solve many problems of decision analysis. Besides, the rough set approach offers new insight into the decision process and offers new efficient algorithms. Several real life decision problems have been successfully solved using the rough set approach. Nevertheless, several problems, both theoretical and practical, need further research, e.g.,

efficient algorithms for data reduction and discretization. Also widely available software for data analysis based on rough set theory is badly needed.

Acknowledgements

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