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The idea of a rough fuzzy controller and its application to the stabilization of a pendulum–car system

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Abstract

This paper presents the idea of a rough fuzzy controller with application to the stabilization of a pendulum–car system. The structure of such a controller based on the concept of a fuzzy controller (fuzzy logic controller) is suggested. The results of a simulation comparing the performance of both controllers are shown. From these results we infer that the performance of the proposed rough fuzzy controller is satisfactory.

Keywords: Fuzzy set; Rough set; Fuzzy controller; Rough fuzzy controller; Inverted pendulum

1. Introduction

Fuzzy set theory introduced by Zadeh [11] in 1965 has provided a mathematical tool useful for modelling uncertain (imprecise) and vague data to be present in many real situations. In his seminal paper [18] (published in 1973) Zadeh recommended a fuzzy rule-based approach to the analysis of complex systems and decision processes, the essential concept presented in that paper being the compositional rule of inference which forms a basis of an important inference method handling uncertain (imprecise) information often called approximate reasoning. The approximate reasoning processes should be automated and it implies directly the necessity of software and even hardware implementation of mechanisms realizing such approximate reasoning. It leads to the construction of algorithms and computer systems being able to manipulate uncertain (imprecise) information.

In many real processes control relies heavily upon human experience. Skilled human operators can control such processes quite successfully without any qualitative models. The control strategy of the human operator is mainly based on linguistic qualitative knowledge concerning the behaviour of an ill-defined process. Numerous applications of the fuzzy controller (fuzzy logic controller) to the control of

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In order to express numerically how a set can be approximated using all equivalence classes of R we will use the accuracy of approximation of X in A_R (accuracy measure)

$$\alpha_R(X) = \frac{card\,\underline{R}X}{card\,\bar{R}X}, \tag{2}$$

where $X \neq 0$.

As we can see, if X is R-exactly approximated in A_R then $\alpha_R(X) = 1$.

If X is R-roughly approximated in A_R then $0 < \alpha_R(X) < 1$.

Below we use another measure related $\alpha_R(X)$ defined as

$$\varrho_R(X) = 1 - \alpha_R(X) \tag{3}$$

and referred to as R-roughness of X.

Roughness, as opposed to accuracy, represents the degree of inexact approximation of X in A_R. Additional numerical characteristics of imprecision, e.g.

- the rough R-membership function of the set X (or rm-function, for short) defined as [15]

$$\mu_X^R(x) = \frac{card([x]_R \cap X)}{card([x]_R)}, \tag{4}$$

- coefficient characterizing the uncertainty of membership of the element to the set with respect to the possessed knowledge

$$\mu_X(x) = \frac{card([x]_R \cap X)}{card(\mathbb{U})}, \tag{5}$$

- the quality of approximation of the family $F = \{X_1, X_2, \dots, X_n\}$, $X_i \subset \mathbb{U}$, $X_i \cap X_j \neq \emptyset$, $(i, j = 1, 2, \dots, n)$ by R

$$\gamma_R(F) = \frac{\sum_{i=1}^{n} card(\underline{R}X_i)}{card(\mathbb{U})} \tag{6}$$

and others are presented in [14] and [15]. The above-mentioned measures may be used in rough fuzzy controller synthesis.

In the next section general structures of fuzzy and rough fuzzy controllers will be described.

3. The structures of fuzzy and rough fuzzy controllers

In this section we will recall a rule-based approach to an approximate reasoning process based on the compositional rule of inference [18], which preserves a maximal amount of information contained in the rules and observations and forms a common basis of both fuzzy and rough fuzzy controllers. The design of the fuzzy and rough fuzzy controllers includes the specification of the collection of control rules consisting of linguistic statements that link the controller inputs with appropriate outputs, respectively. Such knowledge can be collected and delivered by a human expert (e.g. operator of an industrial complex process). This knowledge, expressed by a finite number ($r = 1, 2, \dots, n$) of the heuristic rules of the MISO type (two input single output), may be written in the form

$$R_{\text{MISO}}^{(r)}\text{: } \textbf{if } x \text{ is } E_i^{(r)} \textbf{ and } y \text{ is } DE_j^{(r)} \textbf{ then } u \text{ is } U_k^{(r)}, \tag{7}$$

where $E_i^{(r)}$, $DE_j^{(r)}$ denote values of linguistic variables x, y representing error and change in error (conditions) defined in the universes of discourse X, Y, and $U_k^{(r)}$ stands for the value of linguistic variable u of action (conclusion) in the universe of discourse U. The linguistic values $E_i^{(r)}$ and $DE_j^{(r)}$ may be represented by respective fuzzy or rough sets (terms of linguistic variables).

If we employ a knowledge base of MISO system, the compositional rule of inference may be written symbolically as

$$U' = (DE' \times E') \circ R. \tag{8}$$

The global relation R now aggregating MISO system rules will be expressed as

$$R = \mathbf{also}_r(R^{(r)}), \tag{9}$$

where an implicit sentence connective "also" denotes any t- or s-norm (e.g. **min**, **max** operators) or averages [6, 8]. Symbol $\circ$ stands for the compositional rule of inference operators (e.g. **sup-min**, **sup-prod**, etc.).

An output of the MISO-type controller, which has a knowledge base containing a finite number of rules connected by means of the implicit rule connective "also" interpreted as a union (**max** operator), takes the following form:

$$U' = (DE' \times E') \circ \bigcup_r (E_i^{(r)} \times DE_j^{(r)} \to U_k^{(r)}) = \bigcup_r U'^{(r)}, \tag{10}$$

where in this case $\times$ stands for the explicit sentence connective "and".

Applying **sup-min** operations to the compositional rule of inference, the membership function of the output set may be expressed as follows:

$$U'(u) = \sup_{x,y} \min\left[\min(DE'(y), E'(x)), \max_r (E_i^{(r)} \times DE_j^{(r)} \to U_k^{(r)})(x, y, u)\right]. \tag{11}$$

If we take fuzzy sets E', DE' as singletons (measurements), i.e. $E'(x) = \delta_{x,x0}$ and $DE'(y) = \delta_{y,y0}$, where

$$\delta_{z,z0} = \begin{cases} 1 & \text{for } z = z_0, \\ 0 & \text{otherwise,} \end{cases} \tag{12}$$

the function of the output may be simplified:

$$U'(u) = \max_r [(E_i^{(r)} \times DE_j^{(r)} \to U_k^{(r)})(x_0, y_0, u)]. \tag{13}$$

Now let us consider the rule connective "also" as an intersection. In this case the following inequality is valid:

$$\begin{aligned} \underline{U}'_\cap &= (DE' \times E') \circ \left[\bigcap_r (E_i^{(r)} \times DE_j^{(r)} \to U_k^{(r)})\right] \\ &\subseteq \bigcap_r (DE' \times E') \circ (E_i^{(r)} \times DE_j^{(r)} \to U_k^{(r)}) = \bigcap_r U'^{(r)} = \bar{U}'_\cap. \end{aligned} \tag{14}$$

By means of functions that characterize fuzzy and rough sets the inequality may be rewritten as follows:

$$\begin{aligned} \underline{U}'_\cap(u) &= \sup_{x,y} \min\left[\min(DE'(y), E'(x)), \min_r (E_i^{(r)} \times DE_j^{(r)} \to U_k^{(r)})(x, y, u)\right] \\ &\leqslant \min_r \sup_{x,y} \min[\min(DE'(y), E'(x)), (E_i^{(r)} \times DE_j^{(r)} \to U_k^{(r)})(x, y, u)] = \bar{U}'_\cap(u). \end{aligned} \tag{15}$$

Considering E', DE' as singletons (in this case $U'_\cap(u) = \underline{U}'_\cap(u) = \bar{U}'_\cap(u)$) we get a simple formula:

$$U'_\cap(u) = \min_r [(E_i^{(r)} \times DE_j^{(r)} \to U_k^{(r)})(x_0, y_0, u)]. \tag{16}$$

Assuming the explicit sentence connective "and" as product (**prod**) and **sup-prod** for the compositional rule of inference we obtain formulas analogical to those given above.

Taking into account the fact that none of the operators **max** or **min** are sufficiently "good" as a rule connective "also", we may try to compensate one of them for another one [6, 19]. The convex linear combination of the type

$$(1-p)(x *_t y) + p(x *_s y) \tag{17}$$

may be used ($*_t$ denotes t-norm and $*_s$ s-norm, respectively).

Such a combination can be written in the form

$$U'_C(u) = (1-p)\,U' *_t(u) + p\,U' *_s(u). \tag{18}$$

Taking intersection and union for $*_t$ and $*_s$, we obtain

$$U'_C(u) = (1-p)\,U'_\cap(u) + p\,U'(u). \tag{19}$$

Let us notice that for parameter value $p = 0.5$ we get an arithmetic average that is proportional to the sum (plus) interpreted as the rule connective "also". Of course, for parameter value $p = 0$ we obtain a maximal compensation of **max** operator by **min** operator [6].

Applying the defuzzification operator denoted $DEFUZZ$ to both sides of the last equality we get the following equation:

$$DEFUZZ[U'_C(u)] = DEFUZZ[(1-p)\cdot U'_\cap(u) + p\cdot U'(u)]. \tag{20}$$

Choosing the defuzzification operator as a centre of gravity (COG) we get

$$COG[U'_C(u)] = COG[(1-p)\cdot U'_\cap(u) + p\cdot U'(u)] \tag{21}$$

or

$$u'_C = (1-p)\cdot u'_\cap + p\cdot u', \tag{22}$$

where $u'_\cap$ and u' stand for centres of gravity of intersection and union respectively, i.e.

$$u'_\cap = \frac{\sum u_i U'_\cap(u_i)}{\sum U'_\cap(u_i)}, \tag{23}$$

$$u' = \frac{\sum u_i U'(u_i)}{\sum U'(u_i)}. \tag{24}$$

At this point let us assume for simplicity that $p = 1$ and let us now consider the Cartesian product of the knowledge base rule i.e. $E_i^{(r)} \times DE_j^{(r)} \subseteq \mathbf{X} \times \mathbf{Y}$, the projection of which on the x–y plane represents an element of the input image (as input pattern) of the knowledge base.

The whole input image consisting of input patterns which are the above-mentioned Cartesian products:

$$II^{(r)} = E_i^{(r)} \times DE_j^{(r)} \tag{25}$$

obtained from all rules may be written in the form

$$II = \bigcup_r II^{(r)}. \tag{26}$$

Using alfa-cut of corresponding sets we get

$$II = \bigcup_\alpha \bigcup_r \alpha II_\alpha^{(r)}. \tag{27}$$

The formula written above can be expressed by means of the membership function

$$II(x, y) = \sup_{\alpha} \max_{r} \alpha II_{\alpha}^{(r)}(x, y), \tag{28}$$

where

$$II_{\alpha}^{(r)}(x, y) = (E_i^{(r)}(x) *_t DE_j^{(r)}(y))_{\alpha} \tag{29}$$

and $*_t$ denotes a respective t-norm (e.g. **min**) [6, 8].

It also means that all input patterns projected on the x–y plane create an input image. In other words, the input image is made up of overlapping input patterns obtained by means of respective rules of a knowledge base.

After each input pattern has been formed separately in the implication rule, we can differentiate three stages of processing the input image.

Firstly, the rules are combined by means of the implicit rule connective "also" (at this point the input image can be created).

Secondly, the input image is processed by means of the compositional rule of inference on condition that input information is provided.

Thirdly, a defuzzification procedure is employed to obtain the respective control value.

The idea of the input image has some advantages. It enables us to test the completeness and correctness of the knowledge base. An input image for the knowledge base of a fuzzy controller can be obtained, e.g. by means of an ordinary fuzzy partition of the input space. In this case the number of rules increases exponentially with the number of inputs. Another way of obtaining an input image for this case may be the application of fuzzy clustering (fuzzy c-means) [11]. A way of obtaining an input image for the knowledge base of a rough fuzzy controller is described in Section 5.

Hence, we can say that the input image constitutes a basis for processing in both fuzzy and rough fuzzy controllers.

In the next section we derive a mathematical model of nonlinear system whose control simulation results will be compared using both controllers.

4. The mathematical model of the pendulum–car system

In this section we will consider a simplified mathematical model describing a dynamic behaviour of an inverted pendulum–car system. The pendulum–car system [3] is shown in Fig. 1 and consists of

- a car moving along a line on two rails of limited length,
- a pendulum hinged in the car by means of ball bearings, rotating freely in the plane containing this line,
- a car driving device containing a DC motor, a DC amplifier, and a pulley-belt transmission system.

Such a system is characterized by an unstable equilibrium point in upright position of the pendulum, a stable equilibrium point in pendant position, as well as two uncontrollable points when the pendulum is in horizontal position.

Now let us describe the model mathematically. Assuming that the pendulum is a rigid body, both friction and damping forces are neglected in the system. Thus we obtain differential equations, describing the system, by projecting respective forces on to corresponding axes. Here, however, we will apply the Lagrange method (cf. [3]). Assuming that $L = E - V$, we get

$$L = \tfrac{1}{2}M\dot{x}^2 + \tfrac{1}{2}mv^2 - (-\tfrac{1}{2}\, mgl\cos\theta), \tag{30}$$

where v can be obtained from the cosine formula:

$$v^2 = \dot{x}^2 + (\tfrac{1}{2}\, l\dot{\theta})^2 - 2\dot{x}\tfrac{1}{2}\, l\dot{\theta}\cos(180 - \theta). \tag{31}$$

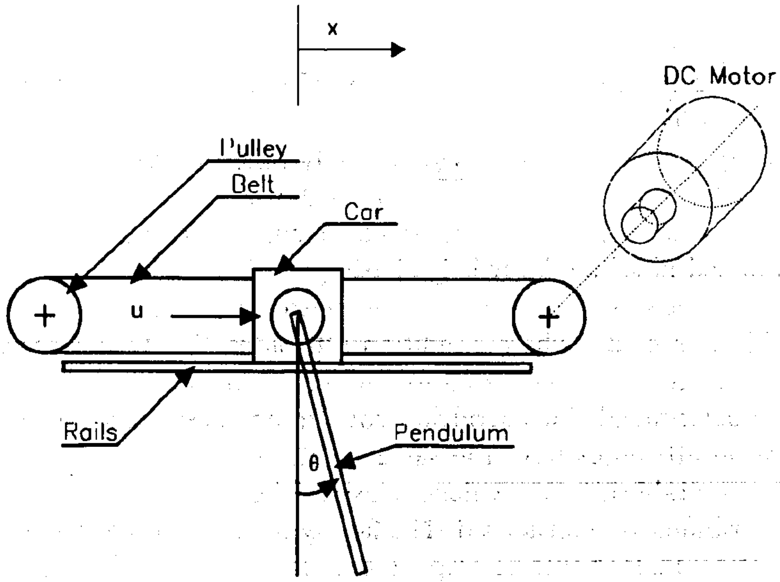


Fig. 1. The pendulum–car system.

After elementary transformations we get

$$v^2 = (\tfrac{1}{2} l \dot{\theta} \cos\theta + \dot{x})^2 + (\tfrac{1}{2} l \dot{\theta} \sin\theta)^2. \tag{32}$$

Applying Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = u, \tag{33}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0, \tag{34}$$

we obtain the following differential equations describing dynamic behaviour of the pendulum–car system:

$$(M + m)\ddot{x} - \tfrac{1}{2} ml \dot{\theta}^2 \sin\theta + \tfrac{1}{2} ml \ddot{\theta} \cos\theta = u, \tag{35}$$

$$\tfrac{1}{2} ml \ddot{x} \cos\theta + \tfrac{1}{3} ml^2 \ddot{\theta} + \tfrac{1}{2} mgl \sin\theta = 0. \tag{36}$$

Rearranging Eqs. (35) and (36) we get

$$\ddot{x} = -g \tan\theta - \frac{2}{3} \frac{l}{\cos\theta} \ddot{\theta} \tag{37}$$

$$\ddot{\theta} = \frac{[(M + m) g \tan\theta + \tfrac{1}{2} ml \sin\theta \cdot \dot{\theta}^2] + u}{-\tfrac{2}{3}(M + m) l \sec\theta + \tfrac{1}{2} ml \cos\theta}. \tag{38}$$

Taking into account subsequent simplification only the last equation is used for stabilizing the system in two positions: upright and slightly deflected from vertical.

5. Simulation results

Numerical results obtained by simulating the control of the pendulum will be presented here. Block diagram of the control system is depicted in Fig. 2.

For simplicity and clarity of the proposed methods only nine-rule knowledge bases were used in this experiment. The knowledge base for a fuzzy controller (Fig. 3) was created using an ordinary fuzzy partition of input space. Each coordinate of the input space was evenly divided into three parts. In this way we obtained the above-mentioned nine-rule knowledge base.

The corresponding knowledge base for a rough fuzzy controller was created in the following way. At the beginning, a decision table was made, where condition attributes $C = \{e, de\}$ corresponded to a decision attribute $D = \{u\}$. In such a decision table an indiscernibility relation with respect to both condition and decision attributes can be determined. Two arbitrary rows in decision table are indiscernible if and only if their condition and decision attributes have the same values. As we can see, the indiscernibility relation divides all rows of the decision table into equivalence classes. The family of equivalence classes is denoted by C^* when the condition attributes are considered. The family of equivalence classes is denoted by D^* while decision attributes are considered. It should be emphasized here that one of the main applicational aspects of rough sets is to approximate elements of D^* with elements of C^*. For the condition attributes the following domain was assumed: $V_e = V_{de} = \{1, 1.5, 2, 2.5, 3\}$ whereas the domain $V_d = \{1, 2, 3, 4, 5\}$ was assumed for the decision attribute. The respective nondeterministic decision table contained 49 decision rules. Division of the universum $\mathbb{U}$ with respect to the indiscernibility relation for decisions gives $D^* = \{X_1, X_2, X_3, X_4, X_5\}$.

Accuracy measure and roughness for the elements of D^* were calculated:

$$\alpha_R(X_1) = 1/9, \qquad \rho_R(X_1) = 8/9,$$

$$\alpha_R(X_2) = 1/13, \qquad \rho_R(X_2) = 12/13,$$

$$\alpha_R(X_3) = 3/35, \qquad \rho_R(X_3) = 32/35,$$

$$\alpha_R(X_4) = 1/13, \qquad \rho_R(X_4) = 12/13,$$

$$\alpha_R(X_5) = 1/9, \qquad \rho_R(X_5) = 8/9.$$

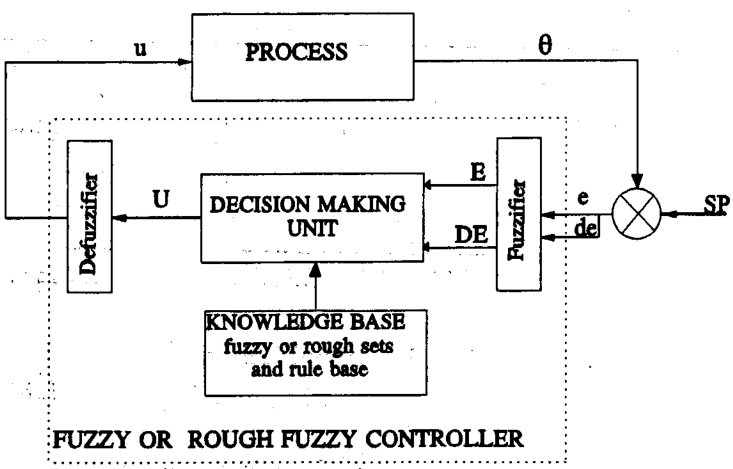


Fig. 2. Block diagram of the control system.

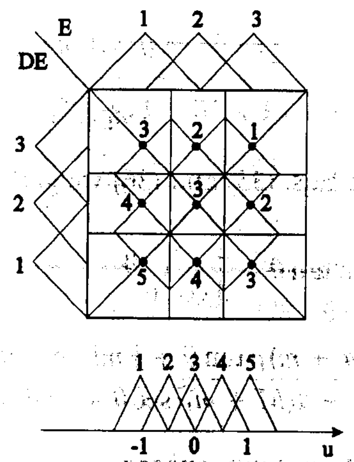


Fig. 3. A scheme of a knowledge base for a fuzzy controller.

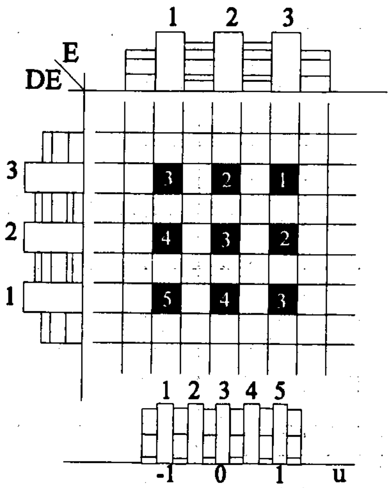
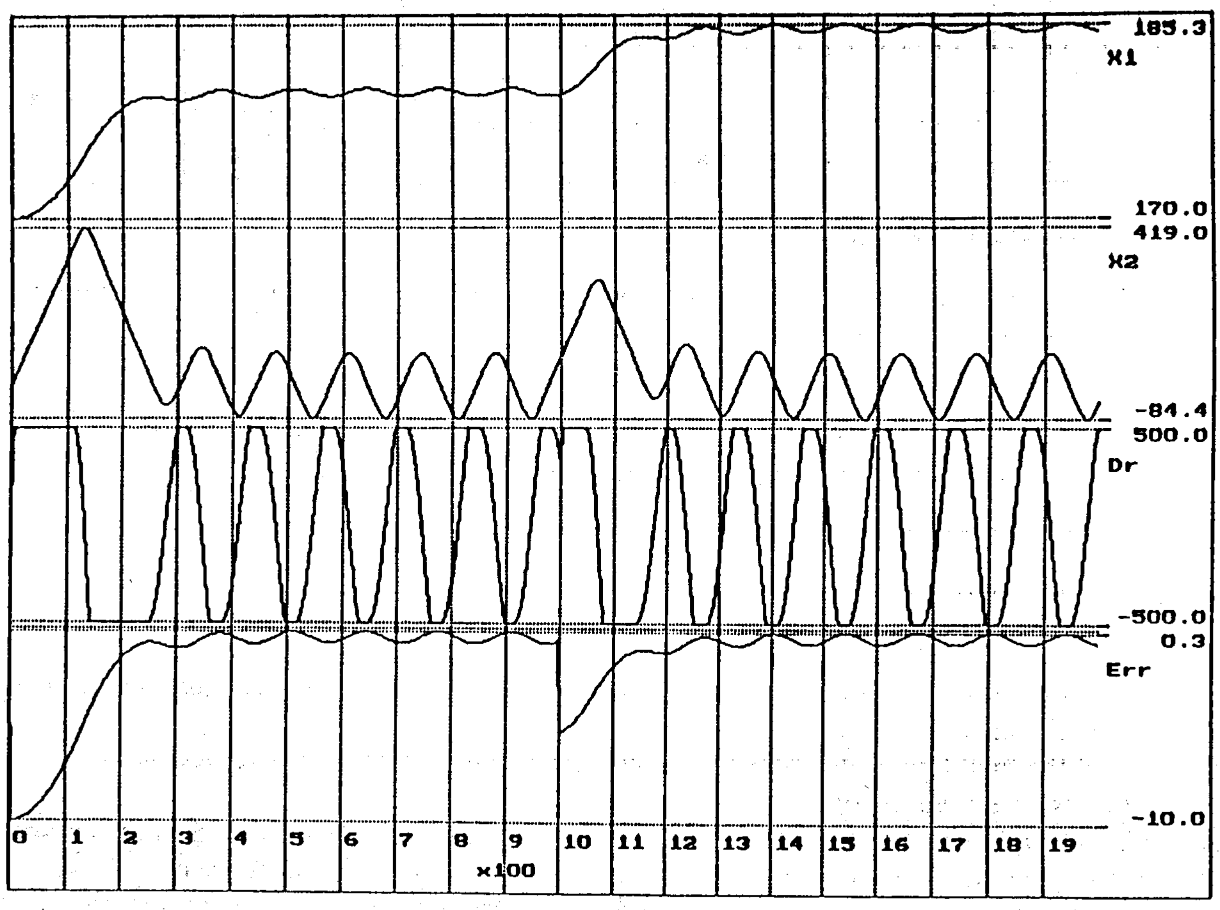
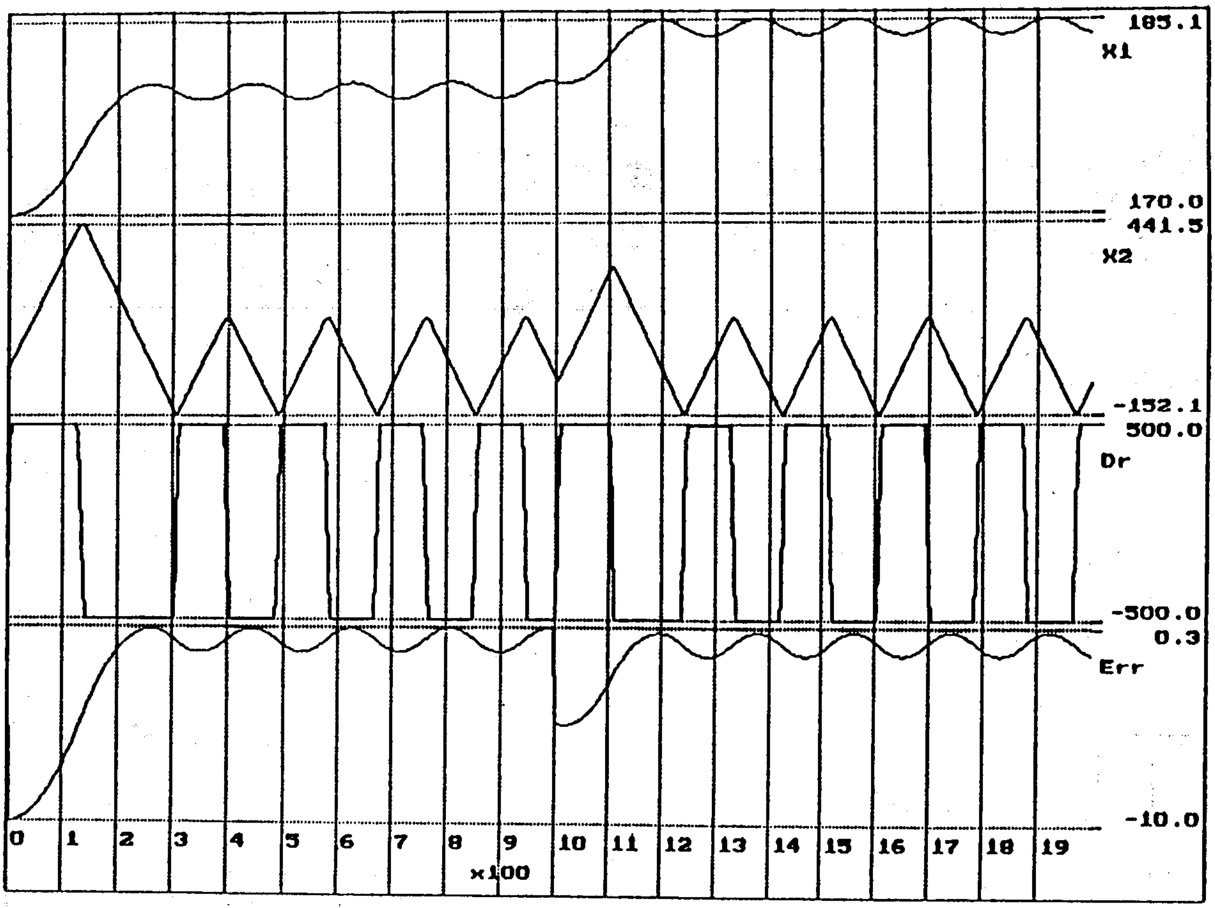


Fig. 4. A scheme of a knowledge base for a rough fuzzy controller.



QI = 582.616107

Fig. 5. Results of control; fuzzy controller applied. X1 = deflection angle, X2 = derivative of deflection angle, Dr = control value, Err = control error.



QI = 620.749039

Fig. 6. Results of control; rough fuzzy controller (using accuracy measure) applied. X1 = deflection angle, X2 = derivative of deflection angle, Dr = control value, Err = control error.

By analogy, the accuracy measure and roughness for the respective rough sets were obtained on the basis of appropriate information systems for the classification of error and change in error:

$$\alpha_R(X_1) = 1/3, \quad \rho_R(X_1) = 2/3,$$
$$\alpha_R(X_2) = 1/5, \quad \rho_R(X_2) = 4/5,$$
$$\alpha_R(X_3) = 1/3, \quad \rho_R(X_3) = 2/3.$$

Using the rm-functions we obtain value 1 for certain regions and 0.5 for all uncertain regions of condition attributes (error, change in error) and a decision attribute as well.

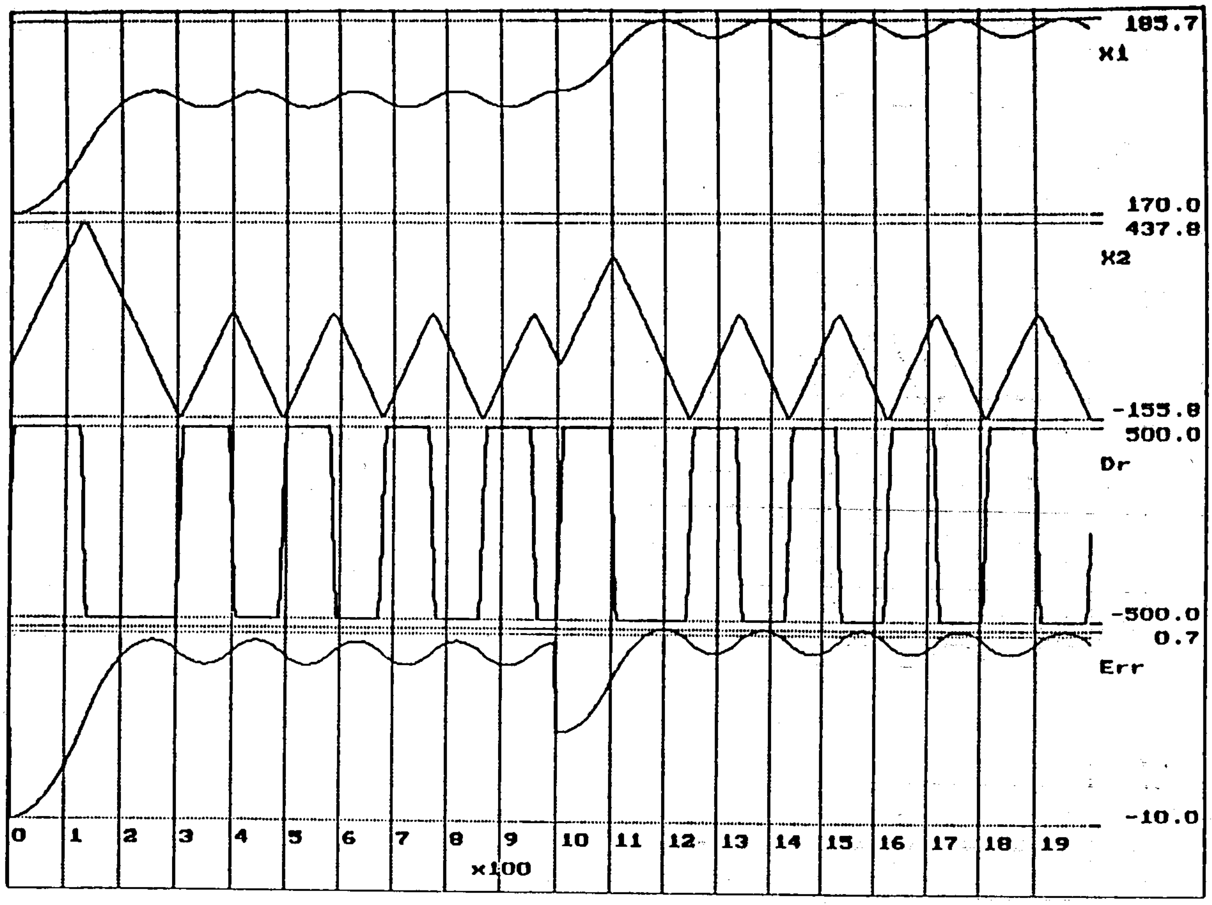
The scheme of the final knowledge base for a rough fuzzy controller using accuracy measure, roughness and rm-function is presented in Fig. 4.

The fuzzy and rough fuzzy controllers used in our experiments employed **sup-prod** as the compositional operator, **prod** for the "and" connective between rule premises, **sum** for the sentence connective "also".

The control objective was: (a) to stabilize the pendulum in upright (180°) position and (b) to stabilize it in a position that would be slightly deflected from vertical, i.e. 185°.

The parameters of the model were taken as follows [3]:

$$M = 2.8\ \text{kg}, \quad m = 0.2\ \text{kg}, \quad l = 0.75\ \text{m}, \quad g = 9.81\ \text{m/s}^2.$$



QI = 632.551801

Fig. 7. Results of control; rough fuzzy controller (using roughness) applied. X1 = deflection angle, X2 = derivative of deflection angle, Dr = control value, Err = control error.

The initial position of the pendulum was 170° and the initial control value was 0.

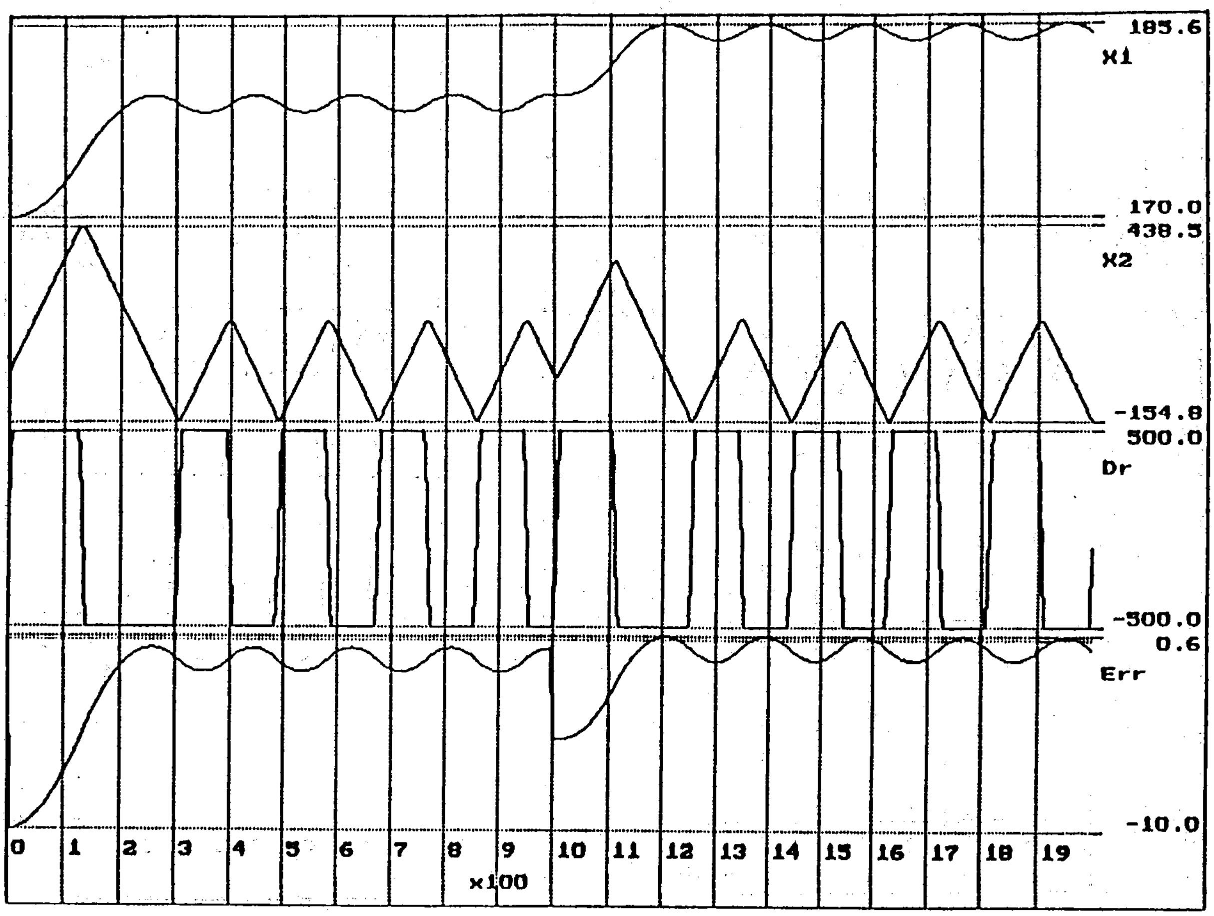
The deflection angle $\theta = \mathrm{X1}$, its derivative $\mathrm{d}\theta/\mathrm{d}t = \mathrm{X2}$, control value $u = \mathrm{Dr}$ and control error $\theta_i - SP_i = \mathrm{Err}$ as functions of time for both types of controllers are shown in Figs. 5–8.

For the purpose of comparative study a quality index ($Q1$) was defined as

$$Q1 = \frac{1}{N+1} \sum_{i=0}^{N} (\theta_i - SP_i)^2, \tag{39}$$

where SP_i is the set point and $N + 1$ is the total number of observation points.

Comparing the controllers we notice that both the fuzzy controller and rough fuzzy controller behave similarly. In case of the fuzzy controller we obtained a slightly better quality index (582.6 versus 620.7 (using accuracy measure), 632.5 (using roughness) and 636.3 (using rm-functions) of the rough fuzzy controller. However, it should be noted that the speed of a rough fuzzy controller is much higher than that of a fuzzy controller. In the case of the defuzzification procedure (COG), the rough fuzzy controller is $n \times 10$ (n fluctuates from 1 to 2 depending on hardware and software used for implementing the rough fuzzy controller) times faster than that of the fuzzy controller.



QI = 636.343620

Fig. 8. Results of control; rough fuzzy controller (using rm-functions) applied. X1 = deflection angle, X2 = derivative of deflection angle, Dr = control value, Err = control error.

6. Concluding remarks

The results of numerical experiments show that a rough fuzzy controller performs more crudely than a conventional fuzzy controller in view of the stabilization task. Its crude performance can be explained by the fact that rough fuzzy controller operates on a finite number of control levels (in our case – 5 levels).

However, a rough fuzzy controller works much faster than a conventional fuzzy controller under the assumption of COG defuzzification procedure. The accuracy of control is satisfactory, taking into account the low number of rules (9 rules) and the low number (3 and 5) of fuzzy and rough sets in input and output spaces.

The difference between the membership functions of fuzzy sets and rm-functions of rough sets should be emphasized. The former are usually intuitively designed whereas the latter, being upper semicontinuous functions, are computable in an algorithmic way [14]. However, from the computational point of view we may consider the rough membership functions of the rough sets as the step-function approximation of the membership functions of fuzzy sets.

While controlling the system it can be observed that using a fuzzy logic controller we get a smooth control value as a function of time; applying a rough fuzzy controller we get a sharp function of time for the control value. Nevertheless, the quality index does not differ very much for both controllers.

Hardware solutions of a rough fuzzy controller based on embedded Intel microprocessors have also been tried out.

As an objective for further research, the reasonable partition of input and output spaces for "fuzzy" and "rough fuzzy" knowledge bases as well as reasonable number of rules should be considered. A grid for a rough set representing linguistic values of error, change of error and control output in a rough fuzzy controller also needs special attention.

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