PRACE IPI PAN • ICS PAS REPORTS

Ewa Orłowska, Zdzisław Pawlak

Representation of nondeterministic information

450

September 1981
WARSZAWA

INSTYTUT PODSTAW INFORMATYKI POLSKIEJ AKADEMII NAUK
INSTITUTE OF COMPUTER SCIENCE POLISH ACADEMY OF SCIENCES
00-901 WARSAW, P. O. Box 22, POLAND

Ewa Orłowska, Zdzisław Pawlak

REPRESENTATION OF NONDETERMINISTIC INFORMATION

450

Warsaw, September 1981

Pracę zgłosił Andrzej Blikle

Mailing address: Ewa Orłowska
Zdzisław Pawlak
Institute of Computer Science
Polish Academy of Sciences
P.O. Box 22
00-901 Warszawa, PKiN

ISSN 0138-0648

Nakład 700 egz. Ark. wyd. 0,90; ark. druk. 1,50. Papier offset. kl. III, 70 g, 70 x 100. Oddano do druku we wrześniu 1981 r. W.D.N. Zam. nr 700/c

Abstract . Содержание . Streszczenie

In the paper we introduce the notion of a nondeterministic information system. We present the modal logic which enables us to define the language of such systems. These languages can be used both to ask queries and to define nondeterministic information.

Представление недетерминистической информации

В работе введено понятие системы недетерминистической информации. Представлена модальная логика, на базе которой определяются языки таких систем. Эти языки могут быть использованы как языки запросы, а также как языки для определения недетерминистической информации.

Reprezentacja niedeterministycznej informacji

W pracy wprowadzono pojęcie systemu niedeterministycznej informacji. Wprowadzona jest logika modalna, w oparciu o którą definiuje się języki takich systemów. Języki te mogą być używane jako języki pytań oraz jako języki do definiowania niedeterministycznej informacji.

1. INTRODUCTION

In the paper we present a mathematical model of information systems in which informations involve nondeterminizm. The basic concepts we use are object, attribute, and generalized value of an attribute. A generalized value of an attribute is a set, whose elements are interpreted as possible values of the attribute. The given model is called nondeterministic information system. We also present the logic which enables us to define a language for a nondeterministic information systems. Formulas of the language are interpreted as sets of objects, satisfying conditions mentioned in the formulas. Besides the usual set-theoretical operations on sets of objects, we admit the special modal operations. Informally speaking these operations relax indeterminizm involved in informations contained in a system.

In connection with information systems we usually deal with the two problems. First, to define a query language enabling us to retrieve informations from a system, and second to define informations, that is to find expressions of the language of the system, providing a description of a given information. The language presented in the paper can be treated as the starting point to the investigations in both of these directions.

A certain subclass of nondeterministic information systems, called incomplete information systems was considered in Lipski [3] and Jaegermann [1].

2. NONDETERMINISTIC INFORMATION SYSTEM

A nondeterministic information system is a generalization of the notions of attribute based information systems introduced in Pawlak [6, 7]. Information system presented in [6] consists of a set Ob of objects, a set At of attributes a family $\{V_a\}_{a\in At}$ of sets of values of attributes and an information function

$\rho: Ob \times At \to V = \bigcup_{a\in At} V_a$ such that for each $o \in Ob$ and each $a \in At$ $\rho(o,a) \in V_a$. The language for such systems was defined in Marek, Pawlak [4]. The generalization of such systems is a many-valued information system [7]. In many valued information systems we assume that $\rho \subseteq Ob \times At \times V$ is not necessarily a function, but an arbitrary relation such that if $(o,a,v) \in \rho$ then $v \in V_a$. However, there are situations when the characteristics of given objects is determined neither by an information function, nor by an information relation. It might be the case that the only information we have for an object o and an attribute a is a set U_{oa} of possible values of a for o. For example, we usually do not know exactly person's age, we can only give its possible values. To deal with such situations Pawlak introduced the notion of an approximate information system [7]. In an approximate information system we consider information function ρ as a function from set $Ob \times At$ into set $P(V)$ of all the subsets of set V such that $\rho(o,a) \subseteq V_a$, and moreover we assume that there exists a relation $\mu \subseteq Ob \times At \times V$ such that if $(o,a,v) \in \mu$ then $v \in \rho(o,a)$. This means that $\rho(o,a)$ consists of all the possible values of attribute a for object o and some of these values are assumed by a for o. This

notion of approximate information system is impredicative, as a result of the assumption about the existence of a relation μ. It causes difficulties in investigating such systems.

In the paper we consider a generalization of many-valued and approximate information systems, called nondeterministic information system.

By a nondeterministic information system we mean a system

$$S = (Ob, At, \{V_a\}_{a\in At}, \{U_{oa}\}_{o\in Ob, a\in At})$$

where

Ob is a non-empty set, whose elements are called objects,

At is a non-empty set, whose elements are called attributes,

V_a is a non-empty set, whose elements are called values of the attribute a,

$U_{oa} \subseteq V_a$ is a non-empty set, whose elements are called possible values of the attribute a for the object o.

The intended interpretation of sets V_a and U_{oa} is as follows. Set V_a consists of all the admited values of the attribute a and set U_{oa} consists of those values of the attribute a which might be taken by the object o. We assume neither the existence of an information relation, nor the existence of any function determining how many values an attribute can take for a given object. Any subset $U \subseteq V_a$ for $a \in At$ is said to be a generalized value of the attribute a.

By using the notion of a nondeterministic information system we can represent informations which involve nondeterminizm.

Examples of such informations are the future states of an object, for instance we do not know which of the university courses students choose next year. We only know some equipossible courses they might choose.

Given a nondeterministic information system $S = (Ob, At, \{V_a\}_{a \in At}, \{U_{oa}\}_{o \in Ob, a \in At})$, we define binary relations R_S and Q_S in the set Ob as follows:

$(o,o') \in R_S$ iff for all $a \in At$ $U_{oa} \subseteq U_{o'a}$

$(o,o') \in Q_S$ iff for all $a \in At$ $U_{oa} \cap U_{o'a} \neq \emptyset$

We say that object o is informationally contained in object o' whenever $(o,o') \in R_S$. Similarly, we say that object o is informationally connected with object o' whenever $(o,o') \in Q_S$.

The following properties of the given relations follow immediately from the respective definitions.

(2.1) For any nondeterministic information system S the following conditions are satisfied:

(a) relation R_S is reflexive and transitive,

(b) relation Q_S is reflexive and symmetric.

The indiscernibility relation $\widetilde{S} \subseteq Ob \times Ob$ of a nondeterministic information system S is defined as follows:

$(o,o') \in \widetilde{S}$ iff for each $a \in At$ $U_{oa} = U_{o'a}$

(2.2) The following conditions are equivalent

(a) $(o,o') \in \widetilde{S}$,

(b) $(o,o') \in R_S$ and $(o', o) \in R_S$

The problem of definability of sets of objects in nondeterministic information systems can be formulated and investigated by using the methods developed in [2,5].

Example 2.1

Consider a system of medical information. Let set Ob of objects be the set of diseases, set At of attributes be the set of some parameters of a patient's body, e.g. temperature, blood pressure, state of throat etc. Set V_a of values of a parameter a is a set of all the possible values of that parameter. For example, if a is temperature then V_a is the set of elements of the interval $35^\circ - 42^\circ$. For a disease o and a parameter a the set U_{oa} is the set of values of a which might occur in the disease o.

We now discuss the meaning of relations of informational inclusion and informational connection in the system. According to the definition a disease o is informationally contained in a disease o' iff for all parameters a the set of possible values of a in the disease o is contained in the set of possible values of a in the disease o'. In other words, the disease o can occur during the disease o'. For example cold can occur during flu. Similarly, a disease o is informationally connected with a disease o' iff for all parameters a there are values of a which are common for the diseases o and o'.

The meaning of the indiscernibility relation in the system of medical information is as follows. Diseases o and o' cannot be distinguished by means of body parameters considered in the system iff for all these parameters sets of their possible values in the diseases o and o' coincide.

In the process of medical diagnosis we are given values of some parameters of a patient's body and we have to decide what does he suffer from. In section 7 we shall show how we can use the language of a nondeterministic information system, to describe informations needed in making a diagnosis.

In the following four sections we present the logic NIL of nondeterministic information.

3. FORMALIZED LANGUAGE OF LOGIC NIL

Formulas of logic NIL are constructed from the symbols of the following disjoint sets:

denumerable set Var At of attribute variables, denoted by $x, y, \ldots$

denumerable set VarPV of generalized attribute values variables, denoted by $X, Y, \ldots$

set $\{\neg, <, >, \Diamond\}$ of the unary information operations,

set $\{\rightarrow\}$ consisting of the binary information operation,

set $\{(,)\}$ of brackets.

Set For of formulas is the least set defined as follows:

$(x,X) \in$ For for each $x \in$ VarAt and each $X \in$ VarPV,

$A \in$ For implies $\neg A$, $<A$, $>A$, $\Diamond A \in$ For,

$A, B \in$ For imply $A \rightarrow B \in$ For.

Formulas of the form (x,X) are called atomic formulas. Let AFor denote the set of all the atomic formulas of the given language. Formulas are intended to be schemes of expressions defining sets of objects of nondeterministic information systems. In the next sections we define the semantics of the presented language and the language of nondeterministic information systems.

4. SEMANTICS

Given a nondeterministic information system $S = (Ob, At, \{V_a\}_{a \in At}, \{U_{oa}\}_{o \in Ob, a \in At})$, by a model we mean the

structure

$$M = (Ob, R, Q, m)$$

where R and Q are binary relations in set Ob such that R is reflexive and transitive and Q is reflexive and symmetric, and $m: AFor \rightarrow P(Ob)$ is a meaning function which assigns sets of objects to atomic formulas.

We say that a model M is standard if R is the relation of informational inclusion in S and Q is the relation of informational connection in S, and the meaning function m is defined as follows:

$$m(x,X) = \{o \in Ob: U_{on(x)} = n(X)\}$$

where $n: VarAt \cup VarPV \rightarrow At \cup P(V)$ is a naming function such that $n(x) \in At$ and $n(X) \in P(V)$ for each $x \in VarAt$ and each $X \in VarPV$.

We now define the notion of satisfaction of a formula in a model. Given a model $M = (Ob,R,Q,m)$ and object $o \in Ob$, we say that formula A is satisfied in model M by the object o ($o\ sat_M A$) if the following conditions are satisfied:

$o\ sat_M (x,X)$ iff $o \in m(x,X)$

$o\ sat_M \neg A$ iff non $o\ sat_M A$

$o\ sat_M < A$ iff there is an $o' \in Ob$ such that $(o',o) \in R$ and $o'\ sat_M A$

$o\ sat_M > A$ iff there is an $o' \in Ob$ such that $(o,o') \in R$ and $o'\ sat_M A$

$o\ sat_M \Diamond A$ iff there is an $o' \in Ob$ such that $(o,o') \in Q$ and $o'\ sat_M A$

$o\ sat_M (A \rightarrow B)$ iff non $o\ sat_M A$ or $o\ sat_M B$.

A formula A is said to be valid in model M iff for each $o \in Ob$ we have $o\ sat_M A$. A set Γ of formulas is valid in M iff every formula $A \in \Gamma$ is valid in M. Set Γ is satisfiable iff there is a model M such that Γ is valid in M. A formula A is a tautology ($\models A$) iff it is valid in any model. A formula A is said to be a semantical consequence of set Γ of formulas ($\Gamma \models A$) iff for any model M A is valid in M whenever all formulas from Γ are valid in M.

The information operations admitted in the language enable us to define information operations corresponding to disjunction, conjunction an necessity operators:

$$A \vee B = \neg A \to B$$

$$A \wedge B = \neg (A \to \neg B)$$

$$\sqsubset A = \neg < \neg A$$

$$\sqsupset A = \neg > \neg A$$

$$\Box A = \neg \Diamond \neg A.$$

The semantical properties of these operations are the following:

$o\ sat_M (A \vee B)$ iff $o\ sat_M A$ or $o\ sat_M B$

$o\ sat_M (A \wedge B)$ iff $o\ sat_M A$ and $o\ sat_M B$

$o\ sat_M \sqsubset A$ iff for all $o' \in Ob$ if $(o,o') \in R$ then $o'\ sat_M A$

$o\ sat_M \sqsupset A$ iff for all $o' \in Ob$ if $(o',o) \in R$ then $o'\ sat_M A$

$o\ sat_M \Box A$ iff for all $o' \in Ob$ if $(o,o') \in Q$ then $o'\ sat_M A$.

In the next sections we give the axiomatization of the presented logic and prove the completeness theorem for consistent theories based on the logic. The problem of axiomatization of the theory of standard models is still open.

5. AXIOMATIZATION

We admit the following axioms for logic NIL:

A1. All formulas having the form of tautologies of the classical propositional calculus

A2 $\sqsubset (A \to B) \to (\sqsubset A \to \sqsubset B)$

A3 $\sqsupset (A \to B) \to (\sqsupset A \to \sqsupset B)$

A4 $\Box (A \to B) \to (\Box A \to \Box B)$

A5 $A \to \sqsupset < A$

A6 $A \to \sqsubset > A$

A7 $\sqsupset A \to A$

A8 $\Box A \to A$

A9 $\sqsupset A \to \sqsupset\sqsupset A$

A10 $A \to \Box \Diamond A$

Axioms A2, A3, A4 assure that logic NIL is a normal modal logic. Axioms A5 and A6 say that operation $<$ is inverse with respect to operation $>$. Axioms A7 and A8, provide the reflexivity of relations R_S and Q_S. Axiom A9 provides the transitivity of relation R_S. Axiom A10 provides the symmetry of relation Q_S.

Rules of inference of logic NIL are modus ponens and necessitation rules:

R1 $$\frac{A,\ A \to B}{B}$$

R2 $\frac{A}{\sqsubset A}$

R3 $\frac{A}{\sqsupset A}$

R4 $\frac{A}{\square A}$

A proof of a formula A from a set Γ of formulas is a finite sequence of formulas each of which is either an axiom or an element of set Γ or else is obtainable from earlier formulas by rules of inference, and the last formula in the sequence is A. We say that a formula A is derivable from a set Γ of formulas ($\Gamma \vdash A$) iff there is a proof of A from Γ. A formula A is a theorem of logic NIL($\vdash A$) whenever there is a proof of A from the empty set of formulas. A set Γ of formulas is consistent if non $\Gamma \vdash A \wedge \neg A$.

(5.1) Soundness theorem

(a) $\Gamma \vdash A$ implies $\Gamma \models A$,

(b) Γ satisfiable implies Γ consistent.

Proof:

The axioms are easily seen to be tautologies and the rules clearly preserve validity. This proves (a), from which (b) follows immediately.

6. COMPLETENESS

Let Γ be a consistent set of formulas and let relation $\sim \subseteq$ For x For be defined as follows:

$A \sim B$ iff $\Gamma \vdash A \to B$ and $\Gamma \vdash B \to A$

(6.1) The following conditions are satisfied:

(a) $\sim$ is an equivalence relation on the set For,

(b) $\sim$ is a congruence with respect to $\vee, \wedge, \neg$,

(c) if $A \sim B$ then $\sqsupset A \sim \sqsupset B$, $\sqsubset A \sim \sqsubset B$ and $\square A \sim \square B$.

The proof of condition (a) and (b) is the same as in the classical propositional logic [8]. The proof of condition (c) uses axioms A2, A3, A4 and rules R2, R3, R4.

Let $For/_\sim$ denote the set of all the equivalence classes of relation $\sim$. For a formula A let $[A]$ denote the class generated by A.

(6.2) The following conditions are satisfied:

(a) The Lindenbaum algebra $(For/_\sim, \cup, \cap, -, 0, 1)$, where

$[A] \cup [B] = [A \vee B]$

$[A] \cap [B] = [A \wedge B]$

$-[A] = [\neg A]$

$0 = [A \wedge \neg A]$

$1 = [A \vee \neg A]$

is a non-degenerate Boolean algebra,

(b) $[A] \leq [B]$ iff $\Gamma \vdash A \to B$,

(c) $\Gamma \vdash A$ iff $[A] = 1$,

(d) $[\neg A] \neq 0$ iff non $\Gamma \vdash A$.

The proof of this theorem is given in [8].

Let $\mathfrak{F}$ be the family of all the maximal filters in the algebra $For/_\sim$. Set $\mathfrak{F}$ is non-empty since the algebra $For/_\sim$ is non-degenerate. We define the relation $R_o \subseteq \mathfrak{F} \times \mathfrak{F}$ as follows:

$(F,G) \in R_o$ iff for any formula A if $[\sqsupset A] \in F$ then $[A] \in G$.

(6.3) For any filters $F, G \in \mathfrak{F}$ the following condti-tions are equivalent:

(a) $(F,G) \in R_o$,

(b) for any formula A if $[\sqsupset A] \in G$ then $[A] \in F$,

(c) for any formula A if $[A] \in F$ then $[< A] \in G$,

(d) for any formula A if $[A] \in G$ then $[> A] \in F$.

Proof:

Assume condition (a), and suppose that $[\sqsupset A] \in G$ and $[A] \notin F$. It follows that $[\neg A] \in F$ and by A5 $[\sqsupset < \neg A] \in F$. By (a) we obtain $[< \neg A] \in G$. Since $\vdash \sqsupset A \wedge < B \rightarrow <(A \wedge B)$, we have $[<(A \wedge \neg A)] \in G$. But G is a proper filter, con-tradiction. Hence condition (b) holds.

Let us now assume that condition (b) holds and suppose that $[A] \in F$ and $[< A] \notin G$. Hence $[\neg A] \in G$ and by (b) we have $[\neg A] \in F$, contradiction. Hence condition (c) holds.

Assume condition (c) and suppose that $[A] \in G$ and $[> A] \notin F$. Then $[\neg > A] \in F$ and by (c) we have $[< \neg > A] \in G$. By A6 $[\neg A] \in G$, contradiction. Hence condition (d) holds.

We also have (d) implies (a). For suppose not, then $[\neg A] \in G$, and by (d) $[> \neg A] \in F$. Since $\vdash \sqsupset A \wedge > B \rightarrow >(A \wedge B)$, we have $[>(A \wedge \neg A)] \in F$, contradiction.

(6.4) Relation R_o is reflexive and transitive.

Proof:

Reflexivity of $\bar{R}_o$ follows from A7 and transitivity from A9.

We define the relation $Q_o \subseteq \mathfrak{F} \times \mathfrak{F}$ as follows:

$(F,G) \in Q_o$ iff for any formula A if $[\Box A] \in F$ then $[A] \in G$.

(6.5) Relation Q_o is reflexive and symmetric.

Proof:

Reflexivity of Q_o follows from A8 and symmetry from A10.

(6.6) For any filter $F \in \mathfrak{F}$ and for any formula A the following conditions are satisfied:

(a) if $[> A] \in F$ then there exists an $G \in \mathfrak{F}$ such that $(F,G) \in R_o$ and $[A] \in G$,

(b) if $[< A] \in F$ then there exists an $G \in \mathfrak{F}$ such that $(G,F) \in R_o$ and $[A] \in G$,

(c) if $[\Diamond A] \in F$ then there exists an $G \in \mathfrak{F}$ such that $(F,G) \in Q_o$ and $[A] \in G$.

Proof:

Let $[> A] \in F$ and consider set $X_F = \{[B] : [\sqsupset B] \in F\}$. Set X_F is non-empty, since $[A \vee \neg A] \in X_F$. Consider filter F' generated by set $X_F \cup \{[A]\}$. We have $F' = \{[B]$: there exist $[A_1], \ldots, [A_n] \in X_F$, $n \geqslant 1$, such that $[A_1] \cap \ldots \cap [A_n] \cap [A] \leqslant [B]\}$. We shall show that for any $[A_1], \ldots, [A_n] \in X_F$ we have $[A_1] \cap \ldots \cap [A_n] \cap [A] \neq 0$. Suppose conversely, then

$$\Gamma \vdash A_1 \wedge \ldots \wedge A_n \rightarrow \neg A.$$

By A3 and R3 we have

$$\Gamma \vdash \sqsupset(A_1 \wedge \ldots \wedge A_n) \rightarrow \sqsupset \neg A.$$

Since $[\sqsupset A_1], \ldots, [\sqsupset A_n] \in F$, we have

$$[\sqsupset A_1 \wedge \ldots \wedge \sqsupset A_n] \in F.$$

Since $\vdash \sqsupset A \wedge \sqsupset B \rightleftarrows \sqsupset(A \wedge B)$, we have

$$[\sqsupset(A_1 \wedge \ldots \wedge A_n)] \in F$$

Hence $[\sqsupset \neg A] \in F$ so $[\neg > A] \in F$, what contradicts the assumption. Thus filter F' is proper. Let G be the maximal filter containing F'. We clearly have $[A] \in G$ and $(F,G) \in R_o$. Hence condition (a) is satisfied. The proof of conditions (b) and (c) is similar.

We now consider the canonical model

$$M_o = (Ob_o, R_o, Q_o, m_o)$$

where

$Ob_o = \mathcal{F}$

$m_o(x,X) = \{F \in \mathcal{F} : [x,X] \in F\}$.

(6.7) For any formula A and for any filter $F \in \mathcal{F}$

$F \text{ sat}_{M_o} A$ iff $[A] \in F$.

Proof:

The proof is by induction with respect to the length of a formula. If A is of the form (x,X) then the theorem holds by the definition of m_o. If A is of the form $\neg B$ or $B \to C$ then the theorem follows easily from the respective definitions. If A is of the form $< B$ or $> B$ then the theorem follows from (6.3) and (6.6) (a) and (b). If A is of the form $\diamond B$ then the theorem follows from (6.6) (c).

(6.8) Completeness theorem

(a) $\Gamma \models A$ implies $\Gamma \vdash A$,

(b) Γ consistent implies Γ satisfiable.

Proof:

Suppose non $\Gamma \vdash A$. By (6.2)(d) we have $[\neg A] \neq 0$. By Rasiowa-Sikorski lemma [8] there is a maximal filter F_o such that $[\neg A] \in F_o$. By (6.7) $F_o \text{sat}_{M_o} \neg A$. For any formula $B \in \Gamma$ we have $[B] = 1$, and hence $F_o \text{sat}_{M_o} B$. Since $\Gamma \models A$, we have $F_o \text{sat}_{M_o} A$, contradiction. Condition (b) follows immediately from (6.7).

7. LANGUAGE OF NONDETERMINISTIC INFORMATION SYSTEM

By using logic NIL we can define a language for a nondeterministic information system. Given a system $S = (Ob, At, \{V_a\}_{a \in At}, \{U_{oa}\}_{o \in Ob, a \in At})$, and a standard model $M = (Ob, R_S, Q_S, m)$ with a naming function n, the set $For(M)$ is the least set satisfying the following conditions:

$(n(x), n(X)) \in For(M)$ for each $x \in VarAt$ and each $X \in VarPV$,

$t \in For(M)$ implies $\neg t, < t, > t, \diamond t \in For(M)$

$t_1, t_2 \in For(M)$ imply $t_1 \vee t_2$, $t_1 \wedge t_2$, $t_1 \to t_2 \in For(M)$.

The expected meaning of formulas from set $For(M)$ is a set of objects from system S. We define the function $val_M: For(M) \to Ob$ which assigns sets of objects to formulas as follows:

$$val_M(n(x), n(X)) = \{o \in Ob: U_{o,n(x)} = n(X)\}$$

$$val_M \neg t = - val_M t$$

$$val_M(t_1 \vee t_2) = val_M t_1 \cup val_M t_2$$

$$val_M(t_1 \wedge t_2) = val_M t_1 \cap val_M t_2$$

$$val_M(t_1 \to t_2) = -val_M(t_1) \cup val_M t_2$$

$$val_M < t = \{o \in Ob: \text{there exists an } o' \in Ob \text{ such that } (o', o) \in R_S \text{ and } o' \in val_M t\}$$

$$\mathrm{val}_M > t = \{ o \in Ob: \text{ there exists an } o' \in Ob \text{ such that } (o,o') \in R_S \text{ and } o' \in \mathrm{val}_M t \}$$

$$\mathrm{val}_M \diamond t = \{ o \in Ob: \text{ there exists an } o' \in Ob \text{ such that } (o,o') \in Q_S \text{ and } o' \in \mathrm{val}_M t \}$$

Thus the value of a formula of the form $(n(x),n(X))$ consists of those objects for which the set $U_{on(x)}$ of possible values of the attribute $n(x)$ coincides with $n(X)$. The value of a formula of the form $> t$ consists of the objects which are informationally contained in objects from the set $\mathrm{val}_M t$. In particular, if t is $(n(x),n(X))$ then $>(n(x),n(X))$ represents the set of such objects o for which $U_{on(x)} \subseteq n(X)$. A formula $< t$ corresponds to a set of objects which informationally contain objects from set $\mathrm{val}_M t$. In particular $<(n(x),n(X))$ is the set of those objects o for which $n(X) \subseteq U_{on(x)}$. A formula $\diamond t$ represents a set of objects which are informationally connected with objects from set $\mathrm{val}_M t$. In particular, $\diamond(n(x),n(X))$ represents the set of those objects o for which $U_{on(x)} \cap n(X) \neq \emptyset$.

Example 7.1

Consider the system of medical information, described in example 2.1, a standard model M connected with this system, and the set $For(M)$ of formulas of the language of this system. Atomic formulas of this language are of the form:

(body parameter, set of values of this parameter)

for example (temperature, $37^\circ - 38^\circ$). If a is a body parameter, and U is a set of values of this parameter, then $\mathrm{val}_M < (a,U)$ is the set of diseases in which a certain disease o' might occur such that set of values of a in the disease o' coincides with U. Hence, the expression $<(a,U)$ can be treated as a query what are diseases which might be accompanied by a disease in which parameter a assumes values from at U. Similarly, $\mathrm{val}_M > (a,U)$ is the set of diseases which might occur together with diseases in which set of possible values of the parameter a coincides with U. Lastly, $\mathrm{val}_M \diamond (a,U)$ is the set of diseases in which for all parameters the sets of values of these parameters have some common elements with the respective sets of values connected with diseases which for the parameter a assume values from U.

Expressions obtained from atomic expressions by using the classical information operations have the usual interpretation. For example, expression $(a_1,U_1) \wedge (a_2,U_2)$ represents the set of diseases in which parameters a_1 and a_2 assume values from U_1 and U_2, respectively. Hence, if U_1 and U_2 are the sets of values of parameters a_1 and a_2, observed for a certain patient, then set $\mathrm{val}_M((a_1,U_1) \wedge (a_2,U_2))$ is the set of diseases he possibly suffer from.

A naming function n induces the mapping from set For into set $For(M)$. For the sake of simplicity this mapping will be denoted by n too:

$$n(x,X) = (n(x),n(X))$$

$$n(\alpha t) = \alpha n(t) \quad \text{for } \alpha = \neg, <, >, \diamond$$

$$n(t_1 \beta t_2) = n(t_1) \beta n(t_2) \quad \text{for } \beta = \vee, \wedge, \rightarrow$$

Thus we can treat formulas from set $For(M)$ as expressions obtained from formulas of logic NIL through assignment of names of attributes and names of generalized values for attribute variables and generalized values variables respectively.

The following fact follows immediately from the respective definitions.

(7.1) For any formula A and for any standard model M the following conditions are equivalent:

(a) $o \in val_M n(A)$

(b) $o\ sat_M A$.

8. SUMMARY

In the paper we presented a method of dealing with nondeterministic information. We introduced the concept of nondeterministic information system as a mathematical model of any collection of such informations. We defined the data definition language intended to describe nondeterministic information. We sketched the possible applications by means of examples concerning systems of medical information.

REFERENCES

1. M. Jaegermann: Information Storage and Retrieval Systems with Incomplete Information.
Part I Fundamenta Informaticae 2(1978) 17-41.
Part II Fundamenta Informaticae 2(1979) 141-166.
2. E. Konrad, E. Orłowska, Z. Pawlak: Knowledge Representation Systems. Definability of Informations.
ICS PAS Reports 433 (1981).
3. W. Lipski: On Semantic Issues Connected with Incomplete Information Databases.
ACM Transactions on Database Systems 4(1979) 262-296.
4. W. Marek, Z. Pawlak: Information Storage and Retrieval Systems. Mathematical Foundations.
Theoretical Computer Science 1(1976) 331-354.

5. E. Orłowska, Z. Pawlak: Expressive power of knowledge representation systems.
ICS PAS Reports 432 (1981).
6. Z. Pawlak: Information systems,
ICS PAS Reports 338 (1978)
7. Z. Pawlak: Systemy informacyjne. Podstawy teoretyczne.
WNT Warsaw, to appear.
8. H. Rasiowa, R. Sikorski: Mathematics of Metamathematics.
PWN Warsaw (1970).

Contents