Second European Congress on Intelligent Techniques and Soft Computing
Aachen, Germany, September 20–23, 1994
Proceedings
Volume 3

EUFIT '94

ELITE-Foundation
Promenade 9
D-52076 Aachen, Germany
Phone: +49-2408-6969
Fax: +49-2408-94582

Acknowledgement:
Partial Support for EUFIT '94 has been provided by the European Commission (ESPRIT Basic Research).

APPLICATION OF A ROUGH FUZZY CONTROLLER TO THE STABILIZATION OF AN INVERTED PENDULUM

*E. Czogala[1], **A. Mrózek and ***Z. Pawlak

*Fachhochschule Wurzburg-Schweinfurt, ZAM
Ignaz-Schon-Str. 11, 97421 Schweinfurt, Germany

**Institute of Theoretical and Applied Computer Science
Polish Academy of Sciences
Bałtycka 5, 44-100 Gliwice, Poland

***Warsaw University of Technology
Institute of Computer Science
Nowowiejska 15/19, 00-665 Warsaw, Poland

Abstract

This paper describes the application of a rough fuzzy controller to the stabilization of an inverted pendulum using two knowledge bases (a nine-rule base and a forty-nine-rule base). In comparison with fuzzy controllers, rough controllers work considerably faster, however, their performance may be cruder. It has been shown that the control results obtained by means of a forty-nine-rule are more satisfactory than those obtained from a nine-rule knowledge base.

Keywords: fuzzy set, rough set, fuzzy controller, rough fuzzy controller, inverted pendulum.

1. Introduction

In 1974 Mamdani published a paper describing the application of a fuzzy controller (fuzzy logic controller) to the control of an ill-defined complex process [6].

Fuzzy controllers, synthesized from a collection of qualitative "rules of thumb", are applicable to the control of the processes (plants) that are mathematically difficult to understand and describe [1,2,5].

The most important advantages of fuzzy controllers are: intuitive design, reflecting the behaviour of human operator, the fact that the model of the controlled process is not necessary (an important feature when ill-defined processes are to be controlled), and good control quality (not worse than that of classical controllers)

The main disadvantages of fuzzy controllers are: the necessity of the acquisition and preprocessing of the human operator's knowledge about the controlled process, sequential search through rule bases, and time consuming defuzzification methods [1].

The alternative approach to manipulating incomplete or imprecise information was presented by Pawlak in 1982 as a rough set theory [7]

The idea of a rough fuzzy controller based on the notion of a rough set [7,8] was introduced in [3], by analogy with the concept of a fuzzy controller. It can be observed that rough fuzzy controllers work faster than fuzzy controller, although they are more crude due to the finite number of levels on which they operate.

In this paper we compare control results obtained by the use of a rough fuzzy controller, in the stabilization of an inverted pendulum employing two knowledge bases. They are nine-rule and forty-nine rule knowledge bases.

---

[1] On leave of Technical University of Silesia

2. The idea of a rough set

Below we recall the fundamental notions and notation of the rough set theory. More detailed considerations on rough sets and their applications can be found in [8].

Let $\mathbb{U}$ be a finite set and let $R \subseteq \mathbb{U} \times \mathbb{U}$ be an equivalence relation called an indiscernibility relation. We denote by $\mathbb{U}/R$ the family of all equivalence classes R, and $[x]_R$ denotes an equivalence class containing $x \in \mathbb{U}$.

An ordered pair $A_R = < \mathbb{U},R >$ will be called an approximation space.

With every $X \subseteq \mathbb{U}$ we associate two sets defined as follows:

$$\underline{R}X = \{x \in \mathbb{U} : [x]_R \subseteq X\} \tag{1}$$

$$\overline{R}X = \{x \in \mathbb{U} : [x]_R \cap X \neq \varnothing\}$$

and called the R-lower and R-upper approximations of X in A_R, respectively.

Set $Bn_R(X) = \overline{R}X \setminus \underline{R}X$ will be called the R-boundary of X in A_R.

If $\underline{R}X = \overline{R}X$, we say that $X \subseteq \mathbb{U}$ is R-exactly approximated in A_R.

We can see that in this case we have $Bn_R(X) = 0$.

If $\underline{R}X \neq \overline{R}X$, we say that $X \subseteq \mathbb{U}$ is R-roughly approximated in A_R.

In this case we have $Bn_R(X) \neq 0$.

In order to express numerically how a set can be approximated using all equivalence classes of R we will use the accuracy of approximation of X in A_R (accuracy measure)

$$\alpha_R(X) = \frac{card\,\underline{R}X}{card\,\overline{R}X} \tag{2}$$

where $X \neq 0$.

Below we use another measure related to $\alpha_R(X)$ defined as

$$\varrho_R(X) = 1 - \alpha_R(X) \tag{3}$$

and referred to as R-roughness of X.

Additional numerical characteristics of imprecision, e.g.

- the rough R-membership function of the set X (or rm-function, for short)[15] defined as [15]:

$$\mu_X^R(x) = \frac{card\left([x]_R \cap X\right)}{card\left([x]_R\right)} \tag{4}$$

- a coefficient characterizing the uncertainty of membership of the element to the set with respect to the possessed knowledge

$$\mu_X(x) = \frac{card\left([x]_R \cap X\right)}{card\,(U)} \tag{5}$$

- the quality of approximation of the family $F = \{X_1, X_2, \dots X_n\}$ by R

$$\gamma_R(F) = \frac{\sum_{i=1}^{n} card(\underline{R}X_i)}{card(U)} \tag{6}$$

and other measures are presented in [8] and [9].

3.The mathematical model of the pendulum-car system.

The pendulum-car system, shown in Fig. 1, consists of

- a car moving along a line on two rails of limited length,
- a pendulum hinged in the car by means of ball bearings, rotating freely in the plane containing this line,
- a car driving device containing a dc motor, a dc amplifier, and a pulley-belt transmission system.

Such a system is characterized by an unstable equilibrium point in upright position of the pendulum, a stable equilibrium point in pendant position, as well as two uncontrollable points when the pendulum is in horizontal position.

Now let us introduce a simple mathematical description of the system. Assuming that the pendulum is a rigid body, both friction and damping forces are neglected in the system. Thus we obtain differential equations, describing the system, by projecting respective forces on to corresponding axes. We obtain the following differential equations describing the dynamic behaviour of the pendulum-car system:

$$(M+m)\ddot{x} - \frac{1}{2}ml\dot{\theta}^2\sin\theta + \frac{1}{2}ml\ddot{\theta}\cos\theta = u \tag{7}$$

$$\frac{1}{2}ml\ddot{x}\cos\theta + \frac{1}{3}ml^2\ddot{\theta} + \frac{1}{2}mgl\sin\theta = 0 \tag{8}$$

Rearranging equations (7) and (8) we get:

$$\ddot{\theta} = \frac{\left[(M+m)g\cdot\tan\theta + \frac{1}{2}ml\sin\theta\cdot\dot{\theta}^2\right] + u}{-\frac{2}{3}(M+m)l\sec\theta + \frac{1}{2}ml\cos\theta} \tag{9}$$

The last equation is used for the stabilization of the system in two positions: upright and slightly deflected from vertical.

5. Simulation results.

Numerical results obtained by simulating the control of the pendulum will be presented here. Block diagram of the control system is depicted in Fig. 2.

A nine-rule and a forty-nine knowledge base were used for comparison in this experiment. The knowledge bases were created using an ordinary fuzzy partition of input space. Each coordinate of the input space was evenly divided into three parts or seven parts respectively.

Using the rm-functions we obtain value 1 for certain regions and 0.5 for all uncertain regions of condition attributes (error, change in error) and a decision attribute as well.

The rough fuzzy controller used in our experiments employed **sup-min** as a compositional operator, **min** for the 'and' connective between rule premises, **max** for the sentence connective 'also'.

The control objective was: a) to stabilize the pendulum in upright (180°) position and b) to stabilize it in a position that would be slightly deflected from vertical, i.e. 185° .

The parameters of the model were taken as follows:

M = 2.8 kg, m = 0.2 kg, l = 0.75 m, g = 9.81 m/s^2

The initial position of the pendulum was 170° and the initial control value was 0.

The deflection angle $\theta = X1$, its derivative $d\theta/dt = X2$, control value $u = Dr$ and control error θ_i-SP_i = Err as functions of time for both types of controllers are shown in Figs. 5, 6, 7 and 8.

For the purpose of comparative study a quality index (QI) was defined as below

$$QI = \frac{1}{N+1}\sum_{i=0}^{N}(\theta_i - SP_i)^2 \tag{10}$$

where SP_i is the set point and $N+1$ is the total number of observation points.

Comparing the results of the control we see that the forty-nine-rule knowledge base operates better than the nine-rule base. For the forty-nine-rule knowledge base the quality index amounts to 605.5 as opposed to 636.3. The occurence of the zero level of control value in corresponding time interval can be observed. In both cases rm-functions were used.

6. Concluding remarks.

It should be pointed out that rough fuzzy controllers may perform more crudely than fuzzy logic controllers (in view of stabilization problems). Their crude performance can be explained by the fact that they operate on a finite number of selected levels. However, they are considerably faster than fuzzy logic controllers on account of a simplified defuzzification procedures.

References

[1] Czogala E., *Fuzzy logic controllers versus conventional controllers*, 16th Seminar on Fundamentals of Electrotechnics and Circuit Theory, Gliwice - Ustroń 1993, Vol.1, 23-29.

[2] Czogala E., *On the modification of rule connective in fuzzy logic controllers using Zimmermann's compensatory operators*, EUFIT'93 September 7-10, 1993, Aachen (Germany), 1329-1333.

[3] Czogala E., Mrozek A., Pawlak Z., *The idea of a rough fuzzy controller and its application to the stabilization of a pendulum-car system*, submitted to Fuzzy Sets and Systems.

[4] Czogala E., *On modelling knowledge bases for fuzzy and rough fuzzy controllers using the concept of an input image*, submitted to EUFIT'94, September 20-23, Aachen (Germany).

[5] Lee C.C., *Fuzzy logic control systems: fuzzy logic controller*, PartI/Part II, IEEE Trans. on Systems, Man and Cybernetics, Vol. 20, No. 2, March/April 1990, 404-435.

[6] Mamdani E.H., *Applications of fuzzy algorithms for simple dynamic plant*, Proc. IEE, Vol. 121, No. 12, 1974, 1585-1588.

[7] Pawlak Z., *Rough sets*, International Journal of Information and Computer Sciences, 11(5), 341-356.

[8] Pawlak Z., *Rough sets. Theoretical aspects of reasoning about data*, Kluwer Academic Publisher, Dordrecht Boston London, 1991.

[9] Pawlak Z., Skowron A., *Rough membership functions: a tool for reasoning with uncertainty*, Algebraic methods in logic and in computer science, Banach Center Publications, Vol. 28, Institute of Mathematics, Polish Academy of Sciences, Warsaw 1993, 135-150.

[10] Zadeh L.A., *Outline of a new approach to the analysis of complex systems and decision processes*, IEEE Trans. on Systems, Man and Cybernetics, Vol. 3, 1973, 28-44.

[11] Zadeh L.A., *Fuzzy sets*, Information and Control, Vol. 8, 1965, 338-353.

[12] Zimmermann H.-J., Zysno P., *Latent connectives in human decision making*, Fuzzy Sets and Systems 4, 1980, 37-51.

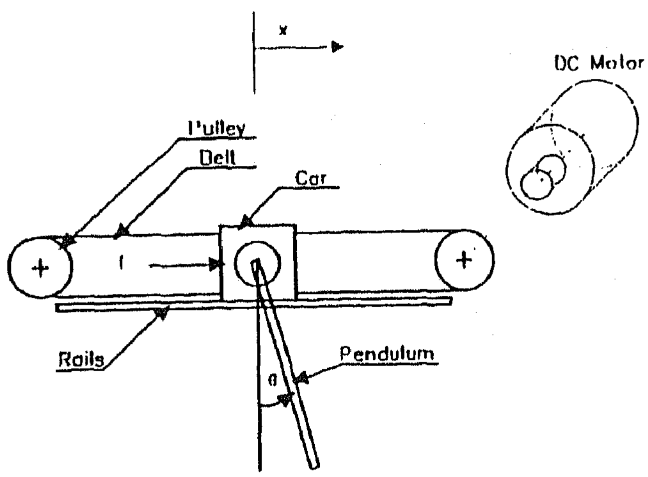


Fig. 1. The pendulum-car system

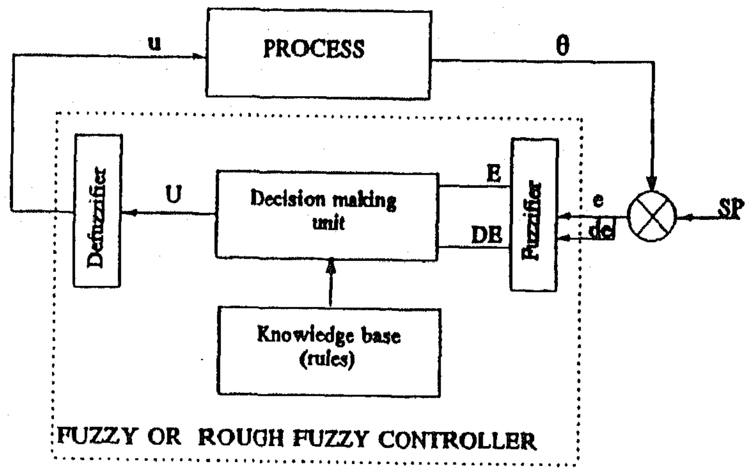
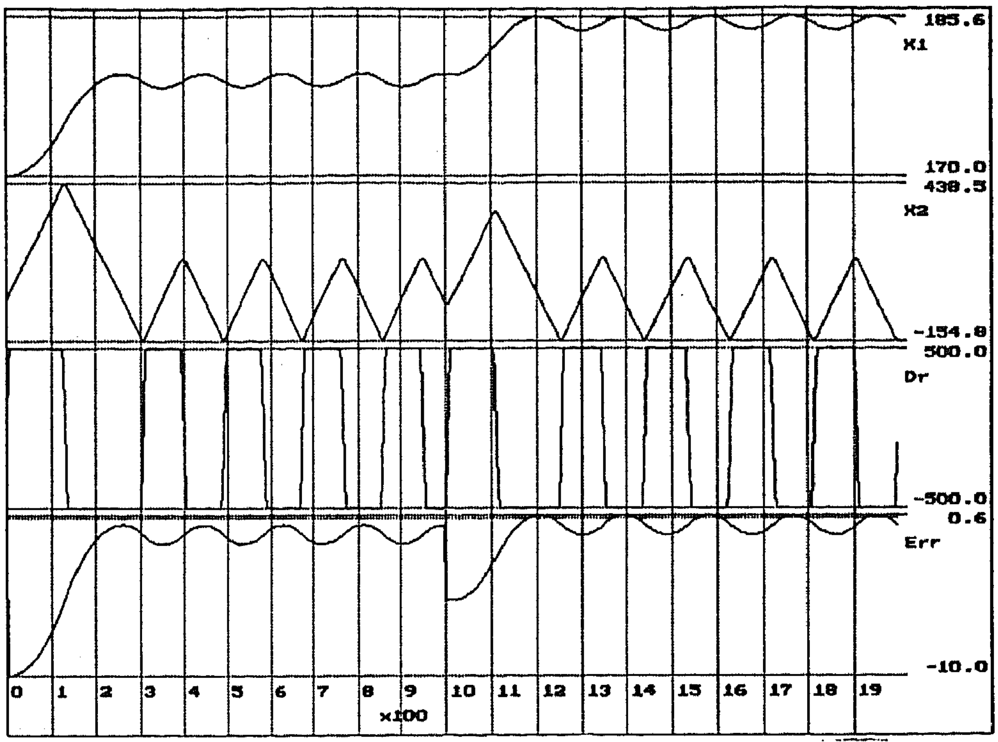
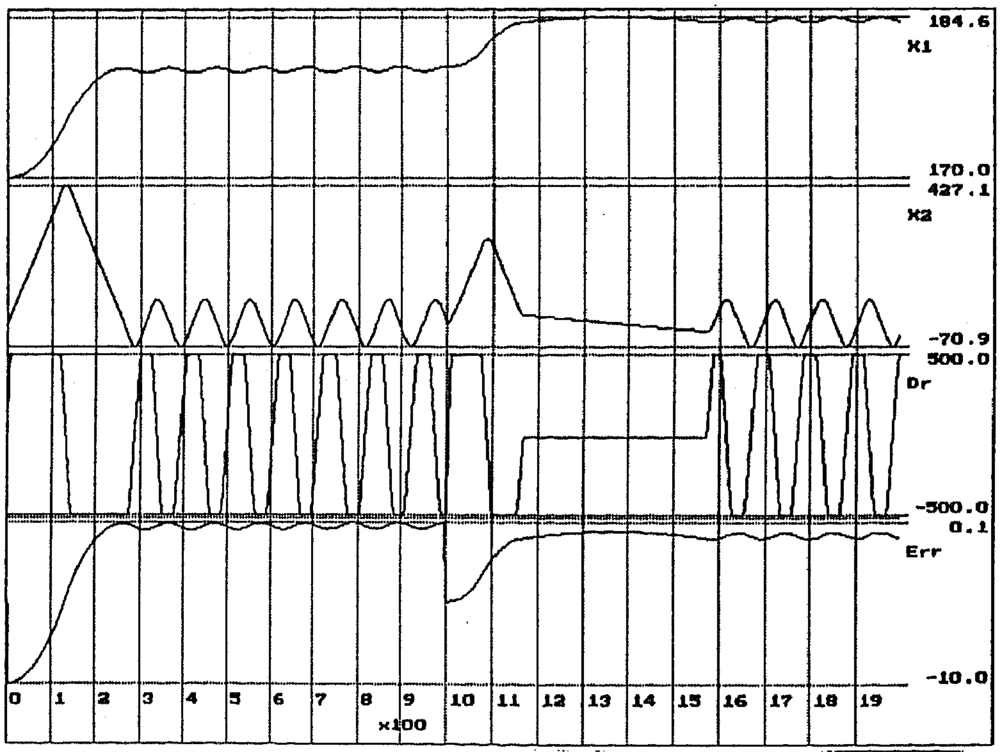


Fig. 2. Block diagram of the control system



QI = 636.343620

Fig. 3. Results of control using rough fuzzy controller (a nine-rule knowledge base applied).
X1 = deflection angle, X2 = derivative of deflection angle, Dr = control value, Err - control error



QI = 605.529853

Fig. 4. Results of control using rough fuzzy controller (a forty-nine-rule knowledge base applied).
X1 = deflection angle, X2 = derivative of deflection angle, Dr = control value, Err - control error