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Wydanie I.
Nakład: 250 egzemplarzy

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On Some Issues Connected with Conflict Analysis

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1 Introduction

Conflict analysis and resolution plays an important role in business, governmental, political and lawsuits disputes, labor-management negotiations, military operations, etc.

There are several formal models of conflicts (cf. e.g. [3], [5], [9, 11], [12]). This article[1] contains extension of some ideas presented in [11].

2 An Example

We assume that in a conflict at least two participants, called in what follows agents, are in dispute over some issues. The agents may be individuals, groups, states, parties etc. The relationship of each agent to a specific issue can be clearly depicted in a form of a table, as shown in an example of the Middle East conflict, which is taken with slight modifications from Casti (cf. [1]).

Consider six agents

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1 – Israel
2 – Egypt
3 – Palestinians
4 – Jordan
5 – Syria
6 – Saudi Arabia

and five issues

a – autonomous Palestinian state on the West Bank and Gaza
b – Israeli military outpost along the Jordan River
c – Israeli retains East Jerusalem
d – Israeli military outposts on the Golan Heights
e – Arab countries grant citizenship to Palestinians who choose to remain within their borders

In the table below the attitude of six nations of the Middle East region to the above issues is presented; -1 means, that the agent is against, 1 - favorable and 0 neutral toward the issue. For the sake of simplicity we will write - and + instead of -1 and 1 respectively.

U	a	b	c	d	e
1	-	+	+	+	+
2	+	0	-	-	-
3	+	-	-	-	0
4	0	-	-	0	-
5	+	-	-	-	-
6	0	+	-	0	+

Table 1

3 Conflict and Information Systems

Tables as shown in the previous section are known as *information systems* (cf. [10]). An information system is a table rows of which are labeled by *objects (agents)*, columns – by *attributes (issues)* and entries of the table are *values of attributes (opinions, beliefs, views, votes, etc)*, which are uniquely assigned to each agent and an attribute, i.e. each entry corresponding to row x and column a represents opinion of agent x about issue a.

Formally an *information system* can be defined as a pair $S = (U, A)$, where

U – is a nonempty, finite set called the *universe*; elements of U will be called *objects (agents)*,
A – is a nonempty, finite set of *attributes (issues)*.

Every attribute $a \in A$ is a total function $a : U \rightarrow V_a$, where V_a - is the set of *values* of a, called the *domain* of a; elements of V_a will be referred to as *opinions*, i.e. $a(x)$ is opinion of agent x about issue a.

The above given definition is general, but for conflict analysis we will need its simplified version, where the domain of each attribute is restricted to three values only, i.e. $V_a = \{-1, 0, 1\}$, for every a - meaning *against, neutral* and *favorable* respectively. For the sake of simplicity we will assume $V = \{-, 0, +\}$. Every information system with the above said restriction will be referred to as a *situation*.

Information system contains explicit information about the attitude of each agent to issues being consider in the debate, and and will be used to derive various implicit information, necessary to conflicts analysis.

4 Conflict, Alliance and Neutrality

In order to express relations between agents we define three basic binary relations on the universe: *conflict, neutrality* and *alliance.* To this end we need the following auxiliary function:

$$\phi_a(x, y) = \begin{cases} 1, & \text{if } a(x).a(y) = 1 \text{ or } x = y \\ 0, & \text{if } a(x).a(y) = 0 \text{ and } x \neq y \\ -1, & \text{if } a(x).a(y) = -1 \end{cases} \qquad (*)$$

This means that, if $\phi_a(x, y) = 1$, agents x and y have the same opinion about issue a (are *allied* on a); if $\phi_a(x, y) = 0$ means that at least one agent x or y has neutral approach to issue a (is *neutral* on a), and if $\phi_a(x, y) = -1$, means that both agents have different opinions about issue a (are in *conflict* on a).

In what follows we will need three basic relations

R_a^+, R_a^0 and R_a^-, called *alliance, neutrality* and *conflict* relations respectively, and defined as follows:

$$\begin{aligned} R_a^+(x, y) &\quad \text{iff} \quad \phi_a(x, y) = 1 \\ R_a^0(x, y) &\quad \text{iff} \quad \phi_a(x, y) = 0 \\ R_a^-(x, y) &\quad \text{iff} \quad \phi_a(x, y) = -1 \end{aligned}$$

It is easily seen that the alliance relation has the following properties:

$$\begin{aligned} &(i) \quad R_a^+(x, x) \\ &(ii) \quad R_a^+(x, y) \quad \text{implies} \quad R_a^+(y, x) \\ &(iii) \quad R_a^+(x, y) \text{ and } R_a^+(y, z) \quad \text{implies} \quad R_a^+(x, z), \end{aligned}$$

i.e R_a^+ is an *equivalence* relation for every a. Equivalence classes of alliance relation will be called *coalition* on a. Let us note that the condition (iii) can be expressed as "friend of my friend is my friend".

For the conflict relation we have the following properties:

$$\begin{array}{rl} (iv) & \text{non } R_a^-(x,x) \\ (v) & R_a^-(x,y) \text{ implies } R_a^-(y,x) \\ (vi) & R_a^-(x,y) \text{ and } R_a^-(y,z) \text{ implies } R_a^+(x,z) \\ (vii) & R_a^-(x,y) \text{ and } R_a^+(y,z) \text{ implies } R_a^-(x,z). \end{array}$$

Conditions (vi) and (vii) refers to well know sayings "enemy of may enemy is my friend" and "friend of my enemy is my friend".

For the neutrality relation we have:

$$\begin{array}{rl} (viii) & \text{non } R_a^0(x,x) \\ (ix) & R_a^0(x,y) = R_a^0(y,x) \end{array}$$

Let us observe that in the conflict and neutrality relations there are no coalitions.

Obviously $R_a^+ \cup R_a^0 \cup R_a^- = U^2$ and all the three relation are pairwise disjoint, i.e. every pair of objects (x,y) belongs to exactly one of the above defined relations (is in conflict, is allied or is neutral).

For example in the Middle East situation Egypt, Palestinians and Syria are allied on issue a (autonomous Palestinian state on the West Bank and Gaza), Jordan and Israel are neutral to this issue whereas, Israel and Egypt, Israel and Palestinian, and Israel and Syria are in conflict about this issue.

5 Degree of Conflict

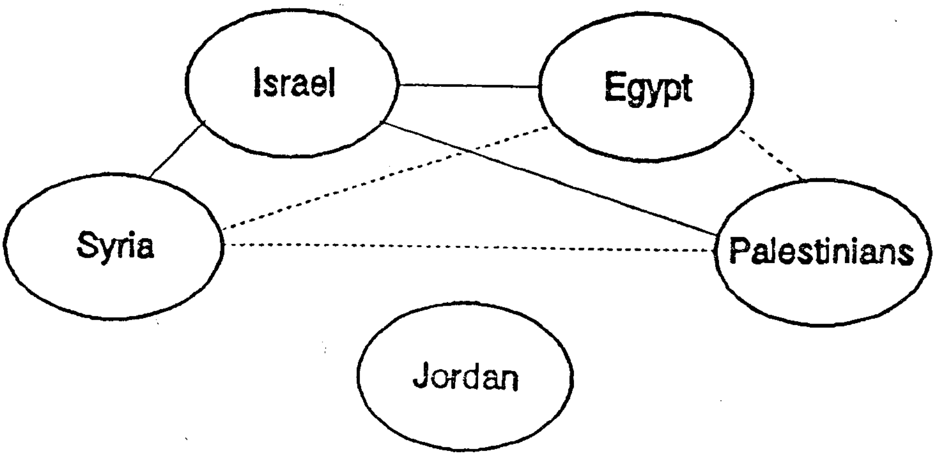
Let $S = (U, A)$ be a situation, and $a \in A$. If there exits a pair (x,y) such that $R_a^-(x,y)$ will will say that the attribute a is *conflicting* (agents), otherwise the attribute is *conflictless*. The following property is obvious.

Proposition 1 *If a is an conflicting attribute, then the relation R_a^+ has exactly two equivalence classes X_a^+ and X_a^-, where $X_a^+ = \{x \in U : a(x) = +\}$, $X_a^- = \{x \in U : a(x) = -\}$, $X_a^+ \cup X_a^- \cup X_a^0 = U$, and $X_a^0 = \{x \in U : a(x) = 0\}$. Moreover $R_a^-(x,y)$ iff $x \in X_a^+$ and $y \in X^-$ for every $x, y \in U$.* ∎

The above proposition says that if a is a conflicting attribute, then all agents are divided into two coalitions (blocks) X_a^+ and X_a^-, and all members of two different coalitions are in conflict, and the remaining (if any) agents are neutral to the issue a.

This can be easily illustrated by a graph as shown in Figure 1.

Fig. 1. An Example



Vertices of the graph are labelled by agents, whereas branches of the graph are representing relation between agents. Solid lines are denoting conflicts, dotted line - alliance, and neutrality, for simplicity is not shown explicity on the graph. The proposition says that the graph shown in Fig 1 can be presented as shown in Fig 2.

The *degree of conflicts* between agents about the issue a can be easily expressed numerically, as follows:

$$Con(a) = \frac{card\, X_a^+ \,.\, card\, X_a^-}{E(n/2).(n - E(n/2))}.$$

The number $card\, X_a^+ \,.\, card\, X_a^-$ is equal to the number of conflicts generated by the issue a (i.e. pairs of agents being in conflicts because of issue a), whereas $E(n/2).(n - E(n/2))$ is the number of maximal conflicts possible between n agents, and $E(n/2)$ denotes the whole part of the division of n by 2. Of course $0 \leq Con(a) \leq 1$.

The coefficient $Con(a)$ can be easily extended to the whole set of attributes as follows:

$$Con(A) = \sum_{a \in A} Con(a)/card(A).$$

Fig. 2. An Example

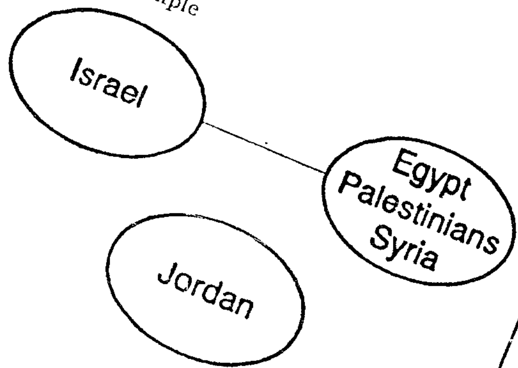


Fig. 3. An Illustration - With permission of the Artist

Evidently $0 \leq Con(a) \leq 1$, and $Con(A)$ can be viewed as an overall measure of conflicts in the situation $S = (U, A)$ and will be referred to as *conflictness* of A, or *tension* in $S = (U, A)$.

The following proposition is true.

Proposition 2

$$Con(A) \geq Con(A - Max(A))$$

where $Max(A)$ is a most conflicting attribute in A. ∎

From the **Proposition 2** it follows that removing from the debate the most conflicting attribute (issue) reduces tension in the situation $S = (U, A)$. This can be used as a guidance for a negotiation process.

For example in the Middle East situation considered previously we have

$$Con(a) = 1/3$$
$$Con(b) = 2/3$$
$$Con(c) = 4/9$$
$$Con(d) = 1/3$$
$$Con(e) = 2/3.$$

The most conflicting attributes are b and e, and $Con(A) \cong 0.49$. If we remove the attribute b we get $Con(\{a, c, d, e\}) = 0.45$, but removing the attribute a we obtain $Con(\{b, c, d, e\}) \cong 0.53$.

6 Distance between Agents

The relations $R_a^+(x, y)$, $R_a^0(x, y)$ and $R_a^-(x, y)$ can be seen as a description of views on the issue a between agents x and y. We will also need an evaluation of views between x and y with respect to the whole set of attributes A. To this end we define the function

$$\rho(x, y) = \sum_{a \in A} \phi_a^*(x, y)/cardA, \qquad (**)$$

where

$$\phi_a^*(x, y) = \frac{1 - \phi_a(x, y)}{2}$$

Obviously $0 \leq \rho(x, y) \leq 1$. If $\rho(x, y) \neq 0$ we will say that x and y are in conflict on A in *the degree* $\rho(x, y)$, and of course if $\rho(x, y) = 0$ x and y are in coalition on A.

In particular, if $\rho(x,y) = 0.5$ x and y are neutral on A. Thus neutrality in this case is considered as a form of a (weak) conflict.

The following properties are obvious

$$1)\ \rho(x,x) = 0$$
$$2)\ \rho(x,y) = \rho(y,x)$$
$$3)\ \rho(x,y) + \rho(y,z) \geq \rho(x,z)$$

thus the $\rho(x,y)$ is a distance function.

For example for the considered Middle East situation we have the following distances matrix between agents

	1	2	3	4	5	6
1	0					
2	0.9	0				
3	0.9	0.2	0			
4	0.8	0.3	0.1	0		
5	1	0.1	0.1	0.2	0	
6	0.4	0.5	0.3	0.4	0.5	0

Table 2

7 Discernibility of Agents

Differences in views of agents concerning specific issues can be also expressed not only in a quantitative way (numerically), as in the previous section, but also in a qualitative way. To this end we will use the concept of a *discernibility matrix* (cf. [15]).

Let $S = (U, A)$, $U = \{x_1, x_2, \ldots, x_n\}$ and $B \subseteq A$. By an *discernibility matrix of B in S*, denoted $M_S(B)$, or $M(B)$, if S is understood, we will mean $n \times n$ matrix defined thus:

$$(c_{ij}) = \{a \in B : a(x_i) \neq a(x_j)\} \quad \text{for } i, j = 1, 2, \ldots, n.$$

Thus entry c_{ij} is the set of all attributes which discern objects x_i and x_j.

Example

The discernibility matrix for conflict presented in Table 1 is given below:

	1	2	3	4	5	6
1						
2	a, b, c, d, e					
3	a, b, c, d, e	b, e				
4	a, b, c, d, e	a, b, d	a, d, e			
5	a, b, c, d, e	b	e	a, d		
6	a, c, d	a, b, e, d	a, b, d, e	b, e	a, b, d, e	

Table 3

We might be also interested in excluding neutral agents from the analysis. In this case the discernibility matrix would have the form

	1	2	3	4	5	6
1						
2	a, c d, e					
3	a, c, d	–				
4	b, c	–	–			
5	a, b, c d, e	–	–	–		
6	c	e	b	b, e	b, e	

Table 4

■

The discernibility matrix $M(B)$ assigns to each pair of objects x and y a subset of attributes $\delta(x, y) \subseteq B$, with the following properties:

$$\begin{array}{ll} i) & \delta(x, x) = \emptyset \\ ii) & \delta(x, y) = \delta(y, x) \\ iii) & \delta(x, z) \subseteq \delta(x, y) \cup \delta(y, z). \end{array}$$

The function δ may be regarded as *qualitative semi-metric* and $\delta(x, y)$ — *qualitative semi-distance*. Thus the discernibility matrix can be seen as a *semi-distance (qualitative)* matrix.

Each entry of the table shows all issues for which the corresponding agents have different opinions. The difference between Table 2 and Table 3 is that in the first table we have numerical evaluation of differences between agents (e.g. $\rho(2, 3) = 0.2$), whereas in the second one differences are expressed literary ($\delta(2, 3) = \{b, e\}$).

Main problem we are interested in regarding discernibility matrices is how to choose minimal set of issues,called reduct, which uniquely characterize each agent. Let us outline this problem below.

To every discernibility matrix $M(B)$ one can assign uniquely a *discernibility (boolean) function* $f(B)$ defined as follows.

With every $a \in B$, we can associate a binary boolean variable $\overline{a}$, and let $\sum \delta(x, y)$ denote boolean sum of all boolean variables assigned to the set of attributes $\delta(x, y)$, provided $\delta(x, y) \neq \emptyset$. Then the discernibility function can be defined by the formula

$$f(B) = \prod_{(x,y)\in U^2} \sum \delta(x,y)$$

The proposition below (cf. [15]) states the relationship between the minimal disjunctive normal form of the function $f(B)$ and the set of all reducts of B.

Proposition 3 *All constituents in the minimal disjunctive normal form of the function $f(B)$ are all reducts of B.* ∎

It is easy to compute that there are two reducts $\{a, b, e\}$ and $\{b, d, e\}$ in the discernibility matrix given in Table 3, whereas there is only one reduct $\{b, c, e\}$ in Table 4. More detail discussion of reduct and related matter can be found in [11].

It is easy to see (cf. Proposition 2) that reduction of attributes (issues) as defined in this section not necessarily reduces the tension in a situation, i.e. it is not general true that if B is a reduct of A, then $Con(B) < Con(A)$. The aim of finding reducts consists in preserving overall structure of the conflict situation, i.e. after reducing attributes general relations (conflict, coalition and neutrality) between agents remain intact.

8 Conclusions

There are many problems concerning structure of conflict situations that can be addressed in the proposed framework. In particular logic of conflict seems to be of great interest. Three valued Lukasiwicz's logic apparently is a natural candidate to this end.

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References

1. Casti, J. L. (1989). Alternate Realities - Mathematical Models of Nature and Man. *John Wiley and Sons.*
2. Coombs. C.H. and Avrunin, G.S. (1988). The Structure of Conflict. *Lawrence Erlbaum Assoiates.*
3. Dorclan, P. (1969). Interaction under Conditions of Crisis: Application of Graph Theory to International Relations. *Peace Research Society International Papers*, **11**, pp. 89-109.
4. French, S. (1986). Decision Theory: An Introduction to the Mathematics of Rationality. *Ellis Horwood Limited*, Chichester.

5. Hart, H. (1974). Structures of Influence and Cooperation-Conflict. *International Interactions*, 1, pp.141-162.
6. Nurmi, H. (1987). Comparing Voting Schemes. *D. Reidel*, Dordrecht.
7. Klein, M. (1991). Supporting Conflict Resolution in Cooperative Design Systems. *IEEE Transactions on Systems, Man and Cybernetics*, 6, pp.1379-130.
8. Klein, M. (1990) and Lu, S.C. Conflict Resolution in Cooperative Design. *International Journal for AI in Engineering*, 4, pp. 168-180.
9. Pawlak, Z. (1984). On Conflicts. *Int. J. of Man-Machine Studies*, 21, pp. 127-134.
10. Pawlak, Z. (1991). Rough Sets - TheoreticaL Aspects of Reasoning about Data. *KLUWER ACADEMIC PUBLISHERS.*
11. Pawlak, Z. (1993). Anatomy of Conflicts. *Bull. EATCS*, 50, pp. 234-246.
12. Roberts, F. (1976). Discrete Mathematical Models with Applications to Social, Biological and Environmental Problems, *Englewood Cliffs, Prince Hall Inc.*
13. Saaty, T.L. and Alexander, J. (1989). Conflict Resolution: The Analytic Hierarchy Process, *Praeger, New York.*
14. Saaty, T.L. (1993). War-Peace, Terrorism and Conflict Resolution. *Manuscript*, pp.1-22.
15. Skowron, A. and Rauszer, C. (1991). The Discernibility Matrices and Functions in Information Systems. *Institute of Computer Science Reports, 1/91, Warsaw University of Technology, and Fundamenta Informaticae (to appear).*
16. Zakowski, W. (1990). On Identification of Graphs of Regular Functional Configurations *Bull. Pol. Ac.: Math.*, 36, pp. 419-423.