Multi-valued Logic, Bayes' Rule and Rough Sets

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Abstract

Inference rules, like e.g. modus ponens, play an essential role in logical reasoning and are fundamental in deductive logic, whereas decision rules are basic tools of reasoning in many branches of AI, particularly in data mining, machine learning decision support and others.

Both inference rules and decision rules are implication, but there are essential differences between these two concepts described by premisses and by conclusions of implication. Inference rules are used to draw true conclusions from true premises, whereas decision rules are prescription of decisions (actions) that must be made when some condition are satisfied. Therefore some probabilistic, fuzzy or rough measures must be associated with decision rules, to measure the closenes of concepts – in contrast to inference rules where truth values are propagated from from premisses to conclusions. The rough set approach bridges somehow both concepts – inference and decision rules.

With every implication two conditional probabilities are associated, called *credibility* and *coverage factors* respectively. The credibility factor may be considered as partial truth value of the implication and was first introduced by Łukasiewicz in 1913 – whereas the covering factor, introduced recently by Tsumoto, shows how strongly a decision rule covers decision of the decision rule.

It can be shown that the relationship between this two factors is disclosed by the Bayes' Theorem. However, the meaning of Bayes' Theorem in this case differs from that postulated in statistical inference. In statistical data analysis based on Bayes' Theorem, we assume that prior probability about some parameters without knowledge about the data is given. The posterior probability is computed next, which tells us what can be said about prior probability in view of the data. In the rough set approach the meaning of Bayes' Theorem is different. It reveals some relationships in the database, without referring to prior and posterior probabilities, and it can be used to reason about data in terms of approximate (rough) implications. Thus, the proposed approach can be seen as a new model for Bayes' Theorem.

Thus the rough set approach combines together both logical and probabilistic aspects of implications. This idea is due to Łukasiewicz who first pointed out the relationship between implications and Bayes' Theorem. In the lecture the above ideas will be formulated precisely and discussed from the rough set perspective.

Key words: Bayes' rule, rough sets, decision rules, multi-valued logic

1 Introduction

Classical deductive reasoning is based on *Modus Ponens* inference rule, which states that if a formula Φ is true and the implication $\Phi \to \Psi$ is true then the formula Ψ is also true. Łukasiewicz first proposed to extend *Modus Ponens* to the case when instead of true values probabilities are associated with Φ, $\Phi \to \Psi$ and Ψ [3, 5]. Later, independently, various probabilistic logics have been proposed and investigated by many logicians and philosophers [1, 6].

Recently the generalization of *Modus Ponens* become a very important issue in connection with knowledge based systems. Particularly interesting in this context is the *Generalized Modus Ponens*, introduced by Zadeh in the setting of fuzzy sets [16, 17], which next has been investigated by various authors [2, 7, 14].

Skowron has proposed generalization of *Modus Ponens* in the framework of rough set theory [13]. In this paper we also propose a generalization of *Modus Ponens* within rough set theory, called a *Rough Modus Ponens* (*RMP*), however different to that given in [13], and refering to Łukasiewcz's ideas. The essence of Łukasiewcz's idea consists in association with the implication $\Phi \to \Psi$ a conditional probability, whereas with Φ and Ψ unconditional probabilities are associated. The assumption that the probability of implication $\Phi \to \Psi$ is a conditional probability is due to Ramsey [1] but similar ideas can be also found in Łukasiewicz, however, not expressed *explicitly* [3]. Association of conditional probability with decision rules in the context of rough sets has been proposed also by other authors (cf. [15[, [18]) but our aim is entirely different. We try to set this issue rather in the frame work of logical research, establish sound logical foundations for this kind of research and show that decision rules used in the rough set approach play different role as *MP* inference rule in logical reasoning, and thus they cannot be in fact treated as a simple generalization of *MP*. Although association of conditional probabilities to implications is quite obvious it leeds to logical and philosophical difficulties. Extensive discussion of this problem can be found in [1].

Implication is strongly related to inclusion, i.e., if $\Phi \to \Psi$ is true then every x satisfying Φ satisfies also Ψ, or in other words $|\Phi| \subseteq |\Psi|$, where $|\Phi|$ denotes the set of all x satisfying Φ i.e., the meaning of Φ. To define *RMP* we will need partial (rough) inclusion of sets and to this aim we will adopt the idea of rough mereology proposed by Polkowski and Skowron [11, 12]. Thus the proposed *RMP* has also connection with rough mereology, which can be understood as a natural theory of rough inclusion, and consequently – rough implication.

This paper contains extended version of some ideas presented in [8, 9].

2 Multivalued logics as probability logics – a Łukasiewicz's approach

In this section we present briefly basic ideas of Łukasiewicz's approach to multivalued logics as probability logics.

Łukasiewicz associates with every so called *indefinite proposition* of one variable x, $\Phi(x)$ a true value $\pi(\Phi(x))$, which is the ratio of the number of all values of x which satify $\Phi(x)$, to the number of all possible values of x. For example, the true value of the proposition "x is greater than 3" for $x = 1, 2, \ldots, 5$ is $2/5$. It turns out that assuming the following three axioms

1) Φ is false if and only if $\pi(\Phi) = 0$;

2) Φ is true if and anly if $\pi(\Phi) = 1$;

3) if $\pi(\Phi \rightarrow \Psi) = 1$ then $\pi(\Phi) + \pi(\sim \Phi \wedge \Psi) = \pi(\Psi)$;

one can show that

4) if $\pi(\Phi \equiv \Psi) = 1$ then $\pi(\Phi) = \pi(\Psi)$;

5) $\pi(\Phi) + \pi(\sim \Phi) = 1$;

6) $\pi(\Phi \vee \Psi) = \pi(\Phi) + \pi(\Psi) - \pi(\Phi \wedge \Psi)$;

7) $\pi(\Phi \wedge \Psi) = 0$ iff $\pi(\Phi \vee \Psi) = \pi(\Phi) + \pi(\Psi)$.

Obviously, the above properties have probabilistic flavour. With every implication $\Phi \rightarrow \Psi$ one can associate conditional probability $\pi(\Psi|\Phi) = \frac{\pi(\Phi \wedge \Psi)}{\pi(\Phi)}$. In what follows the above ideas will be used to define the *Rough Modus Ponens.* Let us mention that in applications we are aften interested in properties more specific than (1)-(7)s related to properties of π defined by data tables.

3 Information system and decision table

Starting point of rough set based data analysis is a data set, called an information system.

An information system is a data table, whose columns are labelled by attributes, rows are labelled by objects of interest and entries of the table are attribute values.

Formally by an *information system* we will understand a pair $S = (U, A)$, where U and A, are finite, nonempty sets called the *universe*, and the set of *attributes,* respectively. With every attribute $a \in A$ we associate a set V_a, of its *values*, called the *domain* of a. Any subset B of A determines a binary relation $I(B)$ on U, which will be called an *indiscernibility relation*, and is defined as follows: $(x, y) \in I(B)$ if and only if $a(x) = a(y)$ for every $a \in A$, where $a(x)$ denotes the value of attribute a for element x. Obviously $I(B)$ is an equivalence relation. The family of all equivalence classes of $I(B)$, i.e., partition determined by B, will be denoted by $U/I(B)$, or simple U/B; an equivalence class of $I(B)$, i.e., block of the partition U/B, containing x will be denoted by $B(x)$.

If (x, y) belongs to $I(B)$ we will say that x and y are *B-indiscernible* or indiscernible with respect to B. Equivalence classes of the relation $I(B)$ (or blocks of the partition U/B) are referred to as *B-elementary sets* or *B-granules.*

If we distinguish in an information system two classes of attributes, called *condition* and *decision attributes*, respectively, then the system will be called a *decision table.*

A simple, tutorial example of an information system (a decision table) is shown in Table 1.

Table 1: An example of a decision table

Car	*F*	*P*	*S*	*M*
1	*med.*	*med.*	*med.*	*poor*
2	*high*	*med.*	*large*	*poor*
3	*med.*	*low*	*large*	*poor*
4	*low*	*med.*	*med.*	*good*
5	*high*	*low*	*small*	*poor*
6	*med.*	*low*	*large*	*good*

The table contains data about six cars, where F, P, S and M denote *fuel consumption*, *selling price, size* and *marketability*, respectively.

Attributes F, P and S are condition attributes, whereas M is the decision attribute. Each row of the decision table determines a decision obeyed when specified conditions are satisfied.

4 Approximations

Suppose we are given an information system (a datat set) $S = (U, A)$, a subset X of the universe U, and subset of attributes B. Our task is to describe the set X in terms of attribute values from B. To this end we define two operations assigning to every $X \subseteq U$ two sets $B_*(X)$ and $B^*(X)$ called the *B-lower* and the *B-upper approximation* of X, respectively, and defined as follows:

$$B_*(X) = \bigcup_{x \in U} \{B(x) : B(x) \subseteq X\},$$

$$B^*(X) = \bigcup_{x \in U} \{B(x) : B(x) \cap X \neq \emptyset\}.$$

Hence, the B-lower approximation of a set is the union of all B-granules that are included in the set, whereas the *B-upper* approximation of a set is the union of all B-granules that have a nonempty intersection with the set. The set

$$BN_B(X) = B^*(X) - B_*(X)$$

will be referred to as the *B-boundary region* of X.

If the boundary region of X is the empty set, i.e., $BN_B(X) = \emptyset$, then X is *crisp* (*exact*) with respect to B; in the opposite case, i.e., if $BN_B(X) \neq \emptyset$, X is referred to as *rough* (*inexact*) with respect to B.

For example, let $C = \{F, P, S\}$ be the set of all condition attributes. Then for the set $X = \{1, 2, 3, 5\}$ of cars with poor marketability we have $C_*(X) = \{1, 2, 5\}$, $C^*(X) = \{1, 2, 3, 5, 6\}$ and $BN_C(X) = \{3, 6\}$.

5 Decision rules

With every information system $S = (U, A)$ we associate a formal language $L(S)$, written L when S is understood. Expressions of the language L are logical formulas denoted by Φ, Ψ etc. built up from attributes and attribute-value pairs by means of logical connectives $\wedge$ (*and*), $\vee$ (*or*), $\sim$ (*not*) in the standard way. We will denote by $||\Phi||_S$ the set of all objects $x \in U$ satisfying Φ in S and refer to as the *meaning* of Φ in S.

The meaning of Φ in S is defined inductively as follows:

1) $||(a,v)||_S = \{v \in U : a(v) = U\}$ for all $a \in A$ and $v \in V_a$,

2) $||\Phi \vee \Psi||_S = ||\Phi||_S \cup ||\Psi||_S$,

3) $||\Phi \wedge \Psi||_S = ||\Phi||_S \cap ||\Psi||_S$,

4) $|| \sim \Phi||_S = U - ||\Phi||_S$.

A formula Φ is *true* in S if $||\Phi||_S = U$.

A *decision rule* in L is an expression $\Phi \rightarrow \Psi$, read *if* Φ *then* Ψ; Φ and Ψ are referred to as *conditions* and *decisions* of the rule, respectively.

An example of a decision rule is given below

$(F, med.) \wedge (P, low) \wedge (S, large) \rightarrow (M, poor)$.

Obviously a decision rule $\Phi \rightarrow \Psi$ is *true* in S if $||\Phi||_S \subseteq ||\Psi||_S$.

With every decision rule $\Phi \rightarrow \Psi$ we associate a conditional probability $\pi_S(\Psi|\Phi)$ that Ψ is true in S given Φ is true in S with the probability $\pi_S(\Phi)\frac{card(||\Phi||_S)}{card(U)}$, called the *certainty factor* and defined as follows:

$$\pi_S(\Psi|\Phi) = \frac{card(||\Phi \wedge \Psi||_S)}{card(||\Phi||_S)},$$

where $||\Phi||_S \neq 0$.

This coefficient is widly used in data mining and is called "confidence coefficient".

Obviously, $\pi_S(\Psi|\Phi) = 1$ if and only if $\Phi \rightarrow \Psi$ is true in S.

If $\pi_S(\Psi|\Phi) = 1$, then $\Phi \rightarrow \Psi$ will be called a *certain decision* rule; if $0 < \pi_S(\Psi|\Phi) < 1$ the decision rule will be referred to as a *possible decision* rule.

Besides, we will also need a *coverage factor*

$$\pi_S(\Phi|\Psi) = \frac{card(||\Phi \wedge \Psi||_S)}{card(||\Psi||_S)},$$

which is the conditional probability that Φ is true in S, given Ψ is true in S with the probability $\pi_S(\Psi)$.

Certainty and coverage factors for decision rules associated with Table 1 are given in Table 2.

Table 2: Certainty and coverage factors

Car	*F*	*P*	*S*	*M*	*Cert.*	*Cov.*
1	*med.*	*med.*	*med.*	*poor*	1	1/4
2	*high*	*med.*	*large*	*poor*	1	1/4
3	*med.*	*low*	*large*	*poor*	1/2	1/4
4	*low*	*med.*	*med.*	*good*	1	1/2
5	*high*	*low*	*small*	*poor*	1	1/4
6	*med.*	*low*	*large*	*good*	1/2	1/2

6 Decision rules and approximations

Let $\{\Phi_i \to \Psi\}_n$ be a set of decision rules such that:

all conditions Φ_i are pairwise mutally exclusive, i.e., $||\Phi_i \wedge \Phi_j||_S = \emptyset$, for any $1 \leq i, j \leq n,\ i \neq j$, and (1)

$$\sum_{i=1}^{n} \pi_S(\Phi_i|\Psi) = 1.$$

Let C and D be condition and decision attributes, respectively, and let $\{\Phi_i \to \Psi\}_n$ be a set of decision rules satisfying (1).

Then the following relationships are valid:

a) $C_*(||\Psi||_S) = ||\bigvee_{\pi(\Psi|\Phi_i)=1} \Phi_i||_S,$

b) $C^*(||\Psi||_S) = ||\bigvee_{0<\pi(\Psi|\Phi_i)\leq 1} \Phi_i||_S,$

c) $BN_C(||\Psi||_S) = ||\bigvee_{0<\pi(\Psi|\Phi_i)<1} \Phi_i||_S = ||\bigvee_{i=1}^{n} ||\Phi_i||_S.$

The above properties enable us to introduce the following definitions:

i) If $||\Phi||_S = C_*(||\Psi||_S)$, then formula Φ will be called the *C-lower approximation* of the formula Ψ and will be denoted by $C_*(\Psi)$;

ii) If $||\Phi||_S = C^*(||\Psi||_S)$, then the formula Φ will be called the *C-upper approximation* of the formula Φ and will be denoted by $C^*(\Psi)$;

iii) If $||\Phi||_S = BN_C(||\Psi||_S)$, then Φ will be called the *C-boundary* of the formula Ψ and will be denoted by $BN_C(\Psi)$.

Let us consider the following example.
The C-lower approximation of (M, *poor*) is the formula

$$\begin{aligned} C_*(M, poor) \quad = \quad & ((F, med.) \wedge (P, med.) \wedge (S, med.)) \vee \\ & ((F, high) \wedge (P, med.) \wedge (S, large)) \vee \\ & ((F, high) \wedge (P, low) \wedge (S, small)). \end{aligned}$$

The C-upper approximation of (M, *poor*) is the formula

$$\begin{aligned} C^*(M, poor) &= ((F, med.) \wedge (P, med.) \wedge (S, med.)) \vee \\ &\quad ((F, high) \wedge (P, med.) \wedge (S, large)) \vee \\ &\quad ((F, med.) \wedge (P, low) \wedge (S, large)) \vee \\ &\quad ((F, high) \wedge (P, low) \wedge (S, small)). \end{aligned}$$

The C-boundary of (M, *poor*) is the formula

$$BN_C(M, poor) = ((F, med.) \wedge (P, low) \vee (S, large)).$$

After simplification we get the following approximations

$$\begin{aligned} C_*(M, poor) &= ((F, med.) \wedge (P, med.)) \vee (F, high), \\ C^*(M, poor) &= (F, med.) \vee (F, high). \end{aligned}$$

The concepts of the lower and upper approximation of a decision allow us to define the following decision rules:

$$C_*(\Psi) \rightarrow \Psi,$$
$$C^*(\Psi) \rightarrow \Psi,$$
$$BN_C(\Psi) \rightarrow \Psi.$$

For example, from the approximations given in the example above we get the following decision rules:

$((F,med.) \wedge (P,med.)) \vee (F,high) \rightarrow (M,\ poor)$,
$(F,med.) \vee (F,high) \rightarrow (M,\ poor)$,
$((F,med.) \wedge (P,low) \wedge (S,large)) \rightarrow (M,poor)$.

From these definitions it follows that any decision Ψ can be uniquely discribed by the following two decision rules:

$$C_*(\Psi) \rightarrow \Psi,$$
$$BN_C(\Psi) \rightarrow \Psi.$$

From the above calculations we can get two decision rules

$((F,med.) \wedge (P,med.)) \vee (F,\ high) \rightarrow (M,poor)$,
$((F,med.) \wedge (P,low.) \wedge (S,large)) \rightarrow (M,poor)$,

which are associated with the lower approximation and the boudary region of the decision (M, *poor*), respectively and describe decision (M, *poor*).

Obviously we can get similar decision rules for the decision (M, *good*) which are as follows:

$(F,low) \rightarrow (M,good)$,
$((F,med.) \wedge (P,low.) \wedge (S,large)) \rightarrow (M,good)$.

This coincides with the idea given by Ziarko [14] to represent decision tables by means of three decision rules corresponding to positive region the boundary region, and the negative region of a decision.

7 Decision rules and Bayes' rules

If $\{\Phi_i \rightarrow \Psi\}_n$ is a set of decision rules satisfying condition (1), then the well known formula for total probability holds:

$$\pi_S(\Psi) = \sum_{i=1}^{n} \pi_S(\Psi|\Phi_i) \cdot \pi_S(\Phi_i). \tag{2}$$

Moreover for any decision rule $\Phi \rightarrow \Psi$ the following Bayes' formula is valid:

$$\pi_S(\Phi_j|\Psi) = \frac{\pi_S(\Psi|\Phi_j) \cdot \pi_S(\Phi_j)}{\sum_{i=1}^{n} \pi_S(\Psi|\Phi_i) \cdot \pi_S(\Phi_i)}. \tag{3}$$

That is, any decision table or any set of implications satisfying condition (1) satisfies the Bayes' formula, without referring to prior and posterior probablities – fundamental in Baysian data analysis philosophy. Bayes' formula in our case says that: if an implication $\Phi \rightarrow \Psi$ is true to the degree $\pi_S(\Psi|\Phi)$ then the implication $\Psi \rightarrow \Phi$ is true to the degree $\pi_S(\Phi|\Psi)$.

This idea can be seen as a generalization of a *modus tollens* inference rule, which says that if the implication $\Phi \rightarrow \Psi$ is true so is the implication $\sim \Psi \rightarrow \sim \Phi$.

For example, for the set of decision rules

((*F*,*med.*) ∧ (*P*,*med.*)) ∨ (*F*, *high*) → (*M*,*poor*),
((*F*,*med.*) ∧ (*P*,*low*) ∧ (*S*,*large*)) → (*M*,*poor*),
(*F*,*low*) → (*M*,*good*),
((*F*,*med.*) ∧ (*P*,*low*) ∧ (*S*,*large*)) → (*M*,*good*),

we get the values of ceratinty and coverage factors shown in Table 3.

Table 3: Initial decision rules

Rule	*Decision*	*Certainty*	*Coverage*
certain	*poor*	1	3/4
boundary	*poor*	1/2	1/4
certain	*good*	1	1/2
boundary	*good*	1/2	1/2

The above set of decison rules can be "reversed" as

(*M*,*poor*) → ((*F*,*med.*) ∧ (*P*,*med.*)) ∨ (*F*, *high*),
(*M*,*poor*) → ((*F*,*med.*) ∧ (*P*,*low*) ∧ (*S*,*large*)),
(*M*,*good*) → (*F*,*low*),
(*M*,*good*) → ((*F*,*med.*) ∧ (*P*,*low*) ∧ (*S*,*large*)).

Due to Bayes' formula the certainty and coverage factors for inverted decision rules are mutually exchanged as shown in Table 4 below.

Table 4: Reversed decision rules

Rule	*Decision*	*Certainty*	*Coverage*
certain	*poor*	3/4	1
boundary	*poor*	1/4	1/2
certain	*good*	1/2	1
boundary	*good*	1/2	1/2

This property can be used to reason about data in the way similar to that allowed by *modus tollens* inference rule in classical logic.

8 Conclusions

It is shown in this paper that any decision table satisfies Bayes' rule. This enables to apply Bayes' rule of inference without referring to prior and posterior probabilities, inherently associated with "classical" Bayesian inference philosophy. From data tables one can extract decision rules – implications labelled by certainty factors expressing their degree of truth. The factors can be computed from data. Moreover, one can compute from data the coverage degrees expressing the truth degrees of "reverse" implications. This can be treated as generalization of *modus tollens* inference rule.

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