CONFLICTS AND DECISIONS II

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Abstract

Conflicts are one of the most characteristic attributes of human nature and study of conflicts is of greatest importance both practically and theoretically. Conflict analysis and resolution play an important role in business, governmental, political and lawsuits disputes, labor-management negotiations, military operations and others. Many formal models of conflict situations have been proposed and studied.

In this paper we outline a new approach to conflict analysis, which will be illustrated by voting analysis in conflict situations.

1 Introduction

Conflicts are one of the most characteristic attributes of human nature and study of conflicts is of greatest importance both practically and theoretically. Conflict analysis and resolution play an important role in business, governmental, political and lawsuits disputes, labor-management negotiations, military operations and others. To this end formal models of conflict situations are necessary. Many theoretical models of conflict situations have been proposed and studied, e.g., Casti, 1989, Coombs et al, 1988, Maeda et al, 1999, Nakamura 1999, Pawlak, 1998 and Roberts, 1976.

Conflict analysis seems to be important for decision making. Rough set based decision support plays important role in decision theory, see e.g. Slowinski, 1995. In this paper we outline new approach to conficts analysis, which will be illustrated by voting analysis in conflict situations.

We start our considerations with a very simple illustrative example. Next basic concepts of the proposed approach will be defined and studied.

2 Conflict, alliance and neutrality

In order to express relations between agents we define three basic binary relations on the universe: *conflict, neutrality* and *alliance*. To this end we need the following auxiliary function:

$$\phi_v(x,y) = \begin{cases} 1, & \text{if } v(x)v(y) = 1 \text{ or } \quad x = y, \\ 0, & \text{if } v(x)v(y) = 0 \text{ and } x \neq y, \\ -1, & \text{if } v(x)v(y) = -1. \end{cases}$$

This means that, if $\phi_v(x,y)=1$, agents x and y have the same opinion about issue v (are *allied* on v); if $\phi_v(x,y)=0$ means that at least one agent x or y has neutral approach to issue a (is *neutral* on a), and if $\phi_v(x,y)=-1$, means that both agents have different opinions about issue v (are in *conflict* on v).

In what follows we will define three basic relations R_v^+, R_v^0 and R_v^- on U^2 called *alliance, neutrality* and *conflict* relations respectively, and defined as follows:

$$R_v^+(x,y) \text{ iff } \phi_v(x,y)=1,$$

$$R_v^0(x,y) \text{ iff } \phi_v(x,y)=0,$$

$$R_v^-(x,y) \text{ iff } \phi_v(x,y)=-1.$$

It is easily seen that the alliance relation has the following properties:

(i) $R_v^+(x,x)$,

(ii) $R_v^+(x,y)$ implies $R_v^+(y,x)$,

(iii) $R_v^+(x,y)$ and $R_v^+(y,z)$ implies $R_v^+(x,z)$,

i.e., R_v^+ is an *equivalence* relation for every v. Each equivalence classe of alliance relation will be called *coalition* on v. Let us note that the condition (iii) can be expressed as "friend of my friend is my friend".

For the conflict relation we have the following properties:

(iv) non $R_v^-(x,x)$,

(v) $R_v^-(x,y)$ implies $R_v^-(y,x)$,

(vi) $R_v^-(x,y)$ and $R_v^-(y,z)$ implies $R_v^+(x,z)$,

(vii) $R_v^-(x,y)$ and $R_v^+(y,z)$ implies $R_v^-(x,z)$.

Conditions (vi) and (vii) refers to well know sayings "enemy of may enemy is my friend" and "friend of my enemy is my enemy".

For the neutrality relation we have:

(viii) non $R_v^0(x,x)$,

(ix) $R_v^0(x,y)=R_v^0(y,x)$.

Let us observe that in the conflict and neutrality relations there are no coalitions.

The following property holds $R_v^+ \cup R_v^0 \cup R_v^- = U^2$ because if $(x,y)\in U^2$ then $\Phi_v(x,y)=1$ or $\Phi_v(x,y)=0$ or $\Phi_v(x,y)=-1$ so $(x,y)\in R_v^+$ or $(x,y)\in R_v^0$ or $(x,y)\in R_v^-$. All the three relations R_v^+, R_v^0 are pairwise disjoint, i.e., every pair of objects (x,y) belongd to exactly one of the above defined relations (is in conflict, is allied or is neutral).

3 Information systems and decision tables

An information system is a data table, whose columns are labeled by attributes, rows are labeled by objects of interest and entries of the table are attribute values.

Formally, by an *information system* we will understand a pair $S = (U, A)$, where U and A, are finite, nonempty sets called the *universe*, and the set of *attributes,* respectively. With every attribute $a \in A$ we associate a set V_a, of its *values*, called the *domain* of a. Any subset B of A determines a binary relation $I(B)$ on U, which will be called an *indiscernibility relation*, and defined as follows: $(x, y) \in I(B)$ if and only if $a(x) = a(y)$ for every $a \in A$, where $a(x)$ denotes the value of attribute a for element x. Obviously $I(B)$ is an equivalence relation. The family of all equivalence classes of $I(B)$, i.e., a partition determined by B, will be denoted by $U/I(B)$, or simply by U/B; an equivalence class of $I(B)$, i.e., block of the partition U/B, containing x will be denoted by $B(x)$.

If (x, y) belongs to $I(B)$ we will say that x and y are *B-indiscernible* (*indiscernible with respect to* B). Equivalence classes of the relation $I(B)$ (or blocks of the partition U/B) are referred to as *B-elementary sets* or *B-granules.*

If we distinguish in an information system two disjoint classes of attributes, called *condition* and *decision attributes*, respectively, then the system will be called a *decision table* and will be denoted by $S = (U, C, D)$, where C and D are disjoint sets of condition and decision attributes, respectively.

Thus the decision table determines decisions which must be taken, when some conditions are satisfied. In other words each row of the of the decision table specifies a decision rule which determines decisions in terms of conditions.

An example of a decision table is given in Table 1. In the decision table the only condition attribute is *Party*, whereas the decision attribute is *Voting.* Each row of the table determines a decision rule.

4 Decision rules and decision algorithms

Every decision table describes decisions determined, when some conditions are satisfied. In other words each row of the decision table specifies a decision rule which determines decisions in terms of conditions.

Let us describe decision rules more exactly.

Let $S = (U, C, D)$ be a decision table. Every $x \in U$ determines a sequence $c_1(x), \ldots, c_n(x), d_1(x), \ldots, d_m(x)$ where $\{c_1, \ldots, c_n\} = C$ and $\{d_1, \ldots, d_m\} = D$.

The sequence will be called a *decision rule induced by* x (in S) and denoted by $c_1(x), \ldots, c_n(x) \rightarrow d_1(x), \ldots, d_m(x)$ or in short $C \rightarrow_x D$.

The number $supp_x(C, D) = |C(x) \cap D(x)|$ will be called the *support* of the decision rule $C \rightarrow_x D$ and the number

$$\sigma_x(C, D) = \frac{supp_x(C, D)}{|U|},$$

will be referred to as the *strength* of the decision rule $C \rightarrow_x D$. With every decision rule $C \rightarrow_x D$ we associate the *certainty factor* of the decision rule, denoted $cer_x(C, D)$ and defined as follows:

$$cer_x(C, D) = \frac{|C(x) \cap D(x)|}{|C(x)|} = \frac{supp_x(C, D)}{|C(x)|} = \frac{\sigma_x(C, D)}{\pi(C(x))},$$

where $\pi(C(x)) = \frac{|C(x)|}{|U|}$.

The certainty factor may be interpreted as conditional probability that y belongs to $D(x)$ given y belongs to $C(x)$, symbolically $\pi_x(D|C)$.

If $cer_x(C,D) = 1$, then $C \rightarrow_x D$ will be called a *certain decision* rule in S; if $0 < cer_x(C,D) < 1$ the decision rule will be referred to as an *uncertain decision rule* in S.

Besides, we will also use a *coverage factor* of the decision rule, denoted $cov_x(C,D)$ and defined as

$$\begin{aligned} cov_x(C,D) &= \frac{|C(x) \cap D(x)|}{|D(x)|} = \frac{supp_x(C,D)}{|D(x)|} = \\ &= \frac{\sigma_x(C,D)}{\pi(D(x))}, \end{aligned}$$

where $\pi(D(x)) = \frac{|D(x)|}{|U|}$.

Similarly

$$cov_x(C,D) = \pi_x(C|D).$$

5 Flow graphs and decision tables

With every decision table we associate a *flow graph*, i.e., a directed, connected, acyclic graph defined as follows: to every decision rule $C \rightarrow_x D$ we assign a *directed branch* x connecting the *input node* $C(x)$ and the *output node* $D(x)$. Strength of the decision rule represents a *throughflow* of the corresponding branch. Thus branches of the flow graph connect C-granules and D-granules of the graph.

More about flow graphs can be found in [7]

6 An example

Consider a parliament containing 500 members grouped in four political parties denoted A, B, C and D. Suppose the parliament discussed certain issue (e.g. membership of the country in European Union) and the voting result is presented in column *voting* in Table 1, where +, 0 and − denoted *yes*, *abstention* and *no* respectively. The column *support* contains the number of voters for each option.

Table 1: Voting result

Fact	*Party*	*Voting*	*Support*
1	A	+	200
2	A	0	30
3	A	–	10
4	B	+	15
5	B	–	25
6	C	0	20
7	C	–	40
8	D	+	25
9	D	0	35
10	D	–	100

Our task is to find difference between parties in view of voting result.

In what follows we will formulate the problem more precisely. We will start our consideration from the concept of an information system and a decision table.

The strength, certainty and the coverage factors for Table 1 are given in Table 2.

Table 2: Certainty and coverage factors

Fact	*Strength*	*Certainty*	*Coverage*
1	0.40	0.833	0.833
2	0.06	0.125	0.063
3	0.02	0.042	0.104
4	0.03	0.375	0.353
5	0.05	0.625	0.235
6	0.04	0.333	0.412
7	0.08	0.667	0.057
8	0.05	0.156	0.143
9	0.07	0.219	0.229
10	0.20	0.625	0.571

The flow graph associated with Table 1 is shown in Figure 1.

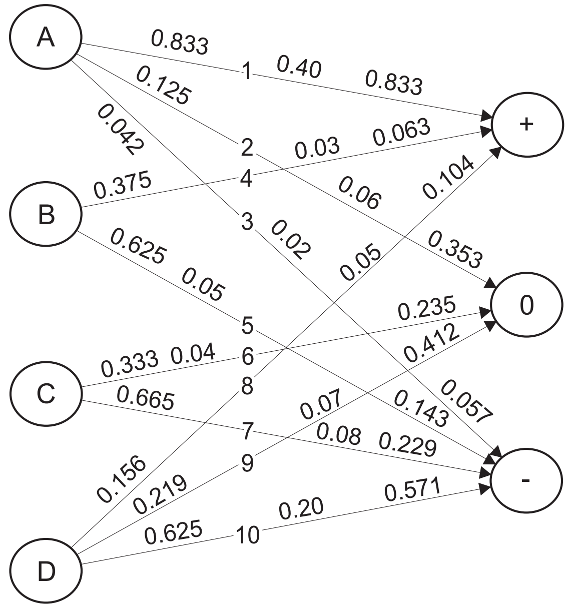

Figure 1: Flow graph

7 Conclusions

In this paper we have shown a new approach to conflict analysis based on the concept of conflict space, determined by voting results of conflicted parties.

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