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Rough Set Theory and Granular Computing

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Rough Measures, Rough Integrals and Sensor Fusion

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Abstract. This paper introduces a family of discrete rough integrals defined relative to rough measures. Rough set theory yields a rough measure based on a recently discovered rough membership set function. The particular form of rough membership function given in this paper is a non-negative set function that is additive. It is an example of a rough measure. The classical rough integral introduced by Z. Pawlak is revisited in the context of rough measure spaces. The family of rough integrals presented in this paper computes a form of ordered, weighted average of the values of a measurable function. Rough integrals are useful in culling from a collection of active sensors those sensors with the greatest relevance in a problem-solving effort such as classification of a perceived phenomenon in the environment of an agent. By way of practical application, an approach to fusion of homogeneous sensors is considered. The form of sensor fusion considered in this paper consists in selecting only those sensors considered relevant in solving a problem.

1 Introduction

This paper presents a measure, a discrete integral, and sensor fusion defined in the context of rough set theory [7]-[12]. In this paper, we investigate measures defined on a family $\wp(X)$ of all subsets of a non-empty, finite set X, i.e. on the power-set of X. A fundamental paradigm in rough set theory is set approximation [7]. Hence, there is interest in discovering a family of measures useful in set approximation. By way of practical application, an approach to fusion of homogeneous sensors deemed relevant in a classification effort is considered (see, e.g. [13]-[14]). Application of rough integrals has also been considered recently relative to sensor signal classification by navigation agents [16] and by web agents [17]. This research also has significance in the context of granular computing [13], [19]-[21]. This paper is organized as follows. Section 2 presents a brief introduction to classical additive set functions. Basic concepts of rough set theory are presented in Section 3. An introduction to rough measures is given in Section 4. A discrete rough integral is defined relative to a rough measure in Section 5. Sensor fusion is covered in Section 6.

2 Classical Additive Set Functions

This section gives a brief introduction to one form of additive set functions in measure theory [6]. Let $card(X)$ denote the cardinality of a finite set X (i.e. the number of elements of set X).

Definition 1. Let X be a finite, non-empty set. A function $\lambda : \wp(X) \rightarrow \Re$ where $\Re$ is the set of all real numbers is called a *set function* on X.

Definition 2. Let X be a finite, non-empty set and let λ be a set function on X. The function λ is said to be *additive* on X iff $\lambda(A \cup B) = \lambda(A) + \lambda(B)$ for every $A, B \in \wp(X)$ such that $A \cap B = \emptyset$ (i.e. A and B are disjoint subsets of X).

Definition 3. Let X be a finite, non-empty set and let λ be a set function on X. A function λ is called to be *non-negative* on X iff $\lambda(Y) \geq 0$ for any $Y \in \wp(X)$.

Definition 4. Let X be a set and let λ be a set function on X. A function λ is called to be *monotonic* on X iff $A \subsetneq B$ implies that $\lambda(A) \leq \lambda(B)$ for every $A, B \in \wp(X)$.

A brief introduction to the basic concepts underlying the design of rough membership function neurons is given in this section.

3 Basic Concepts of Rough Sets

Rough set theory offers a systematic approach to set approximation [7].

3.1 Set Approximation

To begin, let $S = (U, A)$ be an information system where U is a non-empty, finite set of objects and A is a non-empty, finite set of attributes, where $a : U \rightarrow V_a$ for every $a \in A$. For each $B \subsetneq A$, there is associated an equivalence relation $Ind_A(B)$ such that

$$Ind_A(B) = \{(x, x') \in U^2 \mid \forall a \in B.\mathrm{a(x)} = \mathrm{a(x')}\}$$

If $(x, x') \in Ind_A(B)$, we say that objects x and x' are indiscernible from each other relative to attributes from B. The notation $[x]_B$ denotes equivalence classes of $Ind_A(B)$. Further, partition $U/Ind_A(B)$ denotes the family of all equivalence classes of relation $Ind_A(B)$ on U. For $X \subseteq U$, the set X can be approximated only from information contained in B by constructing a B-lower and B-upper approximation denoted by $\underline{B}X$ and $\overline{B}X$ respectively, where $\underline{B}X = \{x \mid [x]_B \subseteq X\}$ and $\overline{B}X = \{x \mid [x]_B \cap X \neq \emptyset\}$.

3.2 Rough Membership Function

In this section, a set function form of the traditional rough membership function is presented.

Definition 5. Let $S = (U, A)$ be an information system, $B \subseteq A$, $u \in U$ and let $[u]_B$ be an equivalence class of an object $u \in U$ of $Ind_A(B)$. A rough membership set function is given in (1).

$$\mu_u^B : \wp(U) \to [0,1], \text{ where } \mu_u^B(X) = \frac{card\left(X \cap [u]_B\right)}{card\left([u]_B\right)} \quad (1)$$

for any $X \in \wp(U)$ is called a *rough membership set function.* A rough membership function provides a classification measure inasmuch as it tests the degree of overlap between the set X in $\wp(U)$ and equivalence class $[u]_B$. The form of rough membership function in Def. 5 is slightly different from the classical definition where the argument of the rough membership function is an object x and the set X is fixed [8].

4 Rough Measures

Let $S = (U, A)$ be an information system, $X \subseteq U, B \subseteq A$, and let $Ind_A(B)$ be the indiscernibility relation on U.

Definition 6. The tuple $(X, \wp(X), U/Ind_A(B))$, where $U/Ind_A(B)$ denotes a set of all equivalence classes determined by $Ind_A(B)$ on U, is called an *indiscernibility space* over X and B.

Definition 7. Let $u \in U$. A non-negative and additive set function $\rho_u : \wp(X) \to [0, \infty)$ defined by $\rho_u(Y) = \rho'(Y \cap [u]_B)$ for $Y \in \wp(X)$, where $\rho' : \wp(X) \to [0, \infty)$ is a set function, is called a *rough measure* relative to $U/Ind_A(B)$ and u on the indiscernibility space $(X, \wp(X), U/Ind_A(B))$.

Definition 8. Let ρ_u for $u \in U$ be a *rough measure* on the indiscernibility space $(X, \wp(X), U/Ind_A(B))$ for $u \in U$. The tuple $(X, \wp(X), U/Ind_A(B), \{\rho_u\}_{u \in U})$ is a *rough measure space* over X and B.

Example 1 (Sample Non-Negative Set Function). The rough membership function $\mu_u^B : \wp(X) \to [0,1]$ is a non-negative set function.

Proposition 1. Let $S = (U, A)$ be an information system, $B \subseteq A$, and let $[u]_B$ be an equivalence class of an object $u \in U$ of $Ind_A(B)$. The rough membership function μ_u^B as defined in Definition 5 (formula 1) is additive on U.

Proposition 2. $(X, \wp(X), U/Ind_A(B), \{\mu_u^B\}_{u \in U})$ is a rough measure space over X and B.

5 Rough Integrals

Rough integrals of discrete functions were introduced in [9], [10] as part of a study of rough functions [11]. In this section, we consider a variation of the Lebesgue integral, the discrete Choquet integral defined relative to a rough measure.

5.1 Discrete Rough Integral

In what follows, let $X = \{x_1, \ldots, x_n\}$ be a finite, non-empty set with n elements. The elements of X are indexed from 1 to n. The notation $X_{(i)}$ denotes the set $\{x_{(i)}, x_{(i+1)}, \ldots, x_{(n)}\}$ where $i \geq 1$ and $n = card(X)$. The subscript (i) is called a permutation index because the indices on elements of $X_{(i)}$ are chosen after a reordering of the elements of X. This reordering is induced by an external mechanism.

Example 2. Let $X = \{x_1, x_2\}$ the function $a : X \to \Re^+$ where $\Re^+$ is the set of non-negative real numbers, be defined such that $a(x_1) = 2001$, $a(x_2) = 44$. That is, $a(x_1) \geq a(x_2)$. Then, after reordering the elements of X and assigning permutation indices to the reordered elements, we obtain $a(x_{(1)}) \leq a(x_{(2)})$ where $x_{(1)} = x_2$ and $x_{(2)} = x_1$; $X_{(1)} = \{x_1, x_2\}, X_{(2)} = \{x_1\}$. Next, we use a functional defined by Choquet in 1953 in capacity theory [3].

Definition 9. Let ρ be a rough measure on X where the elements of X are denoted by $x_1, \ldots, x_n$. The discrete rough integral of $f : X \to \Re^+$ with respect to the rough measure r is defined by

$$\int f \, d\rho = \sum_{i=1}^{n} (f(x_{(i)}) - f(x_{(i-1)}))\rho(X_{(i)})$$

where $\bullet_{(i)}$ specifies that indices have been permuted so that $0 \leq f(x_{(i)}) \leq \cdots \leq f(x_{(n)})$, $X_{(i)} := \{x_{(i)}, \ldots, x_{(n)}\}$, and $f(x_{(0)}) = 0$.

The definition of the rough integral is based on a formulation formulation of the Choquet integral in Grabisch [5], and applied in [11]. The rough measure $\rho(X_{(i)})$ value serves as a weight of a coalition (or combination) of objects in set $X_{(i)}$ relative to $f(x_{(i)})$. It should be observed that in general the Choquet integral has the effect of averaging the values of a measurable function. This averaging closely resembles the well-known Ordered Weighted Average (OWA) operator [16].

5.2 Relevance of an Attribute

In this section, we consider the measurement of the relevance of an attribute using a rough integral. The measure 1 is fundamental in computing an average sensor value. Intuitively, we want to identify those sensors with outputs closest to some threshold.

Example 3. Consider the following decision tables.

Table 1(a)

$X \backslash \{a, e\}$	a	e
$x_1 = 0.203$	0.2	0
$x_2 = 0.454$	0.45	1
$x_3 = 0.453$	0.45	1
$x_4 = 0.106$	0.11	0
$x_5 = 0.104$	0.10	0

Table 1(b)

$X \backslash \{a, e\}$	a	e
$x_2 = 0.454$	0.45	1
$x_9 = 0.455$	0.46	1
$x_{10} = 0.401$	0.4	1
$x_{11} = 0.407$	0.41	1
$x_{12} = 0.429$	0.43	1

Let $X \subseteq U$, $\{a\}$ in A where $a : X \rightarrow [0, 0.5]$ where $a(x)$ is rounded to two decimal places. Let $(Y, U - Y)$ will be a partition defined by an expert e and let $[u]_e$ denote a set in this partition containing u for a fixed $u \in U$. We assume a decision system (X_a, a, e) is given for any considered attribute (sensor) a such that $X_a \subseteq U$, $a : X_a \rightarrow \Re^+$ and e is an expert decision restricted to X_a defining a partition $(Y \cap X_a, (U - Y) \cap X_a)$ of X_a. Moreover, we assume $X_a \cap [u]_e \neq \emptyset$.

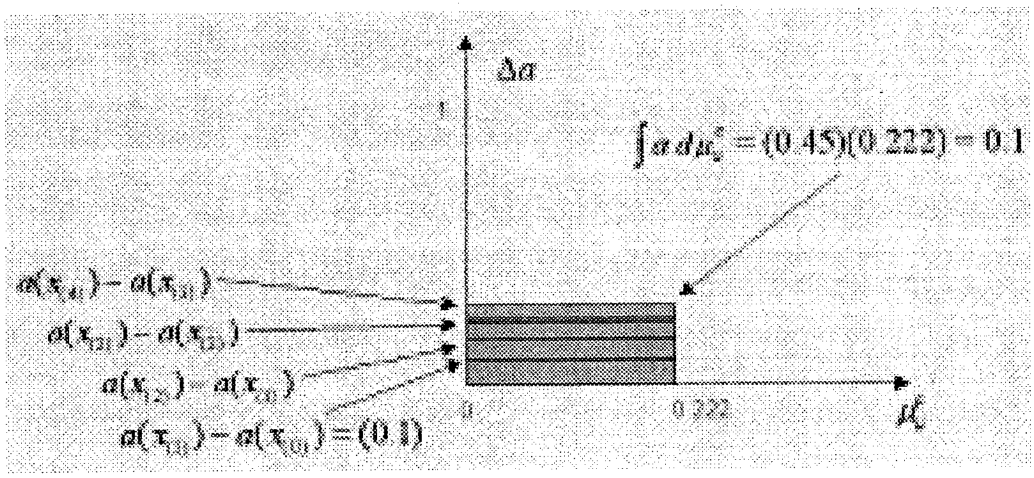

Fig. 1. Geometric Interpretation of Case 1

After reordering the attribute values from Table 1(a), we have the following computation (visualized in Fig. 1):

$0 \leq a(x_{(1)}) \leq a(x_{(2)}) \leq a(x_{(3)}) \leq a(x_{(4)}) \leq a(x_{(5)})$ where

$a(x_{(1)}) = 0.10$, $a(x_{(2)}) = 0.11$, $a(x_{(3)}) = 0.20$, $a(x_{(4)}) = 0.45$, $a(x_{(5)}) = 0.45$, and

$X_{(1)} = \{x_{(1)}, x_{(2)}, x_{(3)}, x_{(4)}, x_{(5)}\} = \{x_5, x_4, x_1, x_2, x_3\}$

$X_{(2)} = \{x_{(2)}, x_{(3)}, x_{(4)}, x_{(5)}\} = \{x_4, x_1, x_2, x_3\}$

$X_{(3)} = \{x_{(3)}, x_{(4)}, x_{(5)}\} = \{x_1, x_2, x_3\}$

$X_{(4)} = \{x_{(4)}, x_{(5)}\} = \{x_2, x_3\}$

$X_{(5)} = \{x_{(5)}\} = \{x_3\} = \{0.45\}$

$\mu_u^e(X_{(1)}) = \mu_u^e(X_{(2)}) = \mu_u^e(X_{(3)}) = \mu_u^e(X_{(4)}) = \frac{2}{9}$

$\mu_u^e(X_{(5)}) = \frac{1}{9}$

$\int a\, d\mu_u^e = 0.1 * 2/9 + (0.11 - 0.1) * 2/9 + (0.2 - 0.11) * 2/9 + (0.45 - 02) * 2/9 + (0.45 - 0.45) * 1/9 = 0.45 * 2/9 = 0.1$

After reordering the attribute values from Table 1b, we similarly obtain $\int a\, d\mu_u^e = 0.239$. From these two cases, it can be seen the relevance of attribute improves as the value of the rough integral increases in value. For a particular $[u]_e$, the rough integral measures the relevance of an attribute for a particular table in a classification effort. One can observe that the following property holds for rough integrals.

Proposition 3. Let $0 < s \leq r$. If $a(x) \in [s, r]$ for all $x \in X_a$, then $\int a\, d\mu_u^e \in (0, r]$ where $u \in U$.

Proposition 4. Let Given a decision table $DT = (U, C, D)$, where U (universe) is a non-empty finite set of objects, C is a set of condition attributes, and D is a set of decision (action) attributes, let $[u]_e$ be a subset of U. A rough Integral value is computed using the following formula:

$$\int_U a \, d\mu^e_{[u]_e} = \frac{\sum_{x_i \in [u]_e \cap X} a(x_i)}{|[u]_e|}$$

6 Multi-Sensor Fusion

Consider, next, the case where there is interest in discovering which sensor is more relevant among a set of sensors. The term relevance in this context denotes the closeness of a set of experimental sensor values relative to a set of a pre-calibrated, target sensor values that are considered important in a classification effort. The identification of relevant sensors provides a form of sensor fusion. The term sensor fusion generally refers to some process of combining sensor readings [1]-[2]. Further, assume that each of the sensors have the same model with essentially the same accuracy. At this stage, we will ignore the issue of the accuracy of a sensor, and trust that each sensor in the set of sensors produces output with low error.

6.1 Relevant Sensors

Consider the decision table (U, A, e). Let $u \in U$, $B = \{e\}$. Further, let $A = \{a_1 \ldots, a_n\}$ be a set of homogeneous sensors. Next, determine $[u]_e$, which is crucial in assessing the relevance of the sensors in B. The set $[u]_e$ consists of objects needed to classify sensor signal values. We can now consider a decision table with real value attributes being integral values for given sensors and given intervals (corresponding to different choice of u) and decision given by an expert. The relevant thresholds and intervals for integral values we can then extract from such a data table by discretization. Then, for example, the selection R of the most relevant sensors in a set of sensors is found using

$R = \{a_i \in B \mid \int a_i \, d\mu^e_u \in [s, r]\}$

where s, r are reals selected by means of discretization. In effect, the integral $\int a_i \, d\mu^e_u$ serves as a filter inasmuch as it filters out all sensors with integral values not belonging to the prescribed interval.

6.2 Sample Sensor Fusion

Consider the case where 46 ultrasonic sensors are used to monitor the environment for an agent and there is interest in determining occasions when the agent wanders into the region [15, 25]. Each sensor monitors the proximity of objects within its field of view. Assume that each sensor has a detection range from 0.1 cm to 100 cm. Assume that sensor readings are made continuously and stored in a queue. Let a_i, $a_i(t)$ be the i^{th} sensor and i^{th} sensor reading at time t, respectively. Sensor queues are analyzed for each collection of 20 readings (see Fig. 2).

As a result, we obtain an information table with 46 columns and 20 rows of sensor values. Each column represents a sensor signal, if we assume that sensor

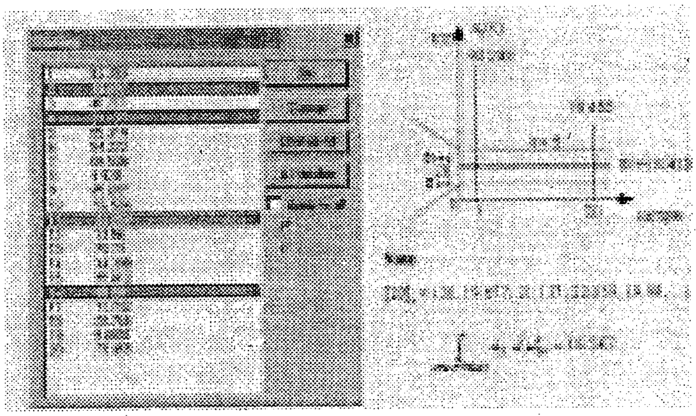

Fig. 2. Sample Sensor Readings

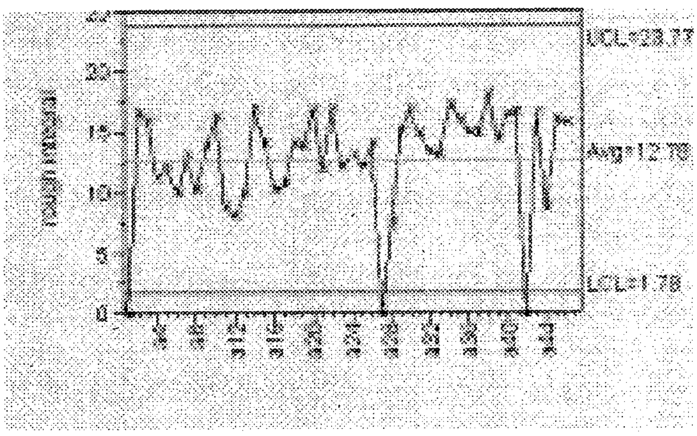

Fig. 3. Control Chart for Integral Values1

readings are made over a time interval $t_0, \ldots, t_{19}$, where t_0 is an instant in time when the first sensor reading is made in a group of 20 readings. Further, assume that we interested in sensor readings in the range 20 cm $\pm$ 5cm. This range provides the basis for a definition of an indiscernibility class represented by the decision e of an expert. That is, $[20]_e = \{15, 15.001, 15.002, \ldots, 25\}$. A sample obtained from sensor a_2 is given in Fig. 2. A summary of the integral values for the 46 sensors is given in table in the control chart in Fig. 3, where the Upper {Lower} Control Limits 23.77{1.78} have been chosen arbitrarily. Thirteen relevant sensors appear in this sample, namely, a_2, a_3, a_{10}, a_{20}, a_{22}, a_{30}, a_{34}, a_{35}, a_{38}, a_{40}, a_{43}, a_{45} and a_{46}.

Remark. Suppose we are interested in estimating for a considered period of time if an agent was walking around region [15, 25]. The meaning of this can be estimated by an expert looking at the plot in Fig. 3 relative to the requirement stipulated by $[20]_e$. We can imagine a decision table with objects represented by sensor signals (not single sample signal values but a set of sample signal values) where a decision is the expert estimation if a wandering agent tends to favor walking in the region [15, 25]. This raises the question: *Does the integral value reflect (to a sufficient degree) an expert decision?* Here, in answer to this question, let us observe that the rough integral computes the aggregative effect reflected by sensor values relative to the "walking region" specified by an indiscernibility class. Notice that the lowest sample values in a signal have considerable influence in the computation, and the higher values (outside the specified region) have little or no influence on the outcome. In effect, whenever the lowest sample values in a signal are clustered in the specified walking region, the value computed by the integral will reflect this fact. In the case where a decision-maker focuses on agents near or above a walking region, the integral does approximate an expert decision. It should also be pointed out that an integral value reflects a decision about a wandering agent not in terms of a single sample sensor value, but rather a decision about the proximity of an agent to a specified region in terms of a set of sample values.

7 Conclusion

Rough set theory provides a variety of set functions that can be studied relative to various measure spaces. In particular, the rough membership function is considered. The particular rough membership function given in this paper is a non-negative set function which is additive and, hence, is an example of a rough measure. We are interested in identifying those sensors considered relevant in a problem-solving effort. The rough integral introduced in this paper serves as a means of classifying sensors. That is, depending on the proximity of the values of a sensor to a region defined by the equivalence class $[u]_e$, the rough integral computes the relevance of a sensor in a classification effort. By careful selection of an equivalence class $[u]_e$, it is possible to use the rough integral given in this paper as a form of filter inasmuch as the integral filters out sensor signals (sets of sample sensor values) with low representation in $[u]_e$. An application of rough integration has been given in the context of identifying one or more homogeneous sensors considered relevant in a classification effort. Two forms of sensor fusion are implicit in the application considered in this paper, namely, aggregation of sensor signal values in computing a rough integral value and culling from a set of sensors those sensors considered relevant in a problem-solving effort. It is left for future work to consider how one might gain by identifying relevant sensors. Notice, also, that rough measure spaces are defined in the context of non-empty, finite universes. There is also interest in capturing the cumulative effect of set approximation relative to an uncountable set of points.

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References

1. R.R. Brooks, S.S. Iyengar, Methods of approximate agreement for multisensor fusion. In: Kadar and Libby (Eds.), Signal Processing, Sensor Fusion and Target Recognition IV, SPIE vol. 2484, 1995, 37-44.
2. R.R. Brooks, S.S. Iyengar, Multi-Sensor Fusion, Upper Saddle River, NJ: Prentice-Hall PTR, 1998.
3. G. Choquet, Theory of capacities. Annales de l'Institut Fourier, 5, 1953, 131-295.
4. M.Grabisch, T. Murofushi, M. Sugeno (Eds.), Fuzzy Measures and Integrals: Theory and Applications, Berlin, Physica-Verlag, 2000.
5. M. Grabisch, Alternative expressions of the discrete Choquet integral. In: Proc. 7th Int. Fuzzy Systems Association World Congress (IFSA'97), Prague, 25-29 June 1997, 472-477.
6. P.R. Halmos, Measure Theory. London: D. Van Nostrand Co., 1950.
7. Z. Pawlak, Rough Sets: Theoretical Aspects of Reasoning About Data, Boston, MA, Kluwer Academic Publishers, 1991.
8. Z. Pawlak, A. Skowron, Rough membership functions. In: R. Yager, M. Fedrizzi, J. Kacprzyk (Eds.), Advances in the Dempster-Shafer Theory of Evidence, NY, John Wiley & Sons, 1994, 251-271.
9. Z. Pawlak, On rough derivatives, rough integrals, and rough differential equations. ICS Research Report 41/95, Institute of Computer Science, Nowowiejska 15/19, 00-665 Warsaw, Poland, 1995.
10. Z. Pawlak, Rough sets, rough function and rough calculus. In: S.K. Pal, A. Skowron (Eds.), Rough Fuzzy Hybridization: A New Trend in Decision-Making, Berlin, Springer-Verlag, 1999, 99-109.
11. Z. Pawlak, Rough functions, Bull. Polish Academy of Sciences, Tech. Series 35/5-6, 1987, 249-251.
12. Z. Pawlak, J.F. Peters, A. Skowron, Z. Surai, S. Ramanna, M. Borkowski, Rough measures: Theory and application. In: Proc. of the Int. Workshop on Rough Set Theory and Granular Computing (RSTGR'2001), Bulletin of the International Rough Set Society, vol. 5, no. 1/2, Matsue, Shimane, Japan, 20-22 May 2001, 177-183.
13. J.F. Peters, S. Ramanna, A. Skowron, J. Stepaniuk, Z. Suraj, M. Borkowsky, Sensor fusion: A rough Petri net model. In: Proc. Joint 9th International Fuzzy Systems Association (IFSA) World Congress and 20th North American Fuzzy Information Processing Society (NAFIPS) Int. Conf., Vancouver, British Columbia, Canada, 25-28 June 2001, 2882-2888.
14. J.F. Peters, S. Ramanna, A. Skowron, J. Stepaniuk, Z. Suraj, M. Borkowsky, Sensor fusion: A rough granular approach. In: Proc. Joint 9th International Fuzzy Systems Association (IFSA) World Congress and 20th North American Fuzzy Information Processing Society (NAFIPS) Int. Conf., Vancouver, British Columbia, Canada, 25-28 June 2001, 1367-1372.
15. J.F. Peters, S. Ramanna, L. Han, The Choquet integral in a rough software cost estimation system. In: M. Grabisch, T. Murofushi, M. Sugeno (Eds.), Fuzzy Measures and Integrals: Theory and Applications. (Springer-Verlag, Heidelberg, Germany, 2000) 392-414.
16. J.F. Peters, S. Ramanna, A. Skowron, M. Borkowski, Approximate sensor fusion in a navigation agent. In: Proc. Intelligent Agents Technology, Japan, October 2001.

17. J.F. Peters, S. Ramanna, A. Skowron, M. Borkowski, Wireless agent guidance of remote mobile robots: Rough integral Approach to Sensor Signal Analysis. In: Proc. Web Intelligence, Japan, October 2001.
18. Z. Pawlak, J.F. Peters, A. Skowron, Z. Suraj, S. Ramanna, M. Borkowski, Rough measures and integrals: A brief introduction. In: T. Terano, T. Nishida, A. Namatame, S. Tsumoto, Y. Ohsawa, T. Washio (Eds.), New Frontiers in Artificial Intelligence, Lecture Notes in Artificial Intelligence No. 2253. Berlin: Springer-Verlag, 2001, 375-279.
19. A. Skowron, Toward Intelligent Systems: Calculi of Information Granules. In: Proc. RSTGC'01, Bulletin of the International Rough Set Society, vol. 5, no. 1/2, Matsue, Shimane, Japan, 20-22 May 2001, 9-30.
20. A. Skowron, J. Stepaniuk, Constructive information granules. In: Proc. of the 15th IMACS World Congress on Scientific Computation, Modelling and Applied Mathematics, Berlin, Germany, 24-29 August 1997. Artificial Intelligence and Computer Science 4, 1997, 625-630.
21. A. Skowron, J. Stepaniuk, Information granules: Towards foundation of granular computing, Int. J. of Intelligent Systems, vol. 16, 2001, 57-86.