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Rough Measures: Theory and Applications

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Abstract: *This paper introduces a measure defined in the context of rough sets. Rough set theory provides a variety of set functions that can be studied relative to various measure spaces. In particular, the rough membership function is considered. The particular rough membership function given in this paper is a non-negative set function that is additive. It is an example of a rough measure. The idea of a rough integral is revisited in the context of the discrete Choquet integral that is defined relative to a rough measure. This rough integral computes a form of ordered, weighted "average" of the values of a measurable function. Rough integrals are useful in culling from a collection of active sensors those sensors with the greatest relevance in a problem-solving effort such as classification of a "perceived" phenomenon in the environment of an agent. The relevance of a sensor is computed using a discrete rough integral relative to a target interval. By way of practical application, an approach to fusion of homogeneous sensors is considered. The form of sensor fusion considered in this paper consists in selecting only those sensors considered relevant in solving a problem.*

Keywords: additivity, fusion, measure, measure space, rough sets, rough membership function, rough measure, rough integral.

1 Introduction

This paper introduces a measure defined in the context of rough sets [1]-[5]. In this paper, we investigate measures defined on a family $\wp(X)$ of all subsets of a finite set X, i.e. on the powerset of X. A fundamental paradigm in rough set theory is set approximation. Hence, there is interest in discovering a family of measures useful in set approximation. By way of practical application, an approach to fusion of homogeneous sensors deemed relevant in a classification effort is considered (see, also, [10]). Application of rough integrals has also been considered recently relative to sensor signal classification by intelligent agents [12] and by web agents [13]. This research also has significance in the context of granular computing [14]-[15] and rough neural networks [7].

This paper is organized as follows. Section 2 presents a brief introduction to classical additive functions. Basic concepts of rough set theory (set approximation and rough membership functions) are presented in Section 3. An introduction to rough measures is given in Section 4.

The discrete Choquet integral is defined relative to a rough measure in Section 5. An application of the rough integral is considered in Section 6 in the context of sensor fusion (aggregating sensor signal values and identifying relevant sensors).

2 Classical Additive Set Functions

This section gives a brief introduction to one form of additive set functions in measure theory [8]. Let *card(X)* denote the cardinality of a finite set X (i.e., the number of elements of set X).

Definition 1 Let X be a finite, non-empty set. A function $\lambda : \wp(X) \rightarrow R$ where R is the set of all real numbers is called a *set function* on X.

Definition 2 Let X be a finite, non-empty set and let λ be a set function on X. The function λ is said to be *additive* on X iff $\lambda(A \cup B) = \lambda(A) + \lambda(B)$ for every $A, B \in \wp(X)$ such that $A \cap B = \emptyset$ (i.e., A and B are disjoint subsets of X).

Definition 3 Let X be a finite, non-empty set and let λ be a set function on X. A function λ is called to be *non-negative* on X iff $\lambda(Y) \geq 0$ for any $Y \in \wp(X)$.

Definition 4 Let X be a set and let λ be a set function on X. A function λ is called to be *monotonic* on X iff $A \subseteq B$ implies that $\lambda(A) \leq \lambda(B)$ for every $A, B \in \wp(X)$.

3 Basic Concepts of Rough Sets

Rough set theory offers a systematic approach to set approximation [1]-[2].

3.1 Set Approximation

To begin, let $S = (U, A)$ be an information system where U is a non-empty, finite set of objects and A is a non-empty, finite set of attributes, where $a: U \rightarrow V_a$ for every $a \in A$. For each $B \subseteq A$, there is associated an equivalence relation $\text{Ind}_A(B)$ such that

$$\text{Ind}_A(B) = \{(x, x') \in U^2 \mid \forall a \in B.\, a(x) = a(x')\}$$

If $(x, x') \in \text{Ind}_A(B)$, we say that objects x and x' are indiscernible from each other relative to attributes from B. The notation $[x]_B$ denotes equivalence classes of $\text{Ind}_A(B)$. Further, partition U/ $\text{Ind}_A(B)$ denotes the family of all equivalence classes of relation $\text{Ind}_A(B)$ on U. For $X \subseteq U$, the set X can be approximated only from information contained in B by constructing a B-lower and B-upper approximation denoted by $\underline{B}X$ and $\overline{B}X$ respectively, where $\underline{B}X = \{x \mid [x]_B \subseteq X\}$ and $\overline{B}X = \{x \mid [x]_B \cap X \neq \emptyset\}$.

3.2 Rough Membership Function

In this section, a set function form of the traditional rough membership function is presented.

Definition 5 Let $S = (U, A)$ be an information system, $B \subseteq A$, $u \in U$ and let $[u]_B$ be an equivalence class of an object $u \in U$ of $\text{Ind}_A(B)$. The set function

$$\mu_u^B : \wp(U) \rightarrow [0,1], \, where \, \mu_u^B(X) = \frac{card(X \cap [u]_B)}{card([u]_B)} \quad (1)$$

for any $X \in \wp(U)$ is called a *rough membership function (rmf)*. A rough membership function provides a classification measure inasmuch as it tests the degree of overlap between the set X in $\wp(U)$ and equivalence class $[u]_B$. The form of rough membership function in Def. 5 is slightly different from the classical definition where the argument of the rough membership function is an object x and the set X is fixed [3].

4 Rough Measures

Let $S = (U, A)$ be an information system, $X \subseteq U$, $B \subseteq A$, and let $\text{Ind}_A(B)$ be the indiscernibility relation on U.

Definition 6 The tuple $(X, \wp(X), U / Ind_A(B))$, where $U / Ind_A(B)$ denotes a set of all equivalence classes determined by $\text{Ind}_A(B)$ on U, is called an *indiscernibility space* over X and B.

Definition 7 Let $u \in U$. A non-negative and additive set function $\rho_u : \wp(X) \rightarrow [0, \infty)$ defined by $\rho_u(Y) = \rho'(Y \cap [u]_B)$ for $Y \in \wp(X)$, where $\rho' : \wp(X) \rightarrow [0, \infty)$ is called a *rough measure* relative to $U / Ind_A(B)$ and u on the indiscernibility space $(X, \wp(X), U / Ind_A(B))$.

Definition 8 Let ρ_u for $u \in U$ be a *rough measure* on the indiscernibility space $(X, \wp(X), U / Ind_A(B))$ (relative to $U / Ind_A(B)$ and $u \in U$). The tuple $(X, \wp(X), U / Ind_A(B), \{\rho_u\}_{u \in U})$ is a *rough measure space* over X and B.

Example 1. [Sample Non-Negative Set Function]. The rough membership function $\mu_u^B : \wp(X) \rightarrow [0, 1]$ is a non-negative set function.

Proposition 1 Let $S = (U, A)$ be an information system, $B \subseteq A$, and let $[u]_B$ be an equivalence class of an object $u \in U$ of $\text{Ind}_A(B)$. The rough membership function μ_u^B as defined in Definition 5 (formula (1)) is additive on U.

Proof. The proof follows from the following facts. Let $X, Y \in \wp(U)$ and $X \cap Y = \emptyset$. Then $(X \cap [u]_B) \cap (Y \cap [u]_B) = \emptyset$. Hence, from formula (1) of Definition 5 we get $\mu_u^B(X \cup Y) = \text{card}((X \cup Y) \cap [u]_B) / \text{card}([u]_B) =$ $\text{card}(X \cap [u]_B) / \text{card}([u]_B) + \text{card}(Y \cap [u]_B) / \text{card}([u]_B)$ $= \mu_u^B(X) + \mu_u^B(Y)$.

Proposition 2 $(X, \wp(X), U / Ind_A(B), \{\mu_u^B\}_{u \in U})$ is a rough measure space over X and B.

5 Rough Integrals

Rough integrals of discrete functions were introduced in [5], [17] as part of a study of rough functions [18]. In this section, we consider a variation of the Lebesgue integral, the discrete Choquet integral defined relative to a rough measure.

5.1 Discrete Rough Integral

In what follows, let $X=\{x_1,...,x_n\}$ be a finite, non-empty set with n elements. The elements of X are indexed from 1 to n. The notation $X_{(i)}$ denotes the set $\{x_{(i)}, x_{(i+1)}, ..., x_{(n)}\}$ where $i \geq 1$ and $n = card(X)$. The subscript (i) is called a permutation index because the indices on elements of $X_{(i)}$ are chosen after a reordering of the elements of X. This reordering is "induced" by an external mechanism.

Example 2. Let $X = \{x_1, x_2\}$ the function $a: X \rightarrow \mathbb{R}^+$ where $\mathbb{R}^+$ is the set of non-negative real numbers, be defined such that $a(x_1) = 2001$, $a(x_2) = 44$. That is, $a(x_1) \geq a(x_2)$. Then, after reordering the elements of X and assigning permutation indices to the reordered elements, we obtain $a(x_{(1)}) \leq a(x_{(2)})$ where $x_{(1)} = x_2$ and $x_{(2)} = x_1$; $X_{(1)} = \{x_1,x_2\}$, $X_{(2)} = \{x_1\}$. Next, we use a functional defined by Choquet in 1953 in capacity theory [6].

Definition 9 Let ρ be a rough measure on X where the elements of X are denoted by $x_1, ..., x_n$. The discrete Choquet integral of $f: X \rightarrow \mathbb{R}^+$ with respect to the rough measure ρ is defined by

$$\int f \, d\rho = \sum_{i=1}^{n} (f(x_{(i)}) - f(x_{(i-1)}))\rho(X_{(i)})$$

where $\bullet_{(i)}$ specifies that indices have been permuted so that $0 \leq f(x_{(i)}) \leq ... \leq f(x_{(n)})$, $X_{(i)} := \{x_{(i)}, ..., x_{(n)}\}$, and $f(x_{(0)}) = 0$.

This definition of the Choquet integral is based on a formulation in Grabisch [7], and applied in [11]. The rough measure ρ($X_{(i)}$) value serves as a "weight" of a coalition (or combination) of objects in set $X_{(i)}$ relative to $f(x_{(i)})$. It should be observed that in general the Choquet integral has the effect of "averaging" the values of a measurable function. This averaging closely resembles the well-known Ordered Weighted Average (OWA) operator [16].

5.2 Relevance of an Attribute

In this section, we consider the measurement of the relevance of an attribute using a rough integral. The measure (1) is fundamental in computing an "average" sensor value. Intuitively, we want to identify those sensors with outputs closest to some threshold.

Example 3. Consider the following decision tables.

Table 1(a)

X_a\{a,d}	a	e
x_1=0.203	0.2	0
x_2=0.454	0.45	1
x_3=0.453	0.45	1
x_4=0.106	0.11	0
x_5=0.104	0.10	0

Table 1(b)

X_a\{a,d}	a	e
x_2= 0.454	0.45	1
x_9= 0.455	0.46	1
x_{10}= 0.401	0.4	1
x_{11}= 0.407	0.41	1
x_{12}= 0.429	0.43	1

Let $X \subseteq U$, $\{a\}$ in A where $a: X \rightarrow [0, 0.5]$ where $a(x)$ is rounded to two decimal places. Let $(Y, U\text{-}Y)$ will be a partition defined by an expert e and let $[u]_e$ denote a set in this partition containing u for a fixed u ∈ U. We assume a decision system (X_a,a,e) is given for any considered attribute (sensor) a such that $X_a \subseteq U$, $a: X_a \rightarrow R^+$ and e is an expert decision restricted to X_a defining a partition $(Y \cap X_a, (U\text{-}Y) \cap X_a)$ of X_a. Moreover, we assume $X_a \cap [u]_e \neq \emptyset$.

After reordering the attribute values from Table 1(a), we have case 1 (see Fig. 1):

$0 \leq a(x_{(1)}) \leq a(x_{(2)}) \leq a(x_{(3)}) \leq a(x_{(4)}) \leq a(x_{(5)})$ where
$a(x_{(1)}) = 0.10$, $a(x_{(2)}) = 0.11$, $a(x_{(3)}) = 0.20$, $a(x_{(4)}) = 0.45$, $a(x_{(5)}) = 0.45$, and
$X_{(1)} = \{x_{(1)}, x_{(2)}, x_{(3)}, x_{(4)}, x_{(5)}\} = \{x_5, x_4, x_1, x_2, x_3\}$
$X_{(2)} = \{x_{(2)}, x_{(3)}, x_{(4)}, x_{(5)}\} = \{x_4, x_1, x_2, x_3\}$
$X_{(3)} = \{x_{(3)}, x_{(4)}, x_{(5)}\} = \{x_1, x_2, x_3\}$
$X_{(4)} = \{x_{(4)}, x_{(5)}\} = \{x_2, x_3\}$
$X_{(5)} = \{x_{(5)}\} = \{x_3\} = \{0.45\}$
$\mu_u^e(X_{(1)}) = \mu_u^e(X_{(2)}) = \mu_u^e(X_{(3)}) = \mu_u^e(X_{(4)}) = 2/9$
$\mu_u^e(X_{(5)}) = 1/9$
$\int a \, d\mu_u^e =$
$0.1*2/9 + (0.11\text{-}0.1)*2/9 + (0.2\text{-}0.11)*2/9 + (0.45\text{-}02)*2/9+(0.45\text{-}0.45)*1/9=$
$0.45*2/9=0.1$

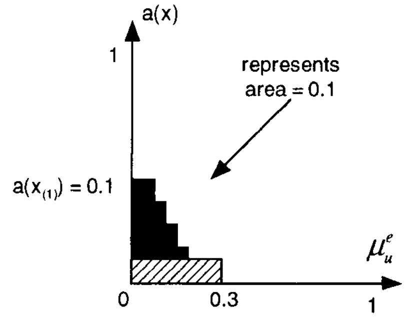

Fig. 1 Geometric Interpretation of Case 1

After reordering the attribute values from Table 1(b), we have case 2 (see Fig. 2):

$0 \leq a(x_{(1)}) \leq a(x_{(2)}) \leq a(x_{(3)}) \leq a(x_{(4)}) \leq a(x_{(5)})$ where

$a(x_{(1)}) = 0.40$, $a(x_{(2)}) = 0.41$, $a(x_{(3)}) = 0.43$, $a(x_{(4)}) = 0.45$, $a(x_{(5)}) = 0.46$, and
$X_{(1)} = \{x_{(1)}, x_{(2)}, x_{(3)}, x_{(4)}, x_{(5)}\} = \{x_{10}, x_{11}, x_{12}, x_2, x_9\}$
$X_{(2)} = \{x_{(2)}, x_{(3)}, x_{(4)}, x_{(5)}\} = \{x_{11}, x_{12}, x_2, x_9\}$
$X_{(3)} = \{x_{(3)}, x_{(4)}, x_{(5)}\} = \{x_{12}, x_2, x_9\}$
$X_{(4)} = \{x_{(4)}, x_{(5)}\} = \{x_2, x_9\}$
$X_{(5)} = \{x_{(5)}\} = \{x_9\}$
$\mu_u^e(X_{(1)}) = 5/9 = 0.56$
$\mu_u^e(X_{(2)}) = 4/9 = 0.4$
$\mu_u^e(X_{(3)}) = 3/9 = 0.3$
$\mu_u^e(X_{(4)}) = 2/9 = 0.2,$
$\mu_u^e(X_{(5)}) = 1/9 = 0.1$
$\int a\, d\mu_u^e =$
$[0.40].56 + [0.41 - 0.40].4 + [0.43 - 0.41].3 +$
$[0.45 - 0.43]0.2 + [0.46 - 0.45]0.1 =$
$0.224 + 0.004 + 0.006 + 0.004 + 0.001 =$
0.239

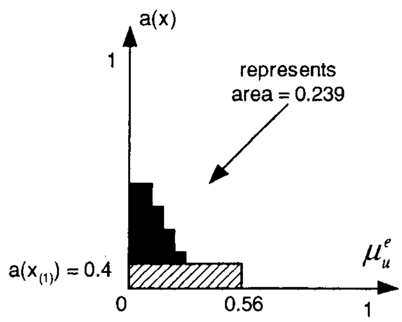

Fig. 2 Geometric Interpretation of Case 2

From these two cases, it can be seen the relevance of attribute improves as the value of the rough integral increases in value. For a particular $[u]_e$, the rough integral measures the relevance of an attribute for a particular table in a classification effort. One can observe that the following property holds for rough integrals.

Proposition 3. Let $0 < s \le r$. If $a(x) \in [s, r]$ for all $x \in X_a$, then $\int a\, d\mu_u^e \in (0, r]$ where $u \in U$.

Proof:
Since $0 < s \le r$, it is enough to show that (1) $\int a\, d\mu_u^e \le r$ and (2) $\int a\, d\mu_u^e > 0$.

(1) We have $\mu_u^e(X_{(i)}) \le 1$, $B = \{e\}$. Hence

$$\int a\, d\mu_u^e = \sum_{i=1}^{n} \left(a(x_{(i)}) - a(x_{(i-1)})\right)\mu_u^e\left(X_{(i)}\right) \le \sum_{i=1}^{n} \left(a(x_{(i)}) - a(x_{(i-1)})\right) = a(x_{(n)}) = r$$

(2) From the assumptions, there exists at least one $x \in [u]_e$. Hence, $\exists_{k \ge 1} : \mu_u^e\left(X_{(k)}\right) > 0$. After reordering of the subsets of $\wp(X)$, we know that μ_u^e is a non-increasing, non-negative set function. Hence, $\mu_u^e(X_{(1)}) > 0$. Consider the case where $a(x_{(1)}) = s$. Then compute

$$\int a\, d\mu_u^e = \sum_{i=1}^{n} \left(a(x_{(i)}) - a(x_{(i-1)})\right)\mu_u^e\left(X_{(i)}\right) \ge \left(a(x_{(1)}) - a(x_{(0)})\right)\mu_u^e\left(X_{(1)}\right) = a(x_{(1)})\mu_u^e\left(X_{(1)}\right) > 0$$

□

Moreover, when the set of sensor a values is close to $[u]_e$, then $\int a\, d\mu_u^e$ is close to the maximal value the integral can take for a sensor. Measures of closeness depend on applications and their parameters can be tuned for specific data cases and targets.

The largest integral value we obtain when a partition defined by singletons is exactly defined by the attribute (sensor). The value of integral is decreasing when the quality of the definability of this special partition (created by singletons) by a given attribute is decreasing. The integral (over all values for a given attribute) reflects in a sense the degree of definability of the partition of objects created by singletons by a partition defined by the values of a given attribute (sensor).

Two sensors a, b are close if the following conditions are satisfied:

$$\max_{1 \le i \le k} \left| a\left(x_{(i)}\right) - b\left(y_{(i)}\right) \right| < \varepsilon\Delta$$

$$\max_{1 \le i \le k} \left| \mu_u^e(X_{(i)}) - \mu_u^e(Y_{(i)}) \right| < \varepsilon$$

where $\Delta = \max(a(x_{(k)}), b(y_{(k)}))$. Then one can prove the following proposition:

Proposition 4. If sensors a, b are close then

$$\frac{1}{k}\left| \int a\, d\mu_u^e - \int b\, d\mu_u^e \right| < 2\Delta\varepsilon\left(1 + \frac{1}{2k}\right) \approx 2\Delta\varepsilon$$

for sufficiently large k.

6 Multi-Sensor Fusion

Consider, next, the case where there is interest in discovering which sensor is more relevant among a set of sensors. The term relevance in this context denotes the "closeness" of a set of experimental sensor values relative to a set of a pre-calibrated, target sensor values that are considered important in a classification effort. The identification of relevant sensors provides a form of sensor fusion. The term sensor fusion generally refers to some process of combining sensor readings [19]-[20]. Further, assume that each of the sensors have the same model with essentially the same accuracy. At this stage, we will ignore the issue of the accuracy of a sensor, and trust that each sensor in the set of sensors produces output with low error.

6.1 Relevant Sensors

Consider the decision table (U, A, e). Let $u \in U$, $B = \{e\}$. Further, let $A = \{a_1, ..., a_n\}$ be a set of homogeneous sensors. Next, determine $[u]_e$, which is crucial in assessing the relevance of the sensors in B. The set $[u]_e$ consists of objects needed to classify sensor signal values. We can now consider a decision table with real value attributes being integral values for given sensors and given intervals (corresponding to different choice of u) and decision given by an expert. The relevant thresholds and intervals for integral values we can then extract from such a data table by discretization. Then, for example, the selection R of the most relevant sensors in a set of sensors is found using

$$R = \left\{ a_i \in B \mid \int a_i \, d\mu_u^e \in [s, r] \right\}$$

where s, r are reals selected by means of discretization. In effect, the integral $\int a_i \, d\mu_u^e$ serves as a filter inasmuch as it "filters" out all sensors with integral values not belonging to the prescribed interval.

6.2 Sample Sensor Fusion

Consider the case where 46 ultrasonic sensors are used to monitor the environment for an agent and there is interest in determining occasions when the agent wanders into the region [15, 25]. Each sensor monitors the proximity of objects within its field of view. Assume that each sensor has a detection range from 0.1 cm to 100 cm. Assume that sensor readings are made continuously and stored in a queue. Let a_i, $a_i(t)$ be the i^{th} sensor and i^{th} sensor reading at time t, respectively. Sensor queues are analyzed for each collection of 20 readings (see Fig. 3).

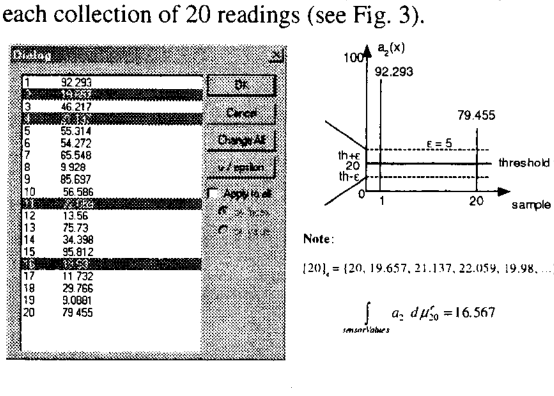

Fig. 3 Sample Sensor Readings

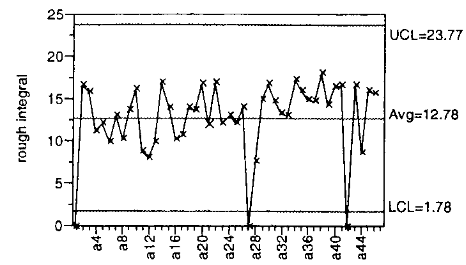

Fig. 4 Control Chart for Integral Values.

As a result, we obtain an information table with 46 columns and 20 rows of sensor values. Each column represents a sensor signal, if we assume that sensor readings are made over a time interval $t_0, ..., t_{19}$, where t_0 is an instant in time when the first sensor reading is made in a group of 20 readings. Further, assume that we interested in sensor readings in the range 20 cm ± 5cm. This range provides the basis for a definition of an indiscernibility class represented by the decision *e* of an expert. That is, $[20]_e = \{15, 15.001, 15.002, \ldots, 25\}$.

A sample obtained from sensor a_2 is given in Fig. 3. A summary of the integral values for the 46 sensors is given in Table in the control chart in Fig. 4, where the Upper {Lower} Control Limits 23.77{1.78} have been chosen arbitrarily. Thirteen relevant sensors appear in this sample, namely, a_2, a_3, a_{10}, a_{20}, a_{22}, a_{30}, a_{34}, a_{35}, a_{38}, a_{40}, a_{43}, a_{45} and a_{46} (see Table 2).

Table 2 Integral Values

	Integral value		Integral value
a_1	0.	a_{12}	8.0851
a_2	16.567	a_{13}	9.9145
a_3	15.828	a_{14}	16.89
a_4	11.214	a_{15}	14.062
a_5	12.131	a_{16}	10.282
a_6	9.8963	a_{17}	10.744
a_7	13.059	a_{18}	13.974
a_8	10.211	a_{19}	13.682
a_9	13.719	a_{20}	16.745
a_{10}	16.184	a_{21}	11.983
a_{11}	8.8464	a_{22}	16.966
	Integral value		Integral value
a_{23}	12.032	a_{35}	16.093
a_{24}	13.002	a_{36}	14.87
a_{25}	12.045	a_{37}	14.689
a_{26}	13.933	a_{38}	18.06
a_{27}	0.	a_{39}	14.258
a_{28}	7.514	a_{40}	16.466
a_{29}	14.929	a_{41}	16.639
a_{30}	16.805	a_{42}	0.
a_{31}	14.729	a_{43}	16.628
a_{32}	13.323	a_{44}	8.7105
a_{33}	12.978	a_{45}	16.069
a_{34}	17.319	a_{46}	15.696

Remark. Suppose we are interested in estimating for a considered period of time if an agent was walking around region [15,25]. The meaning of this can be estimated by an expert looking at the plot in Fig. 4 relative to the requirement stipulated by $[20]_e$. We can imagine a decision table with objects represented by sensor signals (not single sample signal values but a set of sample signal values) where a decision is the expert estimation if a wandering agent "tends to favor" walking in the region [15,25]. This raises the question: *Does the integral value reflect (to a sufficient degree) the expert decision?* This problem will be discussed in our next paper. Here, in answer to this question, let us observe that the rough integral computes the aggregative effect reflected by sensor values relative to the "walking region" specified by an indiscernibility class. Notice that the lowest sample values in a signal have considerable influence in the computation, and the higher values (outside the specified region) have little or no influence on the outcome. In effect, whenever the lowest sample values in a signal are clustered in the specified walking region, the value computed by the integral will reflect this fact. In the case where a decision-maker focuses on agents near or above a walking region, the integral does approximate an expert decision. It should also be pointed out that an integral value reflects a decision about a wandering agent not in terms of a single sample sensor value, but rather a decision about the proximity of an agent to a specified region in terms of a set of sample values.

7 Conclusion

Rough set theory provides a variety of set functions that can be studied relative to various measure spaces. In particular, the rough membership function is considered. The particular rough membership function given in this paper is a non-negative set function which is additive and, hence, is an example of a rough measure. We are interested in identifying those sensors considered relevant in a problem-solving effort. The rough integral introduced in this paper serves as a means of classifying sensors. That is, depending on the proximity of the values of a sensor to an region defined by the equivalence class $[u]_e$, the rough integral computes the relevance of a sensor in a classification effort. By careful selection of an equivalence class $[u]_e$, it is possible to use the rough integral given in this paper as a form of filter inasmuch as the integral "filters" out sensor signals (sets of sample sensor values) with low representation in $[u]_e$. An application of rough integration has been given in the context of identifying one or more homogeneous sensors considered relevant in a classification effort. Two forms of sensor fusion are implicit in the application considered in this paper, namely, aggregation of sensor signal values in computing a rough integral value and culling from a set of sensors those sensors considered relevant in a problem-solving effort. It is left for future work to consider how one might gain by identifying relevant sensors.

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