European Journal of Operational Research 72 (1994) 443–459
North-Holland

Invited Review

Rough set approach to multi-attribute decision analysis *

Zdzisław Pawlak
Institute of Computer-Science, Warsaw University of Technology, 00-665 Warsaw, Poland
and
Institute of Theoretical and Applied Computer Science, Polish Academy of Sciences, 44-100 Gliwice, Poland

Roman Słowinski
Institute of Computing Science, Technical University of Poznan, 60-965 Poznan, Poland
and
Institute of Theoretical and Applied Computer Science, Polish Academy of Sciences, 44-100 Gliwice, Poland

Received August 1993

Abstract: We review the methodology of the rough set analysis of multi-attribute decision problems. Rough set theory proved to be a useful tool for analysis of a vague description of decision situations. It answers two basic questions related to multi-attribute decision problems: one about explanation of a decision situation and, another, about prescription of some decisions basing on analysis of a decision situation. We define four classes of multi-attribute decision problems, depending on the structure of their representation, its interpretation and the kind of questions related. Then, we characterize the rough set methodology for each particular class of decision problems. We use simple practical examples to illustrate this presentation. A review of related literature is made throughout the paper.

Keywords: Decision; Multiple attributes; Rough set theory; Vagueness; Sorting; Conflict analysis; Explanation; Prescription

"The central problem of our age
is how to act decisively in
the absence of certainty"
Bertrand Russel, 1940

1. Introduction

Scientific decision analysis provides various tools for modelling decision situations in view of explaining them or prescribing actions increasing the coherence between the possibilities offered by the

* This research was supported by grant IC 1010 CRIT Project V/92.
Correspondence to: Z. Pawlak, Institute of Computer Science, Warsaw University of Technology, 00-665 Warsaw, Poland.

SSDI 0377-2217(93)E0257-X

situation, and goals and value systems of the agents involved. Both modelling and explanation/prescription stages are also crucial in operations research aiming at elaboration of a systematic and rational approach to modelling and solving complex decision problems [1].

Generally speaking, a *decision problem* involves a set of *objects* (actions, states, competitors, etc.) described or evaluated by a set of *attributes* (criteria, features, issues, etc.) Independently of further interpretation, a decision situation may be represented by a *table* rows of which correspond to objects and columns to attributes; for each pair (object–attribute) there is known a value called *descriptor*. We can also say that the table represents *knowledge* about a decision situation. Typically, one or several *agents* (experts, decision makers, nature, etc.) are also involved in a decision problem. By an agent we understand a person or thing that works to produce a result (observation, decision, evaluation, etc.). The agent may be either identified with an object or an attribute, or may exist 'outside' the description of a decision situation. In the former case, a result produced by the agent is a descriptor in the table while in the latter, the whole table may be set up by the agent although he is not represented in the table.

The attributes used to describe objects are build on some elementary features of the objects. They may be *nominal* (also called *categorical* or *qualitative*, e.g. male or female) or *cardinal* (also called *non-nominal* or *quantitative*, e.g. financial ratios or temperature). Although characterization of an object on the component features may be 'distributional' (distribution in time or space, or probability distribution), it is usually translated into a unique term (qualitative or quantitative) via a 'point-reduction' technique [34]. Another possible quality of an attribute is a *preferential ordering* of its domain. Attributes with domains (finite or not) ordered according to preferences of an agent become *criteria* allowing to compare the objects from particular points of view [6].

An important issue of decision analysis is the kind of questions related to a decision problem. The most general question is probably about *explanation* of a decision situation. Explanation means discovering important facts and dependencies in the table describing a decision situation. A more specific question is about *prescription* of some decisions basing on analysis of information from the table. If this information can be interpreted as *preferential information* of an agent, its analysis tends to synthesis of a *global preference model* which represents a decision policy of the agent and can be used to support new decisions.

There are two major ways of constructing a global preference model upon preferential information obtained from an agent involved in the decision process [42]. The first one comes from mathematical decision analysis and consists in building a functional or a relational model [33]. The functional model has been extensively used within the framework of multi-attribute utility theory [10,16]. The relational model has its most widely known representation in the form of an outranking relation [36,37] or a fuzzy relation [9]. Relationships among these models have been established [48] and some criticisms have been made [5].

The second way comes from artificial intelligence and builds up the global preference model via inductive knowledge acquisition (also called rule induction, inductive learning or learning from examples). The resulting model is a set of *'if ... then ...'* rules or a decision tree [19,31]. This way is motivated by the hypothesis that the global preference model can be inferred by studying global evaluations made by the agents (decision makers, experts) when presented with a set of representative objects from the problem domain of interest (examples). The appeal of this approach is that the agents are typically more confident exercising their evaluations than explaining them. Neural networks also fall into this category, however, the global preference model inferred using this approach is encoded in the structure of the neural network and thus it is unknown explicitly [49]. It seems that such a 'black box' model is not well-suited to decision aid which seeks to give a convincing prescription.

The information about a decision situation is usually vague (inconsistent) because of uncertainty and imprecision coming from many sources (cf. [35]). Vagueness may be caused by *granularity* of representation of the information. Granularity may introduce an ambiguity to explanation or prescription based on the vague information. For example, if a global preference model is assessed in the form of production rules, the ambiguity makes that some rules are non-deterministic, i.e. they are not univocally described by means of 'granules' of the representation of preferential information.

A formal framework for dealing with granularity of information has been given by Pawlak [26,27] and called *rough set theory*. The rough set theory assumes a representation of the information in a *table* form called *information system*. Rows of this table correspond to *objects* and columns to *attributes*. As was pointed out above, the table is just an appropriate form for description of decision situations. Through the last decade, rough set theory has proved to be a useful tool for analysis of a large class of multi-attribute decision problems (cf. [41]). It answers, in an original way, both the questions of explanation and prescription related to decision situations.

In this paper, we review the methodology of the rough set analysis of multi-attribute decision problems. In the next section, we define the considered classes of decision problems. Then, we recall basic concepts of the rough set theory. Next four sections are devoted to the rough set analysis of particular classes of multi-attribute decision problems defined in Section 2. Simple examples serve as illustrations of this presentation. The final section groups some conclusions.

2. Considered classes of decision problems

Given an information system S in which a finite set U of objects is described by a finite set Q of multi-valued attributes, we can distinguish three classes of decision problems:

(i) multi-attribute sorting problems;
(ii) multi-attribute, multi-sorting problems;
(iii) multi-attribute description of objects.

The above classification of decision problems overlaps with the classification made by Roy [34] where multi-criteria sorting and description problems are also distinguished. However, while our understanding of problem (i) is the same as Roy's understanding of the multi-criteria sorting problem, we differ in the understanding of the multi-attribute description. This shows that the multi-criteria analysis and the rough set analysis represent rather different philosophies of analyzing multi-attribute decision problems.

A key feature differentiating the sorting problems (i) and (ii), and the description problem (iii) is a division of the set of attributes Q (in the former ones only) into subset C of condition attributes and subset D of decision attributes. Moreover, in the sorting problems, one or several agents are explicitly involved in the decision situation and represented by decision attributes. In multi-attribute description of objects, depending on interpretation of the information system, agents may be represented by objects or by attributes, or they may not be represented neither by objects nor by attributes, or they may not be represented neither by objects nor by attributes and exist 'outside' the description. Another difference between sorting problems (i) and (ii), and description problem (iii) is that the main question related to the former ones is prescription, while to the latter one, explanation (cf. Introduction).

Problem (i) is a classical multi-attribute sorting problem where there is only one decision attribute. It consists in assignment of each object to an appropriate pre-defined category, for instance: acceptance, rejection or request for an additional information. In this case, the rough set approach will be used to analyse a preferential information consisting of sorting examples. The sorting examples may be tutorial examples constructed by an agent or examples of real decisions, or observations made by him in the past.

Sorting problem (ii) differs from (i) by existence of multiple decision attributes. It means, for instance, that the same set of objects has been sorted by several agents.

As was mentioned above, further distinction of decision problems within the multi-attribute description (iii) is possible on the basis of interpretation of the information system. Specifically, if agents are explicitly represented either by objects or by attributes, we have to deal with:

(iii-α) Multi-attribute description of a decision situation.
In this case, the attribute-values represent opinions of agents on specific issues of a decision situation. If agents are not explicitly represented in the information system but exist 'outside' the description, we are basically interested in:

(iii-β) Discovering dependencies among attributes.
Attributes are interpreted then as consequences of decisions represented by objects.

Application of the rough set approach to the classes of decision problems distinguished above is described in Sections 4 to 7, respectively.

3. Basic concepts of the rough set theory

3.1. Introductory remarks

The observation that objects may be indiscernible in terms of descriptors is a starting point of the rough set philosophy [26]. Indiscernibility of objects by means of attributes prevents generally their precise assignment to a set. Intuitively, a rough set is a set of objects which, in general, cannot be precisely characterized in terms of the values of the set of attributes. In this case, the only sets which can be characterized precisely in these terms are lower and upper approximations of the set of objects. Using a lower and an upper approximation of a set one can define an accuracy and a quality of approximation. These are numbers from interval [0, 1] which define how exactly one can describe the examined set of objects using available information.

The most complete presentation of the rough set theory can be found in [27]. Applications of the rough set theory, some new theoretical developments and comparisons with related approaches have been recently collected by Słowinski [41]. Below, we recall some basic concepts used in the following part of the paper.

3.2. Information system

An *information system* is a finite table, the rows of which are labelled by *objects*, whereas columns are labelled by *attributes* and entries of the table are *attribute-values*. Thus, an information system can be viewed as a collection of objects described by values of attributes. The information system is also called *knowledge representation system*. A formal definition of the information system is given below.

By an information system we understand by 4-tuple $S = \langle U, Q, V, f \rangle$, where U is a finite set of objects, Q is a finite set of attributes, $V = \cup_{q \in Q} V_q$ and V_q is *a domain* of the attribute q, and $f: U \times Q \to V$ is a total function such that $f(x, q) \in V_q$ for every $q \in Q$, $x \in U$, called an *information function*.

Let $S = \langle U, Q, V, f \rangle$ be an information system and let $P \subseteq Q$ and $x, y \in U$. We say that x and y are indiscernible by the set of attributes P in S iff $f(x, q) = f(y, q)$ for every $q \in P$. Thus every $P \subseteq Q$ generates a binary relation on U which will be called an *indiscernibility relation*, denoted by $\mathrm{IND}(P)$. Obviously, $\mathrm{IND}(P)$ is an equivalence relation for any P. Equivalence classes of $\mathrm{IND}(P)$ are called P-elementary sets in S. They correspond to 'granules' of knowledge representation, mentioned in the Introduction. The family of all equivalence classes of relation $\mathrm{IND}(P)$ on U is denoted by $U|\mathrm{IND}(P)$ or, in short, $U|P$, and $[x]_P$ denotes an equivalence class of $\mathrm{IND}(P)$ determined by element $x \in U$.

$\mathrm{Des}_P(x)$ denotes a description of object $x \in U$ in terms of values of attributes from P, and is defined as

$$\mathrm{Des}_P(x) = \{(q, v): f(x, q) = v, \forall q \in P\}.$$

Since all objects being in the same equivalence class are indiscernible, they must have the same description, thus

$$\mathrm{Des}_P([x]_P) = \mathrm{Des}_P(x).$$

According to the definition, description is a set of attribute-value pairs. Sometimes it is more natural to understand description as a linguistic conjunction of attribute-value pairs.

If we distinguish *condition* and *decision attributes* in an information system, we get a *decision table*. The set of condition attributes is then denoted by C and the set of decision attributes by D.

3.3. Approximation of sets

Let $P \subseteq Q$ and $Y \subseteq U$. The P-lower approximation of Y, denoted by $\underline{P}Y$, and the P-upper approximation of Y, denoted by $\overline{P}Y$, are defined as

$$\underline{P}Y = \cup\{X \in U|P : X \subseteq Y\} \quad \text{and} \quad \overline{P}Y = \cup\{X \in U|P : X \cap Y \neq \emptyset\}.$$

The P-boundary (doubtful region) of set Y is defined as

$$\mathrm{Bn}_P(Y) = \overline{P}Y - \underline{P}Y.$$

Set $\underline{P}Y$ is the set of all objects from U which can be certainly classified as elements of Y, employing the set of attributes P. Set $\overline{P}Y$ is the set of objects from U which can be possibly classified as elements of Y, using the set of attributes P. The set $\mathrm{Bn}_P(Y)$ is the set of objects which cannot be certainly classified to Y using the set of attributes P only.

With every set $Y \subseteq U$, we can associate an *accuracy of approximation* of set Y by P in S or, in short, *accuracy* of Y, defined as:

$$\alpha_P(Y) = \mathrm{card}(\underline{P}Y)/\mathrm{card}(\overline{P}Y).$$

We will also need an approximation of a partition of U. Let S be an information system, $P \subseteq Q$, and let $\mathscr{Y} = \{Y_1, Y_2, \ldots, Y_n\}$ be a partition of U. The origin of this partition is independent on attributes from P; for example, it can follow from solving a sorting problem by an expert. Subsets Y_i, $i = 1, \ldots, n$, are *classes* (or blocks) or partition $\mathscr{Y}$. By the P-lower (P-upper) approximation of $\mathscr{Y}$ in S we mean the sets $\underline{P}\mathscr{Y} = \{\underline{P}Y_1, \underline{P}Y_2, \ldots, \underline{P}Y_n\}$ and $\overline{P}\mathscr{Y} = \{\overline{P}Y_1, \overline{P}Y_2, \ldots, \overline{P}Y_n\}$, respectively. The coefficient

$$\gamma_P(\mathscr{Y}) = \sum_{i=1}^{n} \mathrm{card}(\underline{P}Y_i)/\mathrm{card}(U)$$

is called the *quality of approximation of partition* $\mathscr{Y}$ by set of attributes P or, in short, *quality of classification* (or *sorting*). It expresses the ratio of all P-correctly classified objects to all objects in the system.

3.4. Reduction and dependency of attributes

An important issue is that of attribute reduction, in such a way that the reduced set of attributes provides the same quality of classification as the original set of attributes. The minimal subset $R \subseteq P \subseteq Q$ such that $\gamma_P(\mathscr{Y}) = \gamma_R(\mathscr{Y})$ is called $\mathscr{Y}$-reduct of P (or, simply, reduct if there is no ambiguity in the understanding of $\mathscr{Y}$) and denoted by $\mathrm{RED}_{\mathscr{Y}}(P)$. Let us notice that an information system may have more than one $\mathscr{Y}$- reduct. Intersection of all $\mathscr{Y}$-reducts is called the $\mathscr{Y}$-core of P, i.e. $\mathrm{CORE}_{\mathscr{Y}}(P) = \cap \mathrm{RED}_{\mathscr{Y}}(P)$. The core is a collection of the most significant attributes in the system.

Discovering dependencies among attributes is of primary importance in the rough set approach to knowledge analysis.

We will say that set of attributes $R \subseteq Q$ *depends* on set of attributes $P \subseteq Q$, denoted $P \to R$, if each equivalence class of the equivalence relation generated by P is included in some equivalence class generated by R, i.e.

$$P \to R \quad \text{if and only if} \quad \mathrm{IND}(P) \subseteq \mathrm{IND}(R).$$

Intuitively, R depends on P if values of attributes in R are uniquely determined by values of attributes in P, i.e. there is a functional dependency between values of R and P.

Next, we give an important property that establishes relation between reducts and dependency.

If R' is a reduct of R, then $R' \to R - R'$. This kind of relationship will be called *basic dependency* among attributes.

It is obvious that, if $P \to R$ then $P \to q$ for every $q \in R$. This kind of dependency will be called *elementary*.

Employing the above given properties one can discover all dependencies among attributes in any information system.

3.5. Decision tables

Few more notions concerning decision tables will be needed. Decision table is *deterministic* iff $C \to D$; otherwise it is *non-deterministic*. The deterministic decision table uniquely describes the decisions to be made when some conditions are satisfied. In the case of a non-deterministic table, decisions are not uniquely determined by the conditions.

Any row in the decision table is called a *decision rule*. Any decision rule, corresponding to object (row) x, can be viewed as an implication $\varphi \Rightarrow \psi$, where φ is description of x in terms of condition attributes and ψ is description of x in terms of decision attributes.

A decision rule $\varphi \Rightarrow \psi$ is *deterministic* if in the table there is no rule of the form $\varphi \Rightarrow \psi'$, $\psi \neq \psi'$ i.e. no rule with the same conditions but different decisions; otherwise, the decision rule is *non-deterministic*. It is easily seen that the decision table is deterministic if and only if all its decision rules are deterministic.

A decision rule is *reduced* if its conditions are based on the reduced set of attributes. Derivation of minimal decision rules from a decision table is one of the main tasks of the rough set philosophy. Various procedures to solve this problem were given in [4,12,13,43,50,52,53].

4. Multiple-attribute sorting problem

Examples of sorting decisions are given in the form of a decision table where objects correspond to examples. Examples are composed of condition and decision parts. The condition part describes an object in terms of condition attributes and the decision part specifies its assignment to one of categories.

One can expect the following results from the rough set analysis of the decision table:

(a) evaluation of importance of particular attributes [1];
(b) construction of minimal subsets of independent attributes ensuring the same quality of sorting as the whole set, i.e. reducts of the set of attributes;
(c) intersection of those reducts giving a core of attributes which cannot be eliminated without disturbing the ability of approximating the sorting decisions;
(d) elimination of redundant attributes from the decision table;
(e) generation of sorting rules from the reduced decision table; they involve the relevant attributes only and explain a decision policy of the agent (decision maker or expert).

The sorting rules discovered from sorting examples may be used to support new sorting decisions. Specifically, the sorting of a new object can be supported by matching its description to one of the sorting rules. The matching may lead to one of four situations (cf. [42]):

(α) the new object matches exactly one of deterministic sorting rules;
(β) the new object matches exactly one of non-deterministic sorting rules;
(γ) the new object doesn't match any of the sorting rules;
(δ) the new object matches more than one rule.

In (α), the sorting suggestion is direct. In (β), however, the suggestion is no more direct since the matched rule is ambiguous. In this case, the DM is informed about the number of sorting examples which support each possible class. The number is called a strength. If the strength of one class is greater than the strength of other classes occurring in the non-deterministic rule, one can conclude that according to this rule, the considered object most likely belongs to the strongest class.

Situation (γ) is more burdensome. In this case, one can help the DM by presenting him a set of the rules 'nearest' to the description of the new object. The notion of 'nearest' involves the use of a distance measure. Słowinski [42] has proposed a distance measure based on a *valued closeness relation* R having

[1] Attributes in points (a)–(e) mean, in fact, condition attributes.

some good properties. It involves indifference, strict difference and veto thresholds on particular attributes, used in concordance and discordance tests. Due to the definition, the measure does not allow a major difference on one attribute to be compensated by a number of minor differences on other attributes.

Situation (δ) may also be ambiguous if the matched rules (deterministic or not) lead to different classes. Then, the suggestion can be based either on the strength of possible classes, or on an analysis of the sorting examples which support each possible class. In the latter case, the suggested class is that one which is supported by a sorting example being the closest to the new object, in the sense of relation R.

The multi-attribute sorting problem represents probably the largest class of decision problems to which the rough set approach has been used. The applications concern the following domains:

- medicine [11,15,29,39,40,46],
- pharmacology [17,18],
- industry [21,22],
- engineering [2,23–25,32],
- control [20,45,51,54],
- finance [44,53]
- geology [47],
- social sciences [14],

but are not limited to the above list.

Let us stress an important feature of the rough set approach, especially for the sorting problem. The vagueness manifested in the information system is not corrected but the rules produced are categorized into deterministic and non-deterministic. In the context of sorting, the non-deterministic rules mean that, under the corresponding conditions, it is not possible to assign the objects univocally to classes unless one seeks for some additional information. For example, in the case of selection of candidates to a school on the basis of application packages [30], the two original classes correspond to admission and rejection, respectively. The non-deterministic rules create in this case a third class of candidates: those who are invited to an interview.

To illustrate the application of the rough set approach to the multi-attribute sorting problems, let us consider an example of credit card applications, taken from [7].

Eight sorting examples given by an agent (expert) create a training set presented in Table 1. The applications are described by two nominal (qualitative) condition attributes:
c_1 – whether the applicant has an account,
c_3–whether the applicant has an employment,
and by two cardinal (quantitative) condition attributes (in $)
c_2 – applicant's bank balance,
c_4 – applicant's monthly expense.

Table 1
A training set of credit card applications [7]

Applicant	Condition attributes c_1 'account'	c_2 'balance'	c_3 'employed'	c_4 'monthly expense'	Decision d
1	bank	700	yes	200	accept
2	bank	300	yes	600	reject
3	none	0	yes	400	reject
4	other inst.	1200	yes	600	accept
5	other inst.	800	yes	600	reject
6	other inst.	1600	yes	200	accept
7	bank	3000	no	300	accept
8	none	0	no	200	reject

The decision attribute d makes a dichotomic partition of the set of applicants: $d = \text{A}$ means acceptance, $d = \text{R}$ means rejection.

The cardinal attributes will be handled by the rough set analysis after translation of their values into some nominal terms, e.g. low, medium or high. This translation involves a division of the original domain into subintervals and an assignment of nominal terms to them. The boundary values of subintervals are called *norms*. They usually follow from conventions, habits or subjective assessments. The influence of norms on robustness of rough set results has been studied by several authors in [41]. Here, we are adopting the norms defined in [7]:

$$
\begin{array}{ll}
\text{balance} < \$500 & \Rightarrow c_2 = \text{low}, \\
\$500 \le \text{balance} < \$1000 & \Rightarrow c_2 = \text{medium}, \\
\text{balance} \ge \$1000 & \Rightarrow c_2 = \text{high}, \\
\text{montly exp.} < \$250 & \Rightarrow c_4 = \text{low}, \\
\$250 \le \text{montly exp.} < \$500 & \Rightarrow c_4 = \text{medium}, \\
\text{montly exp.} \ge \$500 & \Rightarrow c_4 = \text{high}.
\end{array}
$$

The accuracy of approximation of sets Y_A and Y_R corresponding to the classes of accepted and rejected applications, respectively, is equal to one, and the quality of approximation of the decision by the whole set C of attributes is also equal to one. It means that using all the condition attributes one can perfectly approximate the decision.

The next step of the rough set analysis is construction of minimal subsets of independent attributes ensuring the same quality of classification as the whole set C, i.e. the reducts of C. There are two such reducts:

$$\text{RED}^1_{\mathscr{Y}}(C) = \{c_1, c_2\} \quad \text{and} \quad \text{RED}^2_{\mathscr{Y}}(C) = \{c_2, c_4\}.$$

It can be said that the agent took his decision taking into account only two attributes: 'account' (c_1) and 'balance' (c_2), or 'balance' (c_2) and 'monthly expense' (c_4), and discarded the attribute 'employed' (c_3). So, attribute c_3 has no influence at all on the decision.

The intersection of all reducts is the core of attributes:

$$\text{CORE}_{\mathscr{Y}}(C) = \text{RED}^1_{\mathscr{Y}}(C) \cap \text{RED}^2_{\mathscr{Y}}(C) = \{c_2\}$$

The core is the most essential part of set C, i.e. attribute 'balance' (c_2) cannot be eliminated without disturbing the ability of approximating the decision. It follows, moreover, that attributes 'account' (c_1) and 'monthly expense' (c_4) are mutually exchangeable.

The accuracy and quality coefficients for single condition attributes are given in Table 2. As can be seen, attributes c_1 and c_2 have some possibility of approximating the decision but only c_2 can approximate both decision classes.

Table 2
Accuracy and quality coefficients for C reduced to a singleton

Accuracy and quality coefficients	Condition attributes			
	c_1	c_2	c_3	c_4
$\alpha_{c_i}(Y_A)$	0	0.6	0	0
$\alpha_{c_i}(Y_R)$	0.25	0.6	0	0
$\alpha_{c_i}(\mathscr{Y})$	0.143	0.6	0	0
$\gamma_{c_i}(\mathscr{Y})$	0.25	0.75	0	0

The original decision table (Table 1) can be reduced without any loss of information either to attributes represented in $\mathrm{RED}^1_{\mathscr{Y}}(C)$ or in $\mathrm{RED}^2_{\mathscr{Y}}(C)$. The sorting rules generated from the first reduced decision table have the following form:

Rule No. 1: IF 'balance' = high THEN accept,
Rule No. 2: IF 'balance' = medium AND 'account' = bank THEN accept,
Rule No. 3: IF 'balance' = low THEN reject,
Rule No. 4: IF 'balance' = medium AND 'account' = other inst. THEN reject.

The sorting rules generated from the second reduced decision table are the following:

Rule No. 1: IF 'balance' = high THEN accept,
Rule No. 2: IF 'balance' = medium AND 'monthly exp.' = low THEN accept,
Rule No. 3: IF 'balance' = low THEN reject,
Rule No. 4: IF 'balance' = medium AND 'monthly exp.' = high THEN reject.

The rules No. 1 and No. 3 are the same in both sorting algorithms and all rules are deterministic.

It is interesting to see the sorting rules based on attribute c_2 only:

Rule No. 1: IF 'balance' = high THEN accept,
Rule No. 2: IF 'balance' = low THEN reject,
Rule No. 3: IF 'balance' = medium THEN accept OR reject.

Rule No. 3 is non-deterministic; it means that if the 'balance' is medium, a univocal decision cannot be made unless an additional information about the 'account' or the 'monthly expense' is known.

Let us mention that a decision tree obtained in [7] that correctly classifies the training set has 5 leaves corresponding to decision rules with 7 conditions, while the rough-set sorting algorithm based on one of two reducts has 4 decision rules with 6 conditions only.

5. Multi-attribute, multi-sorting problem

In this case, the set of sorting examples comes from several agents. For the same values of condition attributes, the sorting decisions may be different for some agents, so the global preference models (decision policies) can be different for them.

The sorting examples are given in a decision table form where there is more than one decision attribute. Using the rough set approach to analysis of the decision table one can obtain the same results (cf. (a)–(e) in Section 4) as for problem (i) but related to particular agents (decision attributes). Besides, one can measure the degree of consistency of the agents with the description of the objects by the set of condition attributes, detect and explain discordant and concordant parts of agents' decision policies, evaluate the grade of conflict among the agents, and construct the preference models (sorting rules) expressed in common terms (condition attributes) in order to facilitate a mutual understanding of the agents.

Let us continue the example of credit card applications considered in Section 4. We will augment the original decision table (Table 1) by a column corresponding to decisions of a second agent (expert) on the same set of applications. The added column is shown in Table 3. It can be seen that d' is different from d for applicant No. 8 only. Concerning decisions of the second agent, the accuracy and quality coefficients are equal to one:

$$\alpha_C(Y_A') = \alpha_C(Y_R') = \alpha_C(\mathscr{Y}') = \gamma_C(\mathscr{Y}') = 1.$$

So, the degree of consistency of both agents with the description of the applications by the set of condition attributes C is perfect.

In this case, the reducts of C are the following:

$$\mathrm{RED}^1_{\mathscr{Y}'}(C) = \{c_1, c_2, c_3\}, \qquad \mathrm{RED}^2_{\mathscr{Y}'}(C) = \{c_2, c_4\}.$$

Table 3
Decisions of a second agent on the set of applicants described in Table 1

Applicant	Decision d'
1	accept
2	reject
3	reject
4	accept
5	reject
6	accept
7	accept
8	accept

The core of attributes is the same as for the first agent:

$$\mathrm{CORE}_{\mathscr{Y}'}(C) = \mathrm{RED}^1_{\mathscr{Y}'}(C) \cap \mathrm{RED}^2_{\mathscr{Y}'}(C) = \{c_2\}.$$

Now, attribute c_4 is exchangeable with attributes c_1 and c_3 together. So, attribute 'employed' (c_3) is not completely superfluous for the second agent. Although the core is the same for both agents, attribute 'balance' (c_2) alone approximates the new classes and the decision much worse:

$$\alpha_{c_2}(Y'_A) = 0.375, \quad \alpha_{c_2}(Y'_R) = 0, \quad \alpha_{c_2}(\mathscr{Y}') = 0.231, \quad \gamma_{c_2}(\mathscr{Y}') = 0.375.$$

The same ability of approximation shows attribute 'monthly expense' (c_4) while others are yet worse.

Looking for common characteristics of both agents, one can observe that $\mathrm{RED}^2_{\mathscr{Y}}(C) = \mathrm{RED}^2_{\mathscr{Y}'}(C) = \{c_2, c_4\}$. The sorting rules generated from the decision table reduced to c_2 and c_4 are slightly different, however:

Rule No. 1: IF 'balance' = high THEN accept,
Rule No. 2: IF 'monthly exp.' = low THEN accept,
Rule No. 3: IF 'balance' = low AND 'monthly exp.' = medium THEN reject,
Rule No. 4: IF 'balance' = low AND 'monthly exp.' = high THEN reject,
Rule No. 5: IF 'balance' = medium AND 'monthly exp.' = high THEN reject.

All sorting rules are deterministic and, moreover, rules No. 3 and No. 4 can be aggregated:

Rule No. 3 ∧ 4: IF 'balance' = low AND 'monthly exp.' ≥ medium THEN reject.

Rules No. 1 and No. 4 generated from the second reduced decision table of the first agent are the same as rules No. 1 and No. 5 for the second agent, respectively. Thus, they represent a concordant part of both decision policies. Rules No. 2 and No. 3 for the first agent and rules No. 2 and No. 3 ∧ 4 for the second agent belong to the discordant part of the decision policies. As to rules No. 2, the second agent does not take into account the value of 'balance' when accepting applicants with 'monthly expense' = low, while the first agent does. As to rules No. 3 and No. 3 ∧ 4, the first agent rejects applicants with 'balance' = low, regardless on the 'monthly expense', while the second agent rejects only those with 'balance' = low whose 'monthly expense' is greater or equal to medium.

One can conclude that the grade of conflict between agents is rather low. Their decisions can be fully explained using the same attributes, two on four sorting rules are identical and two others differ by one condition only.

6. Multi-attribute description of decision situations

The primary objective of a multi-attribute description of a decision situation can be formulated in the language of the rough set philosophy as searching for the description of objects of the information system in terms of a minimal set of attribute-values that uniquely discerns all the objects.

As in the case of the multi-attribute sorting problem (i), the rough set approach offers here several advantages, including (a), (b) and (c). Besides, it is worthwhile to mention that by employing the rough set methodology we get all possible solutions to the problem considered, i.e. all minimal descriptions, each using a different set of attributes. This suggests an optimization of the description, for if we have various possibilities of describing objects, we can ask which is the most useful one with regard to some presumed criteria.

The next important issue that can be tackled using the rough set theory is searching what happens if some attributes (or attribute-values) are not available, i.e. how the description of objects will be affected by missing data.

Last but not least, description not of single objects, but collections of objects (subsets of the universe), in terms of attribute-values, can be of interest. It turns out that exact description of a collection of objects is not always possible and approximate description is here a must. In this case, the notions of the lower and the upper approximations can be used.

The rough set approach to the description of decision situations seems to be particularly well suited, especially when minimal description in terms of attributes is of primary concern. This is where the rough set theory shows its strength, and in contrast to other theories offers a full range of techniques to investigate this kind of situations.

The example of the Middle East situation, taken with some modifications from Casti (cf. [8]), will depict some basic ideas presented above. (Note that the example discussed in what follows is a formal one and not necessarily reflects exactly the real life relationships in the Middle East).

Let us consider the information system, where the set of objects consists of the six following agents: 1 – Israel, 2 – Egypt, 3 – the Palestinians, 4 – Jordan, 5 – Syria, 6 – Saudi Arabia, and five attributes a, b, c, d and e representing the following issues:

a – autonomous Palestinian state on the West Bank and Gaza;
b – Israeli military outposts along the Jordan River;
c – Israel retains East Jerusalem;
d – Israeli military outposts on the Golan Heights;
e – Arab countries grant citizenship to Palestinians who choose to remain within their borders.

In Table 4 below, an information system representing the attitude of these six nations of the Middle East region to the above issues is given, where -1 means that the agent is against, 1, he is favorable, and 0, neutral towards the issue. For the sake of simplicity we will write here $-$ and $+$ instead of -1 and 1, respectively.

It is easy to compute that the core is the set $\{e, b\}$ and there are two reducts $\{a, b, e\}$ and $\{b, d, e\}$ of the set of attributes.

This is to mean that the attributes e and b are the most important ones to the debate, for they cannot be omitted without changing the position of the involved parties; attributes a and d can be mutually exchanged, while the attribute c is superfluous in the description of agents. Thus the information systems from Table 4 can be simplified using only reduced sets of attributes, $\{a, b, e\}$ or $\{b, d, e\}$, as for example shown in Table 5.

Table 4
The Middle East situation

U	a	b	c	d	e
1	−	+	+	+	+
2	+	0	−	−	−
3	+	−	−	−	0
4	0	−	−	0	−
5	+	−	−	−	−
6	0	+	−	0	+

Table 5
Simplified representation of the Middle East situation

U	a	b	e
1	−	+	+
2	+	0	−
3	+	−	0
4	0	−	−
5	+	−	−
6	0	+	+

We may further reduce the description of agents by removing superfluous attribute-values from the information system (cf. [27]), and consequently Table 5 can be presented as shown in Table 6 where × denotes 'don't care' values of attributes.

Table 6 contains minimal description of each agent in terms of attributes a, b and e, i.e. their approach to the debated issues. Thus, Israel is uniquely characterized by its negative approach to existence of an autonomous Palestinian state, Egypt by its neutral approach to Israeli military outposts along the Jordan River, the Palestinians by their neutral approach to citizenship issue, etc. Thus each participant of the debate can be uniquely described in terms of his view to at most the three issues a, b and e.

Another important problem that can be tackled by the rough set theory is *conflict analysis*. For example, various views on issues being debated among the nations in the Middle East region lead to conflicts among participants of the debate. In what follows we give some basic ideas concerning this problem. More about this aspect of the rough set theory approach can be found in [27,28].

We assume that in a conflict at least two participants are in dispute over some issues. The agents may be individuals, groups, states, parties, etc. The relationship of each agent to a specific issue is presented in a form of an information system, similar to that representing the Middle East situation. That is, objects are agents taking part in the debate, attributes are *issues* being discussed, and entries of the table are *values of attributes* (*opinions, beliefs, views, votes*, etc.) which are uniquely assigned to each agent and an attribute, i.e. each entry corresponding to row x and column a represents the opinion of agent x about a. As in the considered example, we assume that each attribute can take three values 1, 0, -1, meaning favorable, neutral and against, respectively, towards the issue under consideration.

Having defined this kind of information system we can now define three binary relations among agents: *conflict, neutrality* and *alliance*. Agents x and y are in alliance over issue a if both are favorable or against towards issue a; are in conflict about a if one is favorable and the other is against towards a, and are neutral if at least one agent is neutral towards issue a. The opinion of agents x and y can be extended from one issue to a set of issues, by defining an appropriate function which assigns a 'degree' of conflict between agents considered, on the basis of their views.

Table 6
Minimal representation of the Middle East situation

U	a	b	e
1	−	×	×
2	×	0	×
3	×	×	0
4	0	−	×
5	+	−	−
6	0	+	×

Table 7
Degrees of conflict in the Middle East situation

	1	2	3	4	5	6
1						
2	−0.8					
3	−0.8	0.6				
4	−0.6	0.4	0.4			
5	−1.0	0.8	0.8	0.6		
6	0.2	0.0	0.0	0.2	0.2	

Now, we express the above ideas more formally. First, let us define three basic binary relations among agents: *conflict, neutrality* and *alliance*. To this end we need the following auxiliary function:

$$\phi(x, y) = \begin{cases} 1 & \text{if } f(a, x)\cdot f(a, y) = 1 \text{ or } x = y, \\ 0 & \text{if } f(a, x)\cdot f(a, y) = 0 \text{ and } x \neq y, \\ -1 & \text{if } f(a, x)\cdot f(a, y) = -1. \end{cases}$$

This means that, if $\phi_a(x, y) = 1$, agents x and y have the same opinion about issue a (are allied on a); if $\phi_a(x, y) = 0$, at least one agent x or y has neutral approach to issue a (is neutral on a), and if $\phi_a(x, y) = -1$, both agents have different opinions about issue a (are in conflict on a). The function introduced above uniquely determines the following three binary relations:

$$R_a^+(x, y) \quad \text{iff } \phi_a(x, y) = 1,$$
$$R_a^0(x, y) \quad \text{iff } \phi_a(x, y) = 0,$$
$$R_a^-(x, y) \quad \text{iff } \phi_a(x, y) = -1,$$

called *alliance, neutrality* and *conflict relations,* respectively.

The relations $R_a^+(x, y)$, $R_a^0(x, y)$ and $R_a^-(x, y)$ can be extended to arbitrary subset $B \subseteq Q$ of attributes by defining the function

$$\rho(x, y) = \sum_{a \in P} \phi_a(x, y)/\mathrm{card}(Q). \tag{1}$$

Obviously, $-1 \leq \rho(x, y) \leq 1$. In particular, if $\rho(x, y) > 0$, we will say that x and y are in *alliance (coalition)* on P in degree $\rho(x, y)$, if $\rho(x, y) < 0$, we will say that x and y are in *conflict on P in degree $\rho(x, y)$*, and if $\rho(x, y) = 0$ we will say that x and y are *neutral on P*.

Employing formula (1) to the Middle East situation presented by Table 4, we obtain the degrees of conflict between all participants as shown in Table 7.

It is easily seen from the table that the most conflicting opinions are that of Israel and Syria, whereas Egypt and Saudi Arabia as will as the Palestinians and Saudi Arabia are not in conflict at all.

The ideas outlined above can be extended and generalized in many directions. For example, it is easy to define the most (or the least) conflicting issues and eliminate them from the debate in order to resolve the conflict. We have to bear in mind, however, that removing some issues from the debate can change the relationship between parties involved in the conflict. We can also assign to each object its 'strength', and to each attribute a 'weight' – obtaining thus more interesting and realistic model of conflict situations.

This kind of analysis can be useful in many ways, for example in negotiations and conflict resolution – and contribute essentially to decision making in the presence of conflicts. We will not enter, however, this topic in more detail in this paper.

Finally, let us point out that if attributes would correspond, for example, to referees evaluating some competitors (objects), then the question about conflicts would make no sense in relation to objects but it would be interesting in relation to referees. The above considerations should then be inversed.

7. Discovering dependencies among consequences of decisions

Problem (iii-β) is similar to problem (iii-α), since in both cases attributes are not divided into condition and decision ones – in the case (iii-β), however, we ask completely different questions as in the case (iii-α).

Many various decision situations are possible here, and in what follows we will discuss briefly two exemplary cases.

First, let us consider the case in which objects are understood as decisions and attributes as consequences (outcomes, actions, etc.) of these decisions. For example, assume that a family wants to spend vacations visiting some interesting countries. Each decision concerning visiting a specific country causes some consequences, like, for example, applying for visa, buying special equipment, insurance, booking tickets and hotels in advance, etc. In this case, countries can be interpreted as objects and consequences as attributes. Consider Table 8 – which, for the sake of simplicity, is the same as Table 4 – illustrating the above-mentioned situation. Now, objects in the table are countries numbered 1, 2, 3, 4, 5 and 6, whereas attributes a, b, c, d, and e are representing some consequences of a corresponding decision, i.e. applying for visa, buying equipment etc. Values of attributes in the table now mean: 1 – *necessary*, 0 – *does not matter*, and -1 – *not necessary*. As before, the values will be abbreviated as $+$, 0 and $-$, respectively. Of course, in general, an arbitrary number of attribute values can be assumed.

If attributes are interpreted as consequences of some decisions, we are not allowed to remove some of them, as in the problem of description considered in Section 6, since all consequences are important and cannot be eliminated from the decision situation. Instead, we might be interested in this case in searching for dependencies among consequences, i.e. try to find out how consequences of our decisions are interrelated among each other.

Because there are two reducts $\{a, b, e\}$ and $\{b, d, e\}$ of the set of attributes in the table, one can find that the following basic dependencies among attributes are valid:

$$\{a, b, e\} \to \{c, d\} \text{ and } \{b, d, e\} \to \{a, c\}.$$

Consequently, there are the following elementary dependencies in the system:

$$\{a, b, e\} \to \{c\}, \qquad \{a, b, e\} \to \{d\},$$

and

$$\{b, d, e\} \to \{a\}, \qquad \{b, d, e\} \to \{c\}.$$

Reducing conditions we get the following minimal elementary dependencies valid in Table 8:

$$\{a\} \to \{c\}, \qquad \{a\} \to \{d\}, \qquad \{d\} \to \{a\}, \qquad \{d\} \to \{c\}.$$

Table 8
Vacation planning

U	*a*	*b*	*c*	*d*	*e*
1	−	+	+	+	+
2	+	0	−	−	−
3	+	−	−	−	0
4	0	−	−	0	−
5	+	−	−	−	−
6	0	+	−	0	+

Because $\{a\} \to \{d\}$ and $\{d\} \to \{a\}$, attributes a and d are equivalent, i.e. they define the same partition of the set of objects. Consequently, we have in Table 8 the two following dependencies:

$$\{a\} \to \{c\}, \qquad \{d\} \to \{c\}.$$

Thus we have 'discovered' all minimal elementary dependencies in the system. The result means that attribute c is functionally dependent on attribute a and d, i.e. values of attribute c are uniquely determined by values of attributes a and d. In the example, this is a formal property and not necessarily means that there is a 'cause–effect' relationship between attributes c and a, and c and d, i.e., for example, that buying insurance is somehow determined by visa and hotel issues. But in many cases such relationship can be valid.

The above example may also be interpreted differently. Suppose that we are considering a finite number of decision situations (states) and in each situation (state) specific actions must be undertaken. One can imagine a machine or device which is being controlled by a human or a automatic operator on the basis of the current state. The control process can be presented in the form of a table similar to that considered before. Objects in the table are decision states (e.g. states of a machine) and attributes represent actions to be performed in each specific state. In this case, when a functional dependency between actions occurs, some actions can be performed after other actions are finished, whereas independent actions can be performed concurrently. For example, the dependencies $\{a\} \to \{c\}$ and $\{d\} \to \{c\}$ reveal that action c can be determined by action a or d, but the remaining actions can be performed independently.

It is easily seen from the above two examples that in this case interdependencies of attributes are of primary concern, in contrast to the case (iii-α), where minimal description of objects in terms of attribute-values is the main issue. What are objects and attributes is not important here and many interpretations of these results in terms of decision situations are possible.

8. Conclusions

The review shows a wide range of applications of the rough set theory to multi-attribute decision analysis under vagueness.

The two sorting problems, (i) and (ii), are mainly related with prescription of sorting decisions basing on analysis of sorting examples. We claim that the global preference model in the form of rules derived from a set of examples may have an advantage over a functional or a relational model because it explains the preferential attitude through important and easily understandable facts in terms of significant attributes only. The rules are well-founded by examples and, moreover, inconsistencies manifested in the examples are neither corrected nor aggregated by a global function or relation.

The two problems of description, (iii-α) and (iii-β), are mainly related with explanation of a decision situation. The rough set approach is particulary well suited when minimal description in terms of attribute-values is of primary concern. A minimal description enables a thorough analysis of conflicts which is an important issue of explanation. Finally, if attributes are consequences of some decisions, the rough set analysis permits discovering all minimal elementary dependencies among consequences; in some applications, the dependencies may be interpreted as 'cause–effect' relationships.

The rough set approach does not need any additional information like probability in statistics or grade of membership in fuzzy set theory. It accepts both nominal and cardinal attributes, including those whose domains are not ordered. It is also conceptually simple and needs simple algorithms.

References

[1] Ackoff, R.L. and Sasieni, M.W. *Fundamentals of Operations Research*, Wiley, New York, 1968.

[2] Arciszewski, T. and Ziarko, W., "Structural optimization: Case-based approach", *Computing in Civil Engineering, Journal of the American Society of Civil Engineers* 5/2 (1991).

[3] Bana e Costa, C.A., *Readings in Multiple-Criteria Decision Aid*, Springer-Verlag, Berlin, 1990.
[4] Boryczka, M., and Słowinski, R. "Derivation of optimal decision algorithms from decision tables using rough sets", *Bulletin of the Polish Academy of Sciences. Technical Sciences* 36 (1988) 251–260.
[5] Bouyssou, D., "Expected utility theory and decision-aid: A critical survey", in: O. Hagen and F. Wenstop (eds.), *Progress in Utility and Risk Theory,* D. Reidel, Dordrecht, 1984.
[6] Bouyssou, D., "Building criteria: A prerequisite for MCDA", in: [3], 58–80.
[7] Carter, C., and Catlett, J., "Credit card applications using machine learning", *IEEE Expert*, Fall 1987, 71–79.
[8] Casti, J.L., *Alternative Realities – Mathematical Models and Nature of Man*, Wiley, New York, 1989.
[9] Dubois, D., and Prade, H., "Criteria aggregation and ranking of alternatives in the framework of fuzzy set theory", *TIMS / Studies in the Management Sciences* 20 (1984) 209–240.
[10] Farquahr, P.H., "A survey of multiattribute utility theory and applications", *TIMS / Studies in the Management Sciences* 6 (1977) 59–89.
[11] Fibak, J., Pawlak, Z., Słowinski, K., and Słowinski, R., "Rough sets based decision algorithm for treatment of duodenal ulcer by HSV", *Bulletin of the Polish Academy of Sciences. Biological Sciences* 34, 10/12 (1986), 227–246.
[12] Grzymała-Busse, J.W., "Knowledge acquisition under uncertainty – A rough set approach", *Journal of Intelligent & Robotic Systems* 1/1 (1988) 3–16.
[13] Grzymała-Busse, J.W., "LERS" – A system for learning from examples based on rough sets", in: [41], 3–18.
[14] Hadjimichael, M., and Wasilewska, A. "Rough-sets based study of voter preference in 1988 USA presidential election", in: [41], 137–152.
[15] Kandulski, M., Marciniec, J., and Tukałło, K., "Surgical wound infection – Conductive factors and their mutual dependencies", in: [41], 95–110.
[16] Keeney, R.L., and Raiffa, H. *Decisions with Multiple Objectives – Preferences and Value Tradeoffs*, Wiley, New York, 1976.
[17] Krysinski, J., "Rough set approach to the analysis of structure – activity relationship of quaternary imidazolium compounds", *Arzneimittel Forschung / Drug Research* 40/II (1990) 795–799.
[18] Krysinski, J., "Groß Mengen Theorie in der Analyse der Struktur Wirkungs Beziehungen von quartaren Pyridiniumverbindungen", *Pharmazie* 46/12 (1991) 878–881.
[19] Michalski, R.S., "A theory and methodology of inductive learning", *Artificial Intelligence* 20 (1983) 11–116.
[20] Mrózek, A., "Information systems and control algorithms", *Bulletin of the Polish Academy of Sciences. Technical Sciences* 33 (1985) 195–212.
[21] Mrózek, A., "Rough sets and dependency analysis among attributes in computer implementations of expert's inference models", *International Journal of Man–Machine Studies* 30 (1989) 457–471.
[22] Mrózek, A., "Rough sets in computer implementation of rule-based control of industrial processes", in: [41], 19–31.
[23] Nowicki, R., Słowinski, R., and Stefanowski, J., "Rough sets analysis of diagnostic capacity of vibroacoustic symptoms", *Journal of Computers and Mathematics with Applications* 24/2 (1992) 109–123.
[24] Nowicki, R., Słowinski, R., and Stefanowski, J. "Evaluation of vibroacoustic diagnostic symptoms by means of the rough sets theory", *Journal of Computers in Industry* 20 (1992) 141–152.
[25] Nowicki, R., Słowinski, R., and Stefanowski, J., "Analysis of diagnostic symptoms in vibroacoustic diagnostics by means of the rough sets theory", in: [41], 33–48.
[26] Pawlak, Z., "Rough sets", *International Journal of Information & Computer Sciences* 11 (1982) 341–356.
[27] Pawlak, Z., *Rough Sets. Theoretical Aspects of Reasoning about Data,* Kluwer Academic Publishers, Dordrecht, 1991.
[28] Pawlak, Z., "Anatomy of conflicts", *Bulletin of the European Association for Theoretical Computer Science* 50 (1993).
[29] Pawlak, Z., Słowinski, K. and Słowinski, R. "Rough classification of patients after highly selective vagotomy for duodenal ulcer", *International Journal of Man–Machine Studies* 24 (1986) 413–433.
[30] Pawlak, Z., and Słowinski, R., "Decision analysis using rough sets", *International Transactions in Operational Research* 1 (1994), to appear.
[31] Quinlan, J.R., "Induction of decision trees", *Machine Learning* 1 (1986) 81–106.
[32] Reinhard, A., Stawski, B., Weber, T., and Wybraniec-Skardowska, U., "An application of rough set theory in the control of water conditions on a polder", in: [41], 153–164.
[33] Roubens, M., and Vincke, Ph., *Preference Modelling*, Springer-Verlag, *LNEMS Vol. 250*, Berlin, 1985.
[34] Roy, B., *Méthodologie Multicritère d'Aide à la Décision,* Economica, Paris, 1985.
[35] Roy, B., "Main sources of inaccurate determination, uncertainty and imprecision in decision models", *Mathematical and Computer Modelling* 12 (1989) 1245–1254.
[36] Roy, B., "The outranking approach and the foundations of ELECTRE methods", in: [3], 155–183.
[37] Roy, B., and Bouyssou, D., *Aide Multicritère à la Décision: Méthodes et Cas*, Economica, Paris, 1993.
[38] Słowinski, K., "Rough classification of HSV patients", in: [41], 77–93.
[39] Słowinski, K., and Słowinski, R., "Sensitivity analysis of rough classification", *International Journal of Man–Machine Studies* 32 (1990) 693–705.
[40] Słowinski, K., Słowinski, R., and Stefanowski, J., "Rough sets approach to analysis of data from peritoneal lavage in acute pancreatitis", *Medical Informatics* 13/3 (1988) 143–159.
[41] Słowinski, R., (ed.), *Intelligent Decision Support. Handbook of Applications and Advances of the Rough Sets Theory*, Kluwer Academic Publishers, Dordrecht, 1992.

[42] Słowinski, R., "Rough set learning of preferential attitude in multi-criteria decision making", in: J. Komorowski and Z.W. Raś (eds.), *Methodologies for Intelligent Systems. Lecture Notes in Artificial Intelligence Vol. 689*, Springer-Verlag, Berlin, 1993, 642–651.
[43] Słowinski, R., and Stefanowski, J., "RoughDAS' and RoughClass' software implementations of the rough sets approach", in: [41], 445–456.
[44] Słowinski, R., and Zopounidis, C., "Application of the rough set approach to evaluation of bankruptcy risk", Working Paper 93-08, Decision Support System Laboratory, Technical University of Crete, Chania, June 1993.
[45] Szladow, A.J., and Ziarko, W., "Knowledge-based process control using rough sets", in: [41], 49–60.
[46] Tanaka, H., Ishibuchi, H., and Shigenaga, T., "Fuzzy inference system based on rough sets and its application to medical diagnosis", in: [41], 111–117.
[47] Teghem, J., and Charlet, J.-M., "Use of 'rough sets' method to draw premonitory factors for earthquakes by emphasing gas geochemistry: The case of a low seismic activity context, in Belgium", in: [41], 165–180.
[48] Vincke, Ph., "Basic concepts of preference modelling", in: [3], 101–118.
[49] Wang, J., and Malakooti, B., "A feedforward neural network for multiple criteria decision making", *Computers & Operations Research* 19 (1992) 151–167.
[50] Ziarko, W., "Data analysis and case-based expert system development tool 'Rough', in: *Proc. Case-Based Reasoning Workshop*, Morgan Kaufmann, Los Altos, CA, 1989, 356–361.
[51] Ziarko, W., "Acquisition of control algorithms from operation data", in: [41], 61–75.
[52] Ziarko, W., "Analysis of uncertain information in the framework of variable precision rough sets", *Foundations of Computing and Decision Sciences* 18, 3/4 (1993) 381–396.
[53] Ziarko, W., Golan, R., and Edwards, D., "An application of DATALOGIC/R knowledge discovery tool to identify strong predictive rules in stock market data", in: *Proc. AAAI Workshop on Knowledge Discovery in Databases,* Washington, DC, 1993.
[54] Ziarko, W., and Katzberg, J., "Control algorithms acquisition, analysis and reduction: machine learning approach", in: *Knowledge-Based System Diagnosis, Supervision and Control*, Plenum Press, Oxford, 1989, 167–178.