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as an Approximation of Fuzzy Controller

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ROUGH FUZZY CONTROLLER
AS AN APPROXIMATION OF FUZZY CONTROLLER

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Abstract

This paper presents the idea of a rough fuzzy controller created by analogy with the concept of a fuzzy controller (fuzzy logic controller). The knowledge base of the rough fuzzy controller contains linguistic terms which are modelled by means of the rough sets. In comparison with fuzzy controllers, rough controllers work considerably faster, however, their performance may be cruder.

Keywords: fuzzy set, rough set, fuzzy controller, rough fuzzy controller.

1. Introduction

Numerous applications of the fuzzy controller (fuzzy logic controller) to the control of various ill-defined complex processes have been reported since Mamdani's first paper was published in 1974 (cf. [5,6]).

Fuzzy controllers, synthesized from a collection of qualitative "rules of thumb", are applicable to the control of the processes (plants) that are mathematically difficult to understand and describe [1,2,5].

The most important advantages of fuzzy controllers are: intuitive design, reflecting the behaviour of human operator, the fact that the model of the controlled process is not necessary (an important feature when ill-defined processes are to be controlled), and good control quality (not worse than that of classical controllers). The main disadvantages of fuzzy controllers are: the necessity of the acquisition and preprocessing of the human operator's knowledge about the controlled process, sequential search through rule bases, and time consuming defuzzification methods [5].

The alternative approach to manipulating incomplete or imprecise information was presented by Pawlak in 1982 as a rough set theory [7,8]. The essence of this approach relies on the approximation of incomplete or imprecise information by means of completely and precisely known pieces of information. Such pieces of information constitute equivalence classes of equivalence relation which is called an indiscernibility relation.

Several measures of approximation are connected with the concept of a rough set, e.g. accuracy measure, roughness, rough membership function (rm-function), etc. [9].

By analogy with the concept of a fuzzy controller [6] the idea of a rough fuzzy controller based on the notion of a rough set was introduced in [3]. In this paper we will recall the generic structure of the rough fuzzy controllers.

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2. Rough sets and rough-set-measures of imprecision

Below we recall the fundamental notions and notation concerning the concept of the rough set. More detailed considerations on rough sets and their applications can be found in [8].

Let U be a finite set and let $R \subseteq U \times U$ be an equivalence relation called an indiscernibility relation. We denote by U/R the family of all equivalence classes R, and $[x]_R$ denotes an equivalence class containing $x \in U$.

An ordered pair $A_R = < U,R >$ will be called an approximation space.

With every $X \subseteq U$ we associate two sets defined as follows:

$$\underline{R}X = \{x \in U : [x]_R \subseteq X\} \tag{1}$$

$$\overline{R}X = \{x \in U : [x]_R \cap X \neq \emptyset\}$$

and called the R-lower and R-upper approximations of X in A_R, respectively.

Set $Bn_R(X) = \overline{R}X \setminus \underline{R}X$ will be called the R-boundary of X in A_R.

If $\underline{R}X = \overline{R}X$, we say that $X \subseteq U$ is R-exactly approximated in A_R.

We can see that in this case we have $Bn_R(X) = 0$.

If $\underline{R}X \neq \overline{R}X$, we say that $X \subseteq U$ is R-roughly approximated in A_R.

In this case we have $Bn_R(X) \neq 0$.

In order to express numerically how a set can be approximated using all equivalence classes of R we will use the accuracy of approximation of X in A_R (accuracy measure)

$$\alpha_R(X) = \frac{card\, \underline{R}X}{card\, \overline{R}X} \tag{2}$$

where $X \neq 0$.

Below we use another measure related to $\alpha_R(X)$ defined as

$$\varrho_R(X) = 1 - \alpha_R(X) \tag{3}$$

and referred to as R-roughness of X.

Additional numerical characteristics of imprecision, e.g.

- the rough R-membership function of the set X (or rm-function, for short)[15] defined as [15]:

$$mu_X^R(x) = \frac{card\left([x]_R \cap X\right)}{card\left([x]_R\right)} \tag{4}$$

- a coefficient characterizing the uncertainty of membership of the element to the set with respect to the possessed knowledge

$$mu_X(x) = \frac{card\left([x]_R \cap X\right)}{card\,(U)} \tag{5}$$

- the quality of approximation of the family $F = \{X_1, X_2, \dots X_n\}$ by R

$$\gamma_R(F) = \frac{\sum_{i=1}^{n} card(\underline{R}X_i)}{card(U)} \tag{6}$$

and other measures are presented in [8] and [9].

The above mentioned measures may be used for modelling the values of linguistic input and output variables in the knowledge base of the rough fuzzy controller.

3. The generic structure of fuzzy and rough fuzzy controllers

In this section we will recall a rule-based approach to an approximate reasoning process based on the compositional rule of inference [10], which preserves a maximal amount of information contained in the rules and observations and forms a common basis of both fuzzy and rough fuzzy controllers. The design of the fuzzy and rough fuzzy controllers includes the specification of the collection of control rules consisting of linguistic statements that link the controller inputs with appropriate outputs, respectively. Such knowledge can be collected and delivered by a human expert (e.g. operator of an industrial complex process). This knowledge, expressed by a finite number (i=1,2,..,n) of the heuristic rules of the type MISO (two inputs single output), may be written in the form:

$$R^{i}: \quad \textbf{if } x \textbf{ is } E^{(i)} \textbf{ and } y \textbf{ is } DE^{(i)} \textbf{ then } u \textbf{ is } U^{(i)} \tag{7}$$

where $E^{(i)}$, $DE^{(i)}$ denote values of linguistic variables x,y representing error and change in error (conditions) defined in the universes of discourse X, Y, and $U^{(i)}$ stands for the value of linguistic variable of action (conclusion) in the universe of discourse U.

If we employ a knowledge base of a MISO system, the compositional rule of inference may be written symbolically as:

$$U' = (DE' \times E') \circ R \tag{8}$$

In the last formula R stands for the global relation aggregating all the rules, i.e.

$$R = \text{also}_i (R^i) \tag{9}$$

where an implicit sentence connective "also" denotes any t- or s-norms (e.g. **min**, **max** operators) or averages [3,4]. Symbol $\circ$ stands for the operators of a compositional rule of inference (e.g. **sup-min**, **sup-prod** etc.). Similar operations have to be taken for implication and explicit sentence connective "and".

An output of the fuzzy logic controller (MISO), which has a knowledge base containing a finite number of rules connected by means of the implicit rule connective "also" interpreted as a union (max operator), takes the following form:

$$U' = (DE' \times E') \circ \bigcup_i \left(E^{(i)} \times DE^{(i)} \rightarrow U^{(i)}\right) = \bigcup_i U'^{(i)} \tag{10}$$

where $\times$ stands here for the explicit sentence connective "and".
Applying **sup-min** operations to the compositional rule of inference, the membership function of the output fuzzy set may by expressed as follows:

$$U'(u) = \sup_{x,y} \min\left[\min(DE'(y), E'(x)),\ \max_i (E^{(i)} \times DE^{(i)} \rightarrow U^{(i)})(x,y,u)\right] \tag{11}$$

If we take fuzzy sets E', DE' as singletons (measurements), i.e. $E'(x) = \delta_{x,x0}$ and $DE'(y) = \delta_{y,y0}$ where

$$\delta_{z,z0} = \begin{cases} 1 & \text{for } z = z_0 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

the membership function of the output may be simplified:

$$U'(u) = \max_i \left[(E^{(i)} \times DE^{(i)} \rightarrow U^{(i)})(x_0, y_0, u)\right] \tag{13}$$

Applying the center of gravity as a defuzzification procedure, we get

$$u' = \frac{\sum_{i=1}^{n} u_i U'(u_i)}{\sum_{i=1}^{n} U'(u_i)} \tag{14}$$

The formulas written above constitute the essentials of both the conventional fuzzy controller and rough fuzzy controller [3,5]. Fig. 1 shows the block diagram representing the generic structure of the fuzzy and rough fuzzy controllers incorporated in the closed loop of the control system. The difference between the membership functions of fuzzy sets and rough membership functions of rough sets should be emphasized here. The former are usually intuitively designed whereas the latter are computable in an algorithmic way [9]. However, from the computational point of view in our case we may consider the rough membership functions of the rough sets as the step-function approximation of the membership functions of the fuzzy sets.

4. Modelling knowledge bases for fuzzy and rough fuzzy controllers.

The knowledge base for a fuzzy controller can be created using an ordinary fuzzy partition of input space. Each coordinate of the input space may be evenly divided into a number (e.g. 3, 5, 7, 9) of parts. In this way we may obtain a finite number (e.g. 9, 25, 49, 81 respectively) rule knowledge base for a fuzzy controller (Fig. 2).

The nine-rule knowledge base for a rough fuzzy controller may be created in the following way. Firstly, a decision table has to be established, where condition attributes $C = \{e, de\}$ corresponded to a decision attribute $D = \{u\}$.

For the condition attributes the following domain can be assumed: $V_e = V_{de} = \{1, 1.5, 2, 2.5, 3\}$ whereas the domain $V_d = \{1, 2, 3, 4, 5\}$ is assumed for the decision attribute. The respective nondeterministic decision table contains 49 decision rules. Division of the universum $\mathbb{U}$ with respect to the indiscernibility relation for decisions gives $D^* = \{X_1, X_2, X_3, X_4, X_5\}$.

Accuracy measure and roughness for the elements of D^* can be calculated:

$\alpha_R(X_1) = 1/9 \quad \rho_R(X_1) = 8/9$
$\alpha_R(X_2) = 1/13 \quad \rho_R(X_2) = 12/13$
$\alpha_R(X_3) = 3/35 \quad \rho_R(X_3) = 32/35$
$\alpha_R(X_4) = 1/13 \quad \rho_R(X_4) = 12/13$
$\alpha_R(X_5) = 1/9 \quad \rho_R(X_5) = 8/9$

By analogy, the accuracy measure and roughness for the respective rough sets can be applied on the basis of appropriate information systems for the classification of error and change in error:

$\alpha_R(X_1) = 1/3 \quad \rho_R(X_1) = 2/3$
$\alpha_R(X_2) = 1/5 \quad \rho_R(X_2) = 4/5$
$\alpha_R(X_3) = 1/3 \quad \rho_R(X_3) = 2/3$

Using the rough mmembership functions we obtain value 1 for certain regions and 0.5 for all uncertain regions of condition attributes (error, change in error) and a decision attribute as well.

The scheme of the final nine-rule knowledge base for a rough fuzzy controller using accuracy measure, roughness and rough membership function is presented in Fig. 3.

In the same way we can create a knowledge base where each coordinate of the input space is divided into more than three parts. For example, if the coordinates of the input space are divided into seven parts each, we obtain a forty-nine-rule knowledge base. The schemes of such knowledge bases for fuzzy and rough fuzzy controllers are shown in Fig. 4 and 5. These bases and their modifications can be used for controlling various ill-defined complex processes.

5. Concluding remarks.

In this paper the idea of a rough fuzzy controller created by analogy with the concept of a fuzzy controller (fuzzy logic controller) is presented. The knowledge base of the rough fuzzy controller encompasses linguistic terms (values of linguistic input and output variables) modelled by means of the rough membership functions.

It should be pointed out that rough fuzzy controllers may perform more crudely than fuzzy logic controllers (in view of stabilization problems). Their crude performance can be explained by the fact that they operate on a finite number of selected levels. However, they are considerably faster than fuzzy controllers on account of a simplified defuzzification procedures.

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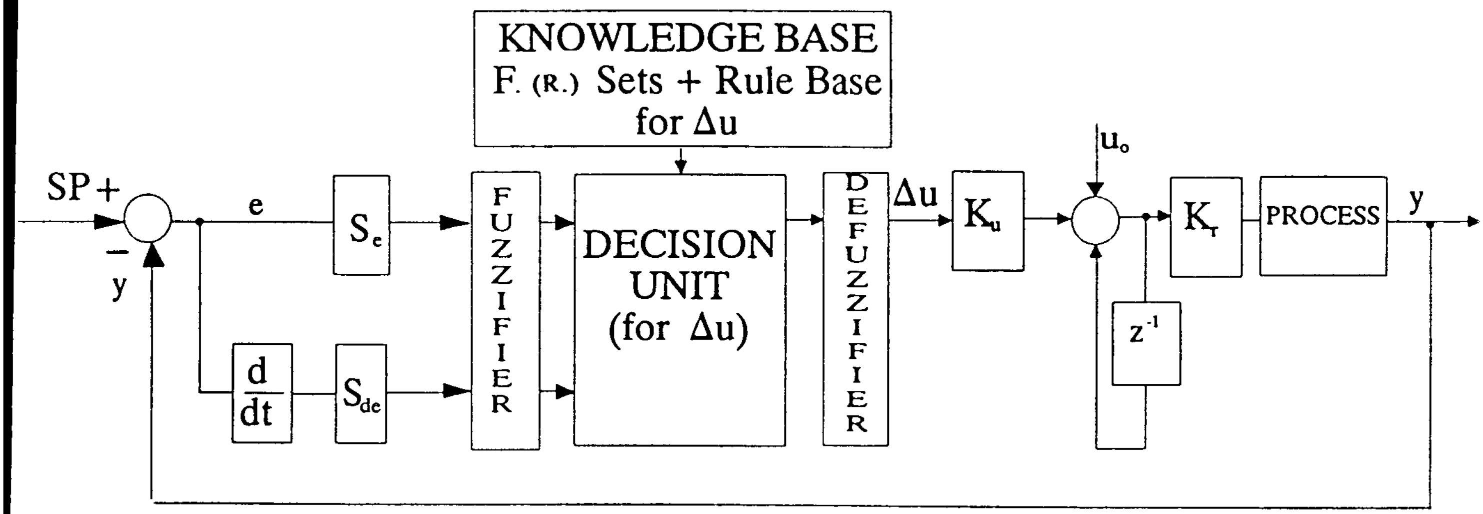

Fig. 1. The block diagram of the control system presenting the generic structure of fuzzy and rough fuzzy controllers

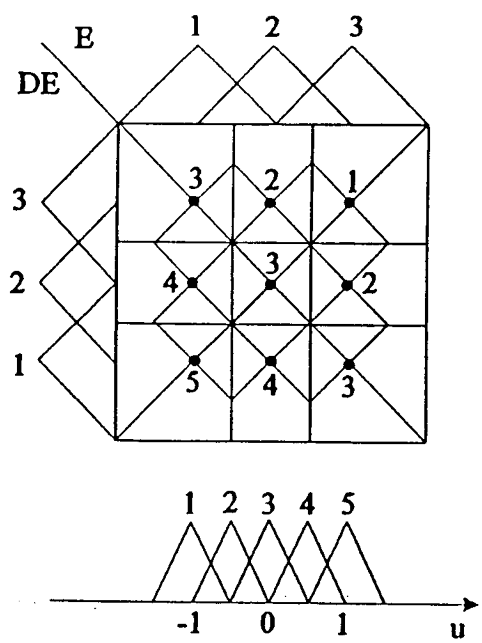

Fig. 2. A graphical representation of a nine-rule knowledge base for a fuzzy controller

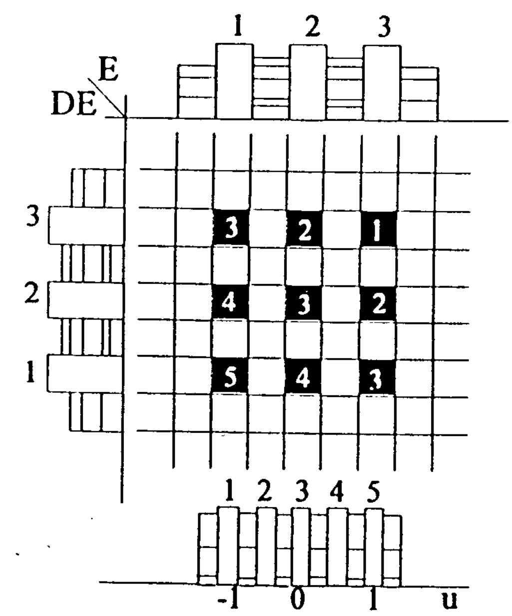

Fig. 3. A graphical representation of a nine-rule knowledge base for a rough fuzzy controller

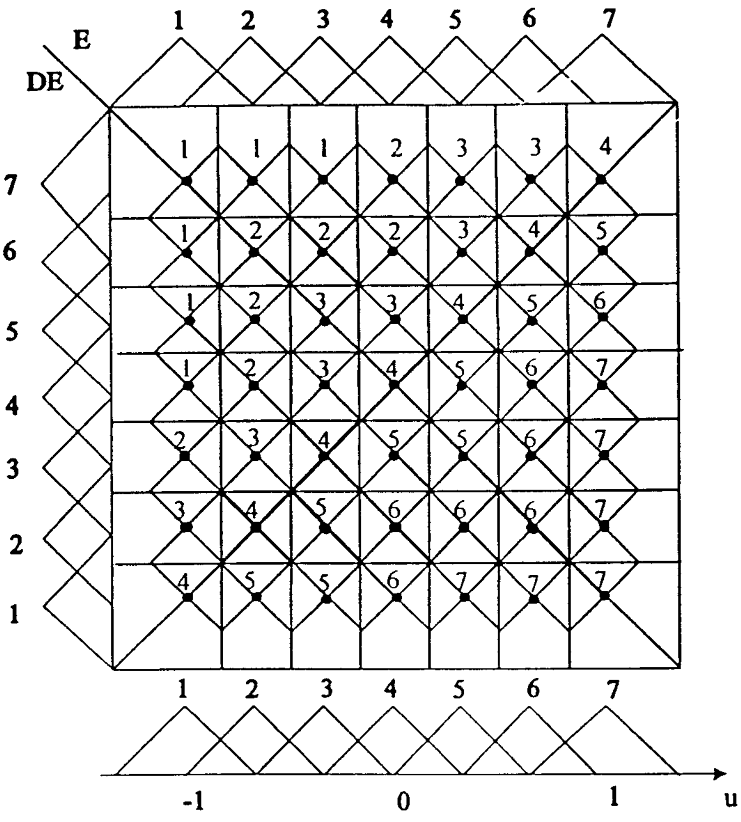

Fig. 4. A graphical representation of a forty-nine-rule knowledge base for a fuzzy controller

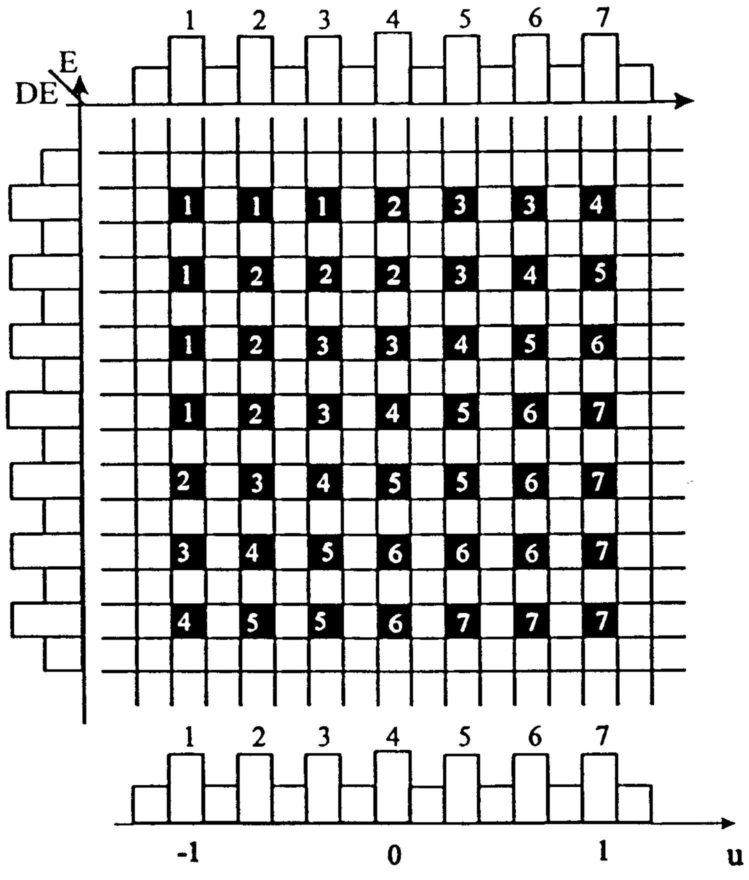

Fig. 5. A graphical representation of a forty-nine-rule knowledge base for a rough fuzzy controller

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