

**HELENA RASIOWA AND CECYLIA RAUSZER RESEARCH ON
LOGICAL FOUNDATIONS OF COMPUTER SCIENCE**

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There has been a long tradition of the interest of logicians in computer science and of computer scientists in logic. On the one hand, many problems raised and investigated by logicians before the emergence of computer science turned out to be relevant for the new science. We may quote here the notion of algorithm, model theory or relational systems. On the other hand, some problems stemming from computer science proved to be inspiring for logic, like algorithm complexity, vagueness of notions, etc.

Besides "pure" logic, Professor Helena Rasiowa was deeply interested in computer science and its applications. She held the conviction that computer science can be of great significance in the development of modern logic because of the entirely new problems it brings forth and due to inspirations leading to new research areas. She was also fully aware of the significance of logic for computer science, and particularly of the theoretical foundations of logic. This point of view was not shared by all the logicians, even if some eminent representatives of the field, like Dana Scott, were of the same opinion on the links between logic and computer science. It seems worthwhile to dwell upon this question a little more before turning to Rasiowa's papers relating logic and computer science.

For simplicity, we shall focus our attention on the work of Polish logicians which had its impact on computer science even before this latter discipline came into being. This should provide a background for a clearer understanding of the achievements of Helena Rasiowa in this respect.

The so-called parenthesis-free notation of Jan Łukasiewicz [L29], [L70] introduced in the twenties of our century was at first considered a harmless curiosity.

Fifty years later, it turned out that this "curiosity" is of great value for computer science. Its merits became evident when computers started to perform computations. The parenthesis-free notation became widely used in the organization of computational processes. To this day it is known among computer scientists as the "Polish notation".

Even before the works of Turing, Stanisław Ulam and Stanisław Mazur [U76] considered the idea of an automaton which could be seen as a forerunner of the abstract computing machine, introduced later by Turing as a formal counterpart of the notion of algorithm.

The first attempts of computer translation from English to Russian were based upon the notion of syntactic coherence of a sentence introduced by Kazimierz Ajdukiewicz [Aj35]. These ideas have been later developed by Bar-Hillel [BH50], [BH60] and used as theoretical foundation for the first translations.

Alfred Tarski's [T41], [T86] work on semantics and relation theory proved to be essential for the definition of programming language semantics as well as for the theory of relational data bases.

Category theory (Samuel Eilenberg [EM42], [EM45], [CE56], [Sc72]), is raising an ever growing interest among computer scientists, both among theorists and those interested in more practical aspects of the field.

Mereology [L16], the theory created at the beginning of our century by Stanisław Leśniewski, competitive with regard to Cantor's set theory, attracts more and more the attention of computer scientists, too. In many applications of computer science, the relation of "being a part of" which lays at the foundations of mereology seems to be more appropriate than the traditional relation of "being an element of", upon which set theory is based.

These notions and investigations, in most cases dating from the beginning of our century without any thought for their practical applications, have now become useful and attractive for computer science.

Professor Rasiowa's works in the domain of logical foundations of computer science were inspired by deep and topical problems of this discipline [AGM92-95], [GHR93-95].

All these papers have one characteristic feature: their Author considers problems belonging to computer science and in their investigation makes full use of her deep and broad knowledge of logic and the foundations of mathematics. Rasiowa's research results were pioneer work in the field and have found worldwide resonance since then.

The first group of her papers concerns automatic theorem proving.

The second group includes papers on programming theory, and in particular, on algorithmic logic [Sa70], [MS87].

Finally, the papers of the third group refer to some questions of artificial intelligence [Gi93], [Bo94]. These are papers on rapidly developing areas of research, namely approximation logics (i.e. reasoning with uncertainty e.g. [Pa91], [SP90]) and on inference methods for groups of agents e.g. [BG88], [H86], [PS95], [FHMV95].

In the lecture a brief description of the results related to the second and the third group is presented.

Helena Rasiowa is the author of more than 30 papers, two lecture notes ([HR63], [HR72]) and an unfinished monograph in which she relates algebraic methods of non-classical logics with applications in the foundations of computer science.

Her contribution to theoretical computer science stems from her conviction that there are deep relations between methods of algebra and logic on one side and essential problems of foundations of computer science on the other. Among these problems she clearly distinguished inference methods characteristic of computer science and its applications. This conviction of hers had been supported by her results on many-valued and nonclassical logics, especially on applications of various generalizations of Post algebras to logics of programs and approximation logics.

Her investigations on logic and algebraic methods in computer science can be divided into two main streams. The first includes many-valued algorithmic logics and their applications to investigation of programs ([HR42-HR44], [HR51], [HR53-HR64], [HR69-HR72]), while the second is concerned with approximation logics in their relation with generalizations of Post algebras ([HR73-HR97]).

In a series of papers on algorithmic logic Rasiowa presents the results of her research on generalizations of classical algorithmic logic introduced by Andrzej Salwicki to complex algorithms, which include programs with stacks or coroutines, programs with recursive coroutines, as well as procedures and recursive procedures (see e.g. [Ma72], [Ja77], [Sa80]). In these papers she makes intensive use of Post algebras of order ω^+ , which she introduced and studied in several publications, some of them written jointly with George Epstein. Rasiowa was particularly interested in axiomatizations of algorithmic logics corresponding to different classes of programs and this problem is represented in her papers belonging to this stream of research. The lecture notes [HR72], based on her lectures at the Istituto per le Applicazioni del Calcolo, reflect her search for a homogeneous approach to a wide class of logics of complex programs, an approach which would be general enough to yield known logics as particular cases. The first lecture notes ([HR63]), formed by lectures delivered at Simon Fraser University, correspond to the same point of view.

In 1984 Helena Rasiowa initiated intensive investigations on methods of inference under incomplete information, which she called approximate reasoning. At present approximation logics have become one of the central topics of research in artificial intelligence and among the algebraic tools used in this research an important role is assigned to those created and developed by Helena Rasiowa. They include generalizations of Post algebras, among them semi-Post algebras ([HR84]) and the so-called plain semi-Post algebras ([HR89]), which served as a basis for the construction of logics of approximate reasoning.

The papers [HR73-HR97] correspond to the period between 1984 and 1994. In [HR74] first order approximation logics are constructed. They are based on

approximation operators in the sense of Zdzisław Pawlak, which are applied to sets and relations defined by first order formulas. In [HR75] these logics have been extended to the case of a chain of equivalence relations which determine lower and upper approximations.

The paper [HR84] introduces a generalization of Post algebras, called semi-Post lattices (algebras). The primitive Post constants are elements of a poset rather than of a chain, complete lattice or connected semilattice as it had been usual in generalized Post algebras (e.g. in papers [Dw67], [T67], [Ng73], [Sp73]).

In [HR82] Rasiowa presents an algebraic approach to approximate reasoning, based on modified information systems of Dana Scott [Sc82]. Semi-Post algebras built over Scott's information systems are used as tools for an analysis of the properties of approximate reasonings.

Approximate reasonings are also treated in [HR76], [HR77], where they are based on a decreasing sequence of equivalence relations, which define a sequence of closure operators. The main result consists in a characterization of those sets X , which are intersections of the family of all closures.

In [HR83] methods of approximate reasoning related to a selection strategy are studied. Here again semi-Post algebras are used as a tool for investigations. The properties of approximate deductions are expressed in a first order logic introduced in the paper. Its semantics is defined via semi-Post algebras and a representation theorem.

The paper [HR86] deals with approximation logics of different types T , where T is a well-founded poset. Such logics have been introduced by Rasiowa in [HR83] and they stem from the idea that a set of objects to be recognized in a process of approximate reasoning is approximated by a family of covering sets and by their intersection and it relies on the notion of rough sets in the sense of Zdzisław Pawlak [Pa91]. The approach is axiomatic; a completeness theorem is proved algebraically, using plain semi-Post algebras. Such algebras have first appeared in literature in [HR89]. Their importance within the class of all semi-Post algebras is due to their simplicity and strong analogies with Post algebras, but also due to their importance in the investigation of approximation logics. The main result of the paper refers to the representability of these algebras. Every element is uniquely represented in a normal form.

In [HR88] an epistemic logic is designed, which formalizes approximating reasonings performed by groups of agents who perceive reality via perception operators, which are their individual attributes, and knowledge operators, which are attributes of subsets of agents when arriving at a consensus. It is assumed that the set of agents is a poset $\mathbf{T} = (T, \leq)$ ordered with respect to the sharpness of perception and the ability to distinguish objects. An axiomatization of the introduced logic is given and a completeness theorem is proved together with several other metalogical theorems. This logic is free of the paradoxes which appear in other epistemic logics. The research initiated in [HR88] has been continued in [HR92] and [HR93].

The problem of axiomatizing fuzzy sets is one of the most interesting and

topical problems in the theory of these sets. In [HR94] Rasiowa introduces the notion of an *LT*-fuzzy set, which is a modification of *L*-fuzzy sets in the sense of J.Goguen [Go67]. This new approach has been based on the theory of semi-Post algebras, which made possible the development of an axiomatic theory of algebras of *LT*-sets ([HR94]) and a representation theorem. *LT*-fuzzy sets are endowed with a rich structure and the classical fuzzy sets of Lofti Zadeh [Za65], [DPY93] appear here as a particular case. The results show numerous advantages of this approach; in particular, it leads to a solution of the axiomatization problem for *LT*-fuzzy sets. The papers [HR94-HR97] introduce and develop the new approach to fuzzy and rough sets, based on semi-Post algebras. *LT*-fuzzy logics formalize approximate reasonings applied to notions which are not totally determined.

In [HR90] a theory of algebras of order $\omega + \omega^*$ is developed and applied to a construction of an approximation logic.

These brief examples of Rasiowa's research results on approximation logics illustrate the variety and wealth of her investigations, on the other - they exhibit the relation between the important topics of today's computer science and the algebraic methods which she constantly developed for the sake of investigations in logic. Helena Rasiowa has greatly contributed to the development of research in Poland on application of logical methods in the foundations of computer science. In the last 20 years of her scientific activity Helena Rasiowa focused her efforts on research related to this field. She organized seminars and supervised research projects. Many of her students and collaborators who attended her lectures or seminars, those who wrote their PhD theses under her supervision, are now continuing the work she initiated. There remains no doubt today that she was right in the appreciation of the field. Among the authors of important research results on logical and algebraic methods in computer science the names of her students can be found quite often. Moreover, some of these important results have appeared in a journal which would not have existed without her dedication and to which she had been Editor-in-Chief for many years, i.e. *FUNDAMENTA INFORMATICA*.

Till the very last moments of Professor Rasiowa's life we admired her youthful enthusiasm towards research, her valuable and original results and her ability to justly appreciate the perspectives of newly arising research directions. It is not easy to accept that we shall have to proceed with our work without her.

CECYLIA RAUSZER, disciple of Helena Rasiowa

In the many years of investigations Cecylia Rauszer obtained numerous valuable results, included in almost 50 published research papers. They reflect the interest that Cecylia Rauszer took in computer science applications of logic as well as in mathematical logic itself. In particular, she obtained important results on relational databases, nonmonotonic logics and knowledge representation methods.

Cecylia Rauszer's research in the field of databases [GM78], [GM84], [M88] covered the following areas: (i) dependencies between information contained in a data base and logics which describe the dependencies [CR26]-[CR29], [CR34], [CR42], (ii) user logics for users with limited access to information on objects describable in a database [CR21], (iii) logical aspects of distributed and communicating knowledge bases [CR33], [CR44], [CR45], [CR47] and [CR48].

Research in the first area included problems such as: the construction of all the dependencies implied by a given initial set of dependencies; the construction of a minimal dependency set equivalent (as a set of theorems) to a given set of dependencies occurring in a relational scheme; the construction of logics for the description of functional and multivalued dependencies.

The second area is represented by [CR17], [CR20], [CR21], where intermediate logics are ingeniously applied to systems used to describe logics allowing for restricted access to databases.

At present a new research area is gaining growing significance in computer science applications. The main topic here is communication between network integrated databases distributed all over the world. The paper [CR33] can be classified within this area; it contains an algorithm for composing "similar" databases, i.e. databases sharing the same scheme, into one large database. The problem finds strong motivation in practical needs for representing many "local" databases by one "global" base. Queries addressed to the "global" base are being decomposed into "local" queries in such a way that all the "local" answers combine into an answer to the original question. In [CR33] Cecylia Rauszer exhibits algorithms both for composing local bases and for decomposing queries. The paper contains a correctness proof for the algorithms.

Investigations in artificial intelligence inspired research on nonmonotonic logic systems, generally called nonmonotonic logics. Many such logics are known today, in particular this concerns Raymond Reiter's default logic [Re80], John McCarthy's circumscription logic [McC80], nonmonotonic modal systems of Drew McDermott [McD] and autoepistemic logic proposed by Robert Moore [Mo74], [Mo85]. To the investigation of the last Cecylia Rauszer has made her own contribution. Autoepistemic logic has become popular among logicians as a research object due to its interesting properties. Rauszer's results are contained in [CR35]-[CR38] and [CR40]. Of particular interest in autoepistemic logic are the so-called stable formula sets which satisfy the condition $S=Cn(S,Y)$, where Cn is the classical consequence operation and Y is a set of formulas. Not all formula sets have such an extension. Thus a natural question arises as to the characterization of those sets which do have an extension of this kind. Next, if such an extension exists at all, then how many exist? Partial answers can be found in the papers cited above.

Cecylia Rauszer was also interested in the algebraic characterization of nonmonotonic logics. Algebraic models of these logics are treated in [CR37], [CR38] and [CR40].

Bertrand Russell's words:"The central problem of our age is how to act

decisively in the absence of certainty” [Ru50] seems to be a perfect motto to Rauszer’s papers [CR28], [CR29], [CR41], [CR43]-[CR48]. In these papers she investigates properties of knowledge seen as the ability to classify. She designs and investigates formal systems to represent such knowledge. The approach is based on the notion of rough set deriving from Zdzisław Pawlak (1982) [Pa91] and used as a tool for the analysis of reasoning with incomplete data. One of the most challenging problems in research on knowledge representation systems is the construction of a logic with the property that its tautologies are formulas describing properties of knowledge. This problem is tackled in [CR41], [CR44]-[CR48]. Special attention is due to one of the last papers by Rauszer [CR47], where she designed a formal system with tautologies representing properties of the knowledge (identified with object classification) of computational units in distributed systems. This logic allows the expression of relations between the knowledge of various agents or teams of agents. Two general knowledge operators are distinguished: the operator of strong common knowledge for teams of computational units and the operator of weak common knowledge of an agent team.

Cecylia Rauszer’s investigations, especially in the more recent period of her activity, were part of the dynamically developing research on mathematical logic applications. It is hard to believe that we must continue these investigations without her guidance and support.

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