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ONE-DIMENSIONAL LEARNING

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Abstract. We study a simple model of learning process over an information system.

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In various applications of computer science and in particular in artificial intelligence, learning process is studied.

Some aspects of this process can be described as follows: we try to determine if an (unknown) notion P can be described in terms of (known) notions P(1),P(2),...,P(n). If so, we try to find an (efficient) definition. Otherwise, when we find that the notion P cannot be defined in terms of P(1),...,P(n), we also get some benefit, namely we recognize the need to introduce a new notion.

A simple model of this process is presented in this paper.

First of all let us explain what the words "one-dimensional learning" mean. One-dimensional means that we deal with the elements of underlying space and not with pairs, triples etc. of such elements. By learning we mean the following process: Given n predicates, P(1),P(2),...,P(n) (of the space X of objects) and an unknown predicate P, we try to decide if the predicate P is expressible in terms of predicates P(1),...,P(n), and if so then in what form.

To this end we visualize the learning process as follows:

There are two "actors" of the learning process: The expert (E) and the student (S). We assume that the student is able to classify each object of the space X according to the predicates P(1),...,P(n), whereas the expert is able to classify the elements of X not only with repsect to P(1),...,P(n), but also with respect to the predicate P.

The learning game G(1) is performed as follows: Players S and E make consecutive moves. Player S at his move picks either an element $x \in X$ or a Boolean expression B(P(1),...,P(n)). The player E answers with a Boolean value T or F.

The answer of the player E is determined as follows: If the player S chooses an element $x \in X$, then the player E answers with the truth value of the sentence P(x). If S chooses B(P(1),....P(n)), then E answers T if P = B(P(1), ...,P(n)) otherwise he answers F. The game is finished after S makes a move of the form B(P(1),...,P(n)). If E answers T to this move, S wins. Otherwise E wins. E also wins if the game lasts interminately.

We have the following fact:

Proposition 1: The player S possesses a winning strategy in the game G(1) if and only if there exists an expression B(P(1),...,P(n)) such that P = B(P(1),...,P(n)).

Proof: 1) Assume, that S possesses a winning strategy. Then in particular S wins; Hence there is a desirable expression B(P(1),...,P(n)). 2) Let us describe a strategy for S. If this strategy fails, there is no winning strategy for S.

Let A be the partition of X into the constituents of P(1),...,P(n) i.e. the partition consisting of sets of the form $e(1)P(1) \cap \ldots \cap e(n)P(n)$ where $e(1),\ldots,e(n) \in \{0,1\}$ and OP = X - P, 1P = P.

The player S plays as follows: He picks one element x(e(1),...,e(n)) from each nonempty constituent $e(1)P(1) \cap \ldots \cap e(n)P(n)$, and plays them consecutively one after another. After he has played them all, he forms an expression of the form $B(P(1),\ldots,P(n)) = \bigvee \{e(1)P(1) \wedge \ldots \wedge e(n)P(n)$: The answer of E for x(e(1),...,e(n)) was T$\}$.

We claim that S wins in our game if he wins after playing B(P(1),...,P(n)). The proof of this claim is left to the reader.

⊠

If the player S played according to our strategy and lost, then the predicate P is not expressible in terms of P(1),...,P(n) and we have to form a new concept.

Notice that the playing of above strategy decides the game in at most exp(2,n)+1 moves.

The learning process may be described also by another learning game between the expert and a student: In this game, $G(2)$, the player S choses either a Boolean expression $B(P(1),\ldots,P(n))$ or a special symbol $*$. The player E possesses now 6 possible answers: 1, if $B(P(1),\ldots,P(n)) \subseteq P$, -1 if $B(P(1),\ldots,P(n)) \subseteq -P$, 0 if $B(P(1),\ldots,P(n)) \nsubseteq P$ and $B(P(1),\ldots,P(n)) \nsubseteq -P$, T if $B(P(1),\ldots,P(n)) = P$ and in the case when S chooses $*$, E answers T if P is not expressible in terms of $P(1),\ldots,P(n)$, F otherwise.

S wins at the move k, if E answers T to his move. S wins if he wins in some move. Otherwise E wins.

In the case of the game $G(2)$, the situation is different from that of the game $G(1)$.

<u>Proposition 2:</u> The player S possesses a winning strategy in the game $G(2)$.

Proof: Here is a winning strategy for S: He consecutive constituents. If at any time he gets the answer T, he has already won. If at some time he gets the answer 0, he plays at his next move. If he gets -1 or 1 for all his moves (at which he plays constituents), then subsequently, he forms the expression: $(*) \bigvee \{e(1)P(1) \wedge \ldots \wedge e(n)P(n)$: E has answered 1 to the move at which S had played $e(1)P(1) \wedge \ldots \wedge e(n)P(n)\}$ and plays it.

We leave it to the reader to check that this is in fact a winning strategy for S. ⊠

Let us look at an example: The student is able - say - to recognize one color (red - non red), one shape (square - non-square) and one material (wood - nonwood). He forms

Boolean expressions like: -[(R -W) (S W)], or (R -S) -W

He gets answers of the following sort:

- correct objects
- wrong objects
- some good, some wrong
- right definition.

After some trying student may convict himself that the unknown predicate is undefinable. He then plays * , getting one of the following answers:

- you are right, P is not definable
- you have guessed wrong, P is definable

In the context of the so-called rough sets (see [P] or [M-P]), the game G(.) has a quite natural interpretation:

If S plays according to the "constituent" strategy and then forms the expression (%), then he gets the associated interior (with respect to the information system associated to P(1),...,P(n)). The expression $B = \bigvee\{e(1)P(1) \wedge \ldots \wedge e(n)P(n)$: E has answered 1 or 0 to the move at which S played $e(1)P(1) \wedge \ldots \wedge e(n)P(n)\}$ corresponds to the closure of the unknown predicate P.

If the universe X changes in time, then only a few things can be said about the learning process. Here is one:

Proposition 3: Consider $\langle X,P(1),\ldots,P(n)\rangle$ and $\langle Y,Q(1),\ldots,Q(n)\rangle$, where P is a restriction of $Q \subseteq Y$ to X, i.e. $P \subseteq X$, $Q \subseteq Y$, $P = Q$ X. Then: if Q is definable in terms of Q(1),...,Q(n), then P is definable in terms of P(1),...,P(n) and the same definition serves as that of P in terms of P(1),...,P(n).

Corollary: Under the assumption of Proposition 3, if P is not definable in <X,P(1),...,P(n)> then Q is not definable in <Y,Q(1),...,Q(n)>.

Conclusions

Unfortunately the predicates P(1),...P(n) may happen to be somewhat fuzzy (not necessarily in the sense of "fuzzy" sets but, perhaps they are defined with respect to some other set of truth values). Can we then extend our learning process? The situation seems to remind one of the Lipski's scheme of incomplete information (in case when P(i)'s are only approximated).

In case of two-, and generally many-dimensional learning processes, when quantification (and, hence, full power of relational algebra is available), learning cannot be decidable.

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