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ROUGH CALCULUS

Zdzislaw Pawlak

Institute of Computer Science
Warsaw University of Technology
ul. Nowowiejska 15/19, 00665 Warsaw, Poland
e-mail : zpw@ii.pw.edu.pl

and

Institute of Theoretical and Applied Informatics
Polish Academy of Sciences
ul. Baltycka 5, 44 000 Gliwice, Poland

Abstract

Physical phenomena are usually described by differential equations. Solutions of these equations are real valued-functions, i.e. functions which are defined and valued on continuum of points. However, real valued functions cannot be either measured or computed, and can be treated with some approximation, determined by the accuracy of measurement or computation. Consequently, in practice, we deal rather with approximate rather than exact solutions, i.e. we use discrete and not continuous variables and functions.
Thus abstract mathematical models of physical systems are expressed in terms of real functions, whereas observed or computational models are described by data sets obtained as a result of measurements or computations - which use not real but rational numbers or integers. Hence an important question arises - what is the relationship between these two approaches, i.e. based on continuous or discrete mathematics philosophy.?
Many mathematical techniques, like numerical and approximation methods, have been developed to bridge the gap between abstract and computational models. In fact these methods are based also on real function theory and they are not related directly to the discrete mathematics needed in computer simulation. Independently of practical problems caused by the "continuous versus discrete" antinomy, the philosophical question of how to avoid the concept of infinity in mathematical analysis has been tackled for a long time by logicians. Non-standard analysis finistic analysis and infinitesimal analysis provide various views on these topics.

In the lecture we are going to discuses the relationship between real and discrete functions based on the rough set philosophy. In particular we define rough (discrete) lower and upper representation of real functions and define some properties of these representations, such as rough continuity, rough derivatives, rough integral and rough differential equations - which can be viewed as discrete counterparts of real functions. It turned out, that some of the introduced concepts display similar properties to those of real functions, but this is not always the case. Because the proposed approach is based on the rough set philosophy, in which the indiscernibility relation, understood as a discretization of the real line, plays a crucial role - we are interested how the discretization effects basic properties of real functions, such as continuity, differentiability, etc..
Our approach differs essentially from numerical and approximation methods, even though we use in some cases similar terminology (e.g. approximation of function by another function) - for our attempt is based on functions defined and valued in the set of integers - however it has some overlaps with non-standard, finistic and infinitesimal analysis, mentioned above. Last but not least the proposed philosophy can be seen as a generalisation of qualitative reasoning , where three-valued (+,0,-, i.e. increasing, not changing, decreasing) qualitative derivatives are replaced by more general concept of multi-valued qualitative derivatives, so that expressions such as "slowly increasing", "fast increasing", "very fast increasing" etc. can be used instead of only "increasing".
We realise that the "rough calculus' outlined here does not include many important issues and many

important problems connected with the proposed approach still remain open. We did not cover much of material needed a serious consideration in connection with the outlined philosophy. Nevertheless we hope that some fundamental notions have been clarified and sound foundations for further research and applications have been laid down.

The proposed approach is intended to be used as a theoretical basis of discrete dynamic systems. Particularly, we hope that the "rough calculus" could be used as a theoretical foundation for "rough control".

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