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Abstract

This paper presents the idea of a rough fuzzy controller with application to the stabilization of a pendulum-car system. The structure of such a controller based on the concept of a fuzzy logic controller (fuzzy controller) is suggested. The results of a simulation comparing the performance of both controllers are shown. From these results we infer that the performance of the proposed rough fuzzy controller is satisfactory.

Keywords: fuzzy set, rough set, fuzzy controller, rough fuzzy controller, inverted pendulum.

1. Introduction

Many decisions in real situations have to be made on the basis of imprecise, incomplete, uncertain and/or vague information. Fuzzy set theory introduced by Zadeh [18] in 1965 has provided a decision maker with a mathematical tool useful for modelling uncertain (imprecise) and vague data to be present in many real decision problems. Zadeh in his seminal paper [17] (published in 1973) recommended a fuzzy rule-based approach to the analysis of such complex systems and decision processes. The essential concept presented in that paper is the compositional rule of inference which preserves a maximal amount of information contained in the rules and the observations. The compositional rule of inference forms a basis of an important inference method handling uncertain (imprecise) information often called approximate reasoning. The automation of the approximate reasoning processes is unquestionable and it implies directly the necessity of software and even hardware implementation of mechanisms realizing such approximate reasoning. It leads to the construction of computer systems being able to manipulate uncertain (imprecise) information

In many real processes control relies heavily upon human experience. Skilled human operators can control such processes quite successfully without any quantitative models. The control strategy of the human operator is mainly based on linguistic qualitative knowledge concerning the behaviour of an ill-defined process. Numerous applications of the fuzzy controller (fuzzy logic controller) to the control of various ill-defined complex processes have been reported since Mamdani's first paper was published in 1974 (cf. [8,9]).

Fuzzy controllers, synthesized from a collection of qualitative "rules of thumb", are applicable to the control of the processes (plants) that are mathematically difficult to understand and describe [2,4,5].

The most important advantages of fuzzy controllers are: intuitive design, reflecting the behaviour of human operator, the fact that the model of the controlled process is not necessary (an important feature when ill-defined processes are to be controlled), and good control quality (not worse than that of classical controllers) [4].

The main disadvantages of fuzzy controllers are: the necessity of the acquisition and preprocessing of the human operator's knowledge about the controlled process, sequential search through rule bases, and time consuming defuzzification methods [5].

The alternative approach to manipulating incomplete or imprecise information was presented by Pawlak in 1982 as a rough set theory [13]

The essence of this approach relies on the approximation of incomplete or imprecise information by means of completely and precisely known pieces of information. Such pieces of information constitute equivalence classes of equivalence relation which is called an indiscernibility relation.

The theory of rough sets turned out to be applicable in many cases [14]. For example in [11,12] rough sets were applied to the generation of decision rules from the data describing decision process of a human operator.

By analogy with the concept of a fuzzy controller [8] the idea of a rough fuzzy controller based on the notion of a rough set [13,14] will be introduced below.

This paper consists of five sections: the first one presents introductory remarks, next, the idea of a rough set is presented, the third section introduces the structures of fuzzy and rough fuzzy controllers, the fourth section presents the mathematical model of the inverted pendulum-car system, the results of numerical experiments are shown in fifth section , while the last one presents the most important concluding remarks.

2. The idea of a rough set

Below we recall the fundamental notions and notation of the rough set theory. More detailed considerations on rough sets and their applications can be found in [13].

Let $\mathbb{U}$ be a finite set. The set $\mathbb{U}$ will be called a universe and elements of $\mathbb{U}$ will be referred to as states (situations or objects).

Let $R \subseteq \mathbb{U} \times \mathbb{U}$ be an equivalence relation called an indiscernibility relation. We denote by $\mathbb{U}/R$ the family of all equivalence classes R, and $[x]_R$ denotes an equivalence class containing $x \in \mathbb{U}$.

An ordered pair $A_R = < \mathbb{U},R >$ will be called an approximation space.

With every $X \subseteq \mathbb{U}$ we associate two sets defined as follows:

$$\underline{R}X = \{x \in \mathbb{U} : [x]_R \subseteq X\}$$

(1)

$$\overline{R}X = \{x \in \mathbb{U} : [x]_R \cap X \neq \emptyset\}$$

and called the R-lower and R-upper approximations of X in A_R, respectively.

A set $Bn_R(X) = \overline{R}X \setminus \underline{R}X$ will be called the R-boundary of X in A_R.

If $\underline{R}X = \overline{R}X$, we say that $X \subseteq \mathbb{U}$ is R-exactly approximated in A_R.

It is easy to see that in this case we have $Bn_R(X) = 0$.

If $\underline{R}X \neq \overline{R}X$, we say that $X \subseteq \mathbb{U}$ is R-roughly approximated in A_R.

In this case we have $Bn_R(X) \neq 0$.

In order to express numerically how a set can be approximated using all equivalence classes of R we will use the accuracy of approximation of X in A_R (accuracy measure)

$$\alpha_R(X) = \frac{card\,\underline{R}X}{card\,\overline{R}X} \tag{2}$$

where $X \neq 0$.

It is easy to see that if X is R-exactly aproximated in A_R then $\alpha_R(X) = 1$.

If X is R-roughly approximated in A_R then $0 < \alpha_R(X) < 1$.

Below we use another measure related $\alpha_R(X)$ defined as

$$\varrho_R(X) = 1 - \alpha_R(X) \tag{3}$$

and referred to as a R-roughness of X.

Roughness, as opposed to accuracy, represents the degree of inexact approximation of X in A_R. Additional numerical characteristics of imprecision, e.g.

- the rough R-membership function of the set X (or rm-function, for short)[15] defined as [15]:

$$\mu_X^R(x) = \frac{card\left([x]_R \cap X\right)}{card\left([x]_R\right)} \tag{4}$$

- coefficient characterizing the uncertainty of membership of the element to the set with respect to the possessed knowledge

$$\mu_X(x) = \frac{card\left([x]_R \cap X\right)}{card\ (U)} \tag{5}$$

- the quality of approximation of the family $F = \{X_1, X_2, \ldots X_n\}$ by R

$$\gamma_R(F) = \frac{\sum_{i=1}^{n} card\ (\underline{R}X_i)}{card\ (U)} \tag{6}$$

and others are presented in [14] and [15].

The above mentioned measures may be used in rough fuzzy controller synthesis.

In the next general structures of fuzzy and rough fuzzy controllers will be described.

3. The structures of fuzzy and rough fuzzy controllers

In this section we will recall a rule-based approach to an approximate reasoning process based on the compositional rule of inference [17], which forms a common basis of both the fuzzy and rough fuzzy controllers. The design of the fuzzy and rough fuzzy controllers includes the specification of the collection of control rules consisting of linguistic statements that link the controller inputs with appropriate outputs, respectively. Such knowledge can be collected and delivered by a human expert (e.g. operator of an industrial complex process). This knowledge, expressed by a finite number (r=1,2,..,n) of the heuristic rules of the type MISO (two input single output), may be written in the form:

$$R^{(r)}_{MISO} : \quad \textbf{if } x \textbf{ is } E_i^{(r)} \textbf{ and } y \textbf{ is } DE_j^{(r)} \textbf{ then } u \textbf{ is } U_k^{(r)} \tag{7}$$

where $E_i^{(r)}$, $DE_j^{(r)}$ denote values of linguistic variables x,y representing error and change in error (conditions) defined in the universes of discourse **X**, **Y**, and $U_k^{(r)}$ stands for the value of linguistic variable u of action (conclusion) in the universe of discourse **U**. The linguistic values $E_i^{(r)}$ and $DE_j^{(r)}$ may be represented by the respective fuzzy or rough sets (terms of linguistic variables).

If we employ a knowledge base of MISO system, the compositional rule of inference may be written symbolically as:

$$U' = (DE', E') \circ R \tag{8}$$

The global relation R now aggregating MISO system rules will be expressed as:

where an implicit sentence connective "also" denotes any *t*- or *s*-norm (e.g. **min, max**

$$R = \text{also}_r\,(R^{(r)}) \tag{9}$$

operators) or averages [6,8]. Symbol $\circ$ stands for the compositional rule of inference operators (e.g. **sup-min**, **sup-prod** etc.).

An output of the controller (MISO), which has a knowledge base containing a finite number of rules connected by means of the implicit rule connective "also" interpreted as a union (**max** operator), takes the following form:

$$U' = (DE', E') \circ \bigcup_r \left(E_i^{(r)} \times DE_j^{(r)} \rightarrow U_k^{(r)}\right) = \bigcup_r U'^{(r)} \tag{10}$$

where $\times$ stands in this case for the explicit sentence connective "and".

Applying **sup-min** operations to the compositional rule of inference, the membership function of the output set may by expressed as follows:

$$U'(u) = \sup_{x,y} \min \left[\min\left(DE'(y), E'(x)\right), \max_r \left(E_i^{(r)} \times DE_j^{(r)} \rightarrow U_k^{(r)}\right)(x,y,u) \right] \tag{11}$$

If we take fuzzy sets *E'*, *DE'* as singletons (measurements), i.e. $E'(x) = \delta_{x,x0}$ and $DE'(y) = \delta_{y,y0}$ where

$$\delta_{z,z0} = \begin{cases} 1 & \text{for } z = z_0 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

the function of the output may be simplified:

$$U'(u) = \max_r \left[\left(E_i^{(r)} \times DE_j^{(r)} \rightarrow U_k^{(r)}\right)(x_0, y_0, u) \right] \tag{13}$$

Now let us consider the rule connective "also" as an intersection. In this case the inequality mentioned above takes the following form:

$$\underline{U}'_{\cap} = (DE', E') \circ \left[\bigcap_r \left(E_i^{(r)} \times DE_j^{(r)} \rightarrow U_k^{(r)}\right)\right] \subseteq$$
$$\subseteq \bigcap_r (DE', E') \circ \left(E_i^{(r)} \times DE_j^{(r)} \rightarrow U_k^{(r)}\right) = \bigcap_r U'^{(r)} = \bar{U}'_{\cap} \tag{14}$$

By means of functions that characterize fuzzy and rough sets the inequality may be rewritten as follows:

$$\underline{U}'_{\cap}(u) = \sup_{x,y} \min\left[\min\left(DE'(y), E'(x)\right), \min_r\left(E_i^{(r)} \times DE_j^{(r)} \rightarrow U_k^{(r)}\right)(x, y, u)\right] \tag{15}$$
$$\leq \min_r \sup_{x,y} \min\left[\min\left(DE'(y), E'(x)\right), \left(E_i^{(r)} \times DE_j^{(r)} \rightarrow U_k^{(r)}\right)(x, y, u)\right] = \bar{U}'_{\cap}(u)$$

Considering E', DE' as singletons (in this case $\underline{U}'_{\cap}(u) = \bar{U}'_{\cap}(u) = U'_{\cap}(u)$) we get a simple formula:

$$U'_{\cap}(u) = \min_r\left[\left(E_i^{(r)} \times DE_j^{(r)} \rightarrow U_k^{(r)}\right)(x_0, y_0, u)\right] \tag{16}$$

Assuming the explicit sentence connective "and" as product (**prod**) and **sup-prod** for the compositional rule of inference we obtain formulas analogical to those given above.

Taking into account the fact that none of the operators **max** or **min** are sufficiently "good" as a rule connective "also", we may try to compensate one of them for another one [6], [19]. The convex linear combination of the type:

$$(1-p)(x *_t y) + p(x *_s y) \tag{17}$$

may be used ($*_t$ denotes t-norm and $*_s$ s-norm respectively).

Such a combination can be written in the form:

$$U'_C(u) = (1-p)\,U'_{*_t}(u) + pU'_{*_s}(u) \tag{18}$$

Taking intersection and union for $*_t$ and $*_s$, we obtain:

$$U'_C(u) = (1-p)\,U'_\cap(u) + pU'(u) \tag{19}$$

Let us notice that for parameter value $p=0.5$ we get an arithmetic average that is proportional to the sum (plus) interpreted as the rule connective "also". Of course, for parameter value p=0 we obtain a maximal compensation of **max** operator by **min** operator [6].

Applying the defuzzification operator denoted *DEFUZZ* to both sides of the last equality we get the following expression:

$$DEFUZZ\;[U'_C(u)] = DEFUZZ\;[(1-p)\cdot U'_\cap(u) + p\cdot U'(u)] \tag{20}$$

Choosing the defuzzification operator as a center of gravity (*COG*) we get:

$$COG\;[U'_C(u)] = COG\;[(1-p)\cdot U'_\cap(u) + p\cdot U'(u)] \tag{21}$$

or

$$u_C = (1-p)\cdot u_\cap + p\cdot u \tag{22}$$

where $u_\cap$ and u stand for centers of gravity of intersection and union respectively.

Let us notice that the *x-y* plane projection of the Cartesian product $E_i^{(r)}\times DE_j^{(r)}$ (defined in the input space) represents an element of the input image, here called an input pattern.

All input patterns projected on the *x-y* plane create an input image.

In other words, the input image is made up of overlapping input patterns obtained by means of respective rules of a knowledge base. Consequently, we can infer that the input image constitutes a basis for processing in both the fuzzy and the rough fuzzy controllers.

4.The mathematical model of the pendulum-car system.

The pendulum-car system [3], shown in Fig. 1, consists of

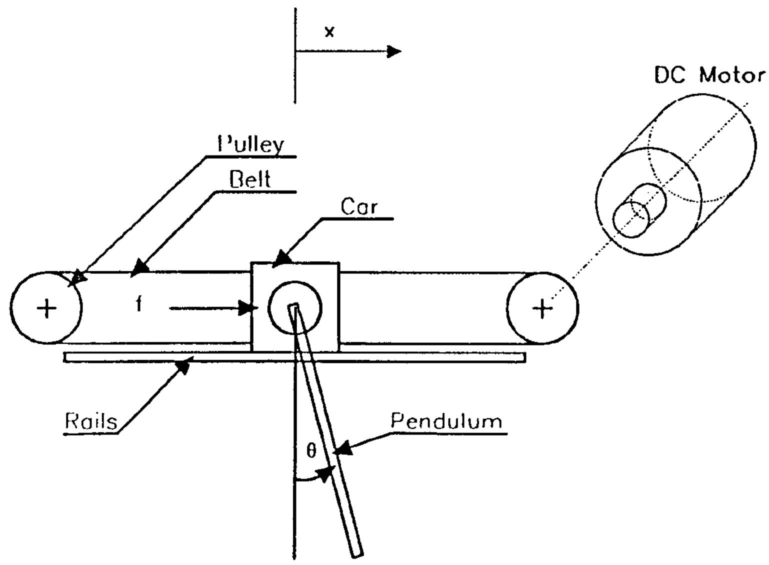

Fig. 1. The pendulum-car system

- a car moving along a line on two rails of limited length,
- a pendulum hinged in the car by means of ball bearings, rotating freely in the plane containing this line,
- a car driving device containing a dc motor, a dc amplifier, and a pulley-belt transmission system.

Such a system is characterized by an unstable equilibrium point in upright position of the pendulum, a stable equilibrium point in pendant position, as well as two uncontrollable points when the pendulum is in horizontal position.

Now let us introduce a simple mathematical description of the system. Assuming that the pendulum is a rigid body, both friction and damping forces are neglected in the system. Thus we obtain differential equations, describing the system, by projecting respective forces on to corresponding axes. Here, however, we will apply the Lagrange method (cf.[3]). Assuming that $L = E - V$, we get

$$L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}mv^2 - \left(-\frac{1}{2}mgl\cos\theta\right) \tag{23}$$

where v can be obtained from the cosine formula:

$$v^2 = \dot{x}^2 + \left(\frac{1}{2}l\dot{\theta}\right)^2 - 2\dot{x}\frac{1}{2}l\dot{\theta}\cos(180-\theta) \tag{24}$$

After elementary transformations we get

$$v^2 = \left(\frac{1}{2}l\dot{\theta}\cos\theta + \dot{x}\right)^2 + \left(\frac{1}{2}l\dot{\theta}\sin\theta\right)^2 \tag{25}$$

Applying the Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = u \tag{26}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \tag{27}$$

we obtain the following differential equations describing the dynamic behaviour of the pendulum-car system:

$$(M+m)\ddot{x} - \frac{1}{2}ml\dot{\theta}^2\sin\theta + \frac{1}{2}ml\ddot{\theta}\cos\theta = u \tag{28}$$

$$\frac{1}{2}ml\ddot{x}\cos\theta + \frac{1}{3}ml^2\ddot{\theta} + \frac{1}{2}mgl\sin\theta = 0 \tag{29}$$

Rearranging equations (28) and (29) we get:

$$\ddot{\theta} = \frac{\left[(M+m)\,g\cdot\tan\theta + \frac{1}{2}ml\sin\theta\cdot\dot{\theta}^2\right] + u}{-\frac{2}{3}(M+m)\,l\sec\theta + \frac{1}{2}ml\cos\theta} \tag{30}$$

The last equation is used for the stabilization of the system in two positions: upright and slightly deflected from vertical.

5. Simulation results.

Numerical results obtained by simulating the control of the pendulum will be presented here. Block diagram of the control system is depicted in Fig. 2.

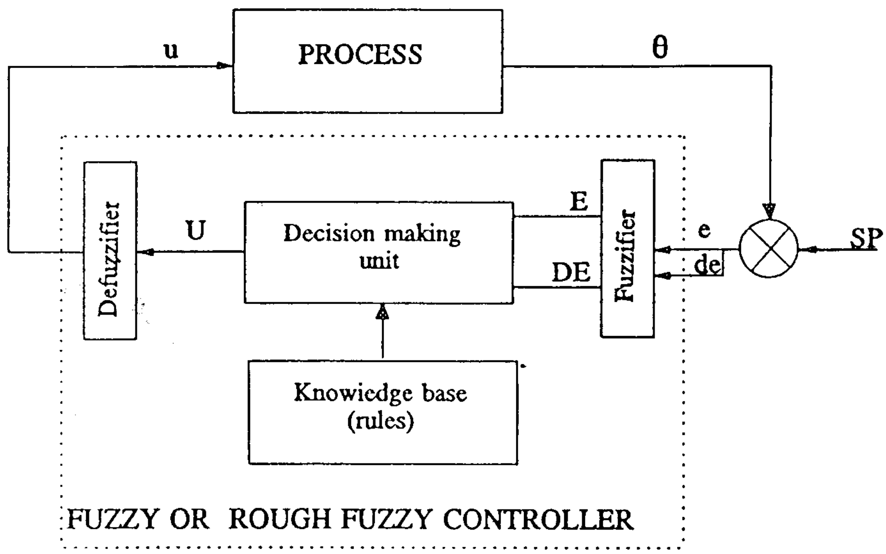

Fig. 2. Block diagram of the control system

For simplicity only nine-rule knowledge bases were used in this experiment. The knowledge base for a fuzzy controller (Fig. 3) was created using an ordinary fuzzy partition of input space. Each coordinate of the input space was evenly divided into three parts. In this way we obtained the above mentioned nine-rule knowledge base.

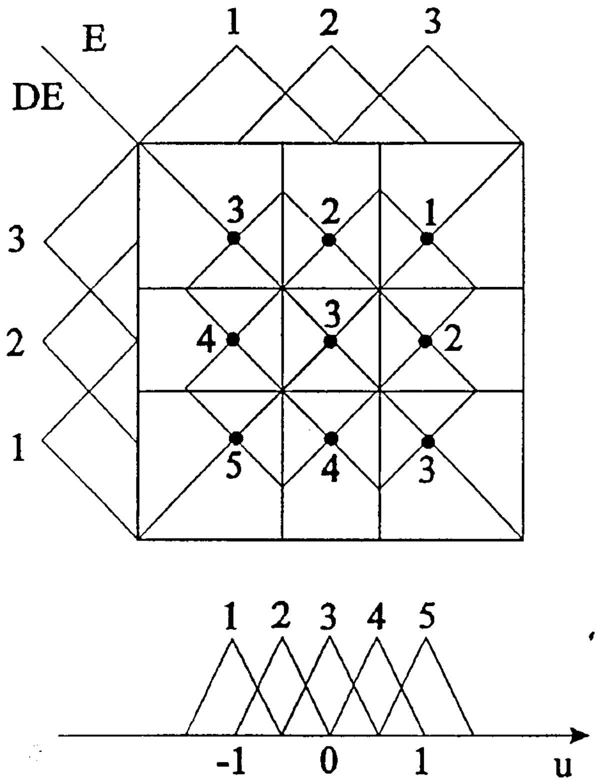

Fig. 3. **A scheme of a knowledge base for a fuzzy controller**

The knowledge base for a rough fuzzy controller was created in the following way. Firstly, a decision table was made, where condition attributes $C = \{e, de\}$ corresponded to a decision attribute $D = \{u\}$.

For the condition attributes the following domain was assumed: $V_e = V_{de} = \{1, 1.5, 2, 2.5, 3\}$ whereas the domain $V_d = \{1, 2, 3, 4, 5\}$ was assumed for the decision attribute. The respective nondeterministic decision table contained 49 decision rules. Division of the universum $\mathbb{U}$ with respect to the indiscernibility relation for decisions gives $D^* = \{X_1, X_2, X_3, X_4, X_5\}$.

Accuracy measure and roughness for the elements of D^* were calculated:

$\alpha_R(X_1) = 0.111 \ \rho_R(X_1) = 0.889$

$\alpha_R(X_2) = 0.077 \ \rho_R(X_2) = 0.923$

$\alpha_R(X_3) = 0.086 \quad \rho_R(X_3) = 0.914$

$\alpha_R(X_4) = 0.077 \quad \rho_R(X_4) = 0.923$

$\alpha_R(X_5) = 0.111 \quad \rho_R(X_5) = 0.889$

By analogy, the accuracy measure and roughness for the respective rough sets were obtained on the basis of appropriate information systems for the classification of error and change in error:

$\alpha_R(X_1) = 0.333 \qquad \rho_R(X_1) = 0.667$

$\alpha_R(X_2) = 0.2 \qquad \rho_R(X_2) = 0.8$

$\alpha_R(X_3) = 0.333 \qquad \rho_R(X_3) = 0.667$

Using the rm-functions we obtain value 1 for certain regions and 0.5 for all uncertain regions of condition attributes (error, change in error) and a decision attribute as well.

The scheme of the final knowledge base for a rough fuzzy controller using accuracy measure, roughness and rm-function is presented in Fig. 4.

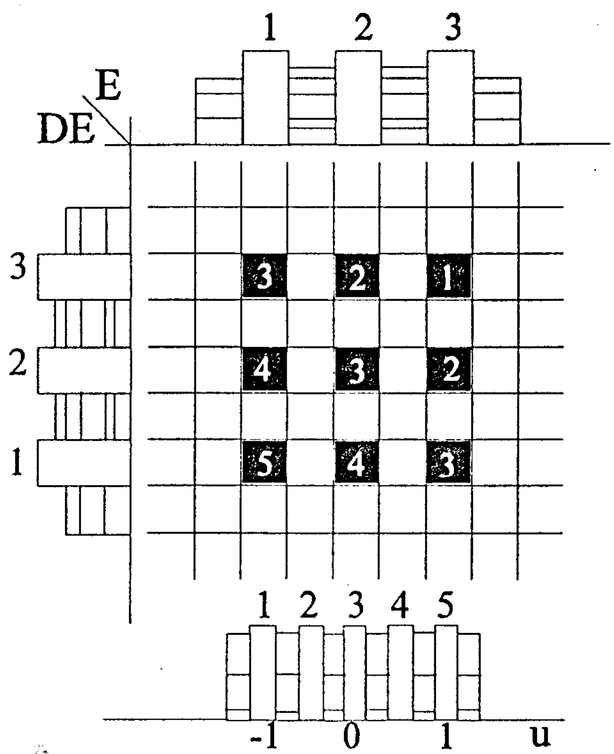

Fig. 4. **A scheme of a knowledge base for a rough fuzzy controller**

The fuzzy and rough fuzzy controllers used in our experiments employed **sup-prod** as the compositional operator, **prod** for the 'and' connective between rule premises, **sum** for the sentence connective 'also'.

The control objective was: a) to stabilize the pendulum in upright (180°) position and b) to stabilize it in a position that would be slightly deflected from vertical, i.e. 185° .

The parameters of the model were taken as follows [3]:

M = 2.8 kg, m = 0.2 kg, l = 0.75 m, g = 9.81 m/s^2

The initial position of the pendulum was 170° and the initial control value was 0.

The deflection angle θ = $X1$, its derivative $d\theta/dt$ = $X2$, control value u = Dr and control error θ_i-SP_i = Err as functions of time for both types of controllers are shown in Figs. 5, 6, 7 and 8.

For the purpose of comparative study a quality index (QI) was defined as below

$$QI = \frac{1}{N+1} \sum_{i=0}^{N} (\theta_i - SP_i)^2 \quad (31)$$

where SP_i is the set point and $N+1$ is the total number of observation points.

Comparing the controllers we notice that both the fuzzy logic controller and rough fuzzy controller behave similarly. In case of the fuzzy logic controller we obtained a slightly better quality index (582.6 versus 620.7 (using accuracy measure), 632.5 (using roughness) and 636.3 (using rm-functions) of the rough fuzzy controller. However, it should be noted that the speed of a rough fuzzy controller is much greater than that of a fuzzy logic controller. In our case the defuzzification procedure (*COG*) of the rough fuzzy controller is ca. 40 times faster than that of the fuzzy controller.

6. Concluding remarks.

The results of numerical experiments show that a rough fuzzy controller performs similarly to a conventional fuzzy logic controller in almost the same conditions. Moreover, it is much faster than the latter. The accuracy of control is satisfactory, taking into account the low number of rules (9 rules) and the low number (3 and 5) of fuzzy and rough sets in input and output spaces.

The difference between the membership functions of fuzzy sets and rm-functions of rough sets should be emphasized. The former are usually intuitively designed whereas the latter are computable in an algorithmic way [14].

While controlling the system it can be observed that using a fuzzy logic controller we get a smooth control value as a function of time; applying a rough fuzzy controller we get a sharp function of time for the control value. Nevertheless, the quality index does not differ very much for both controllers.

As an objective for further research, the reasonable partition of input and output spaces for "fuzzy" and "rough fuzzy " knowledge bases as well as a reasonable number of rules should be considered.

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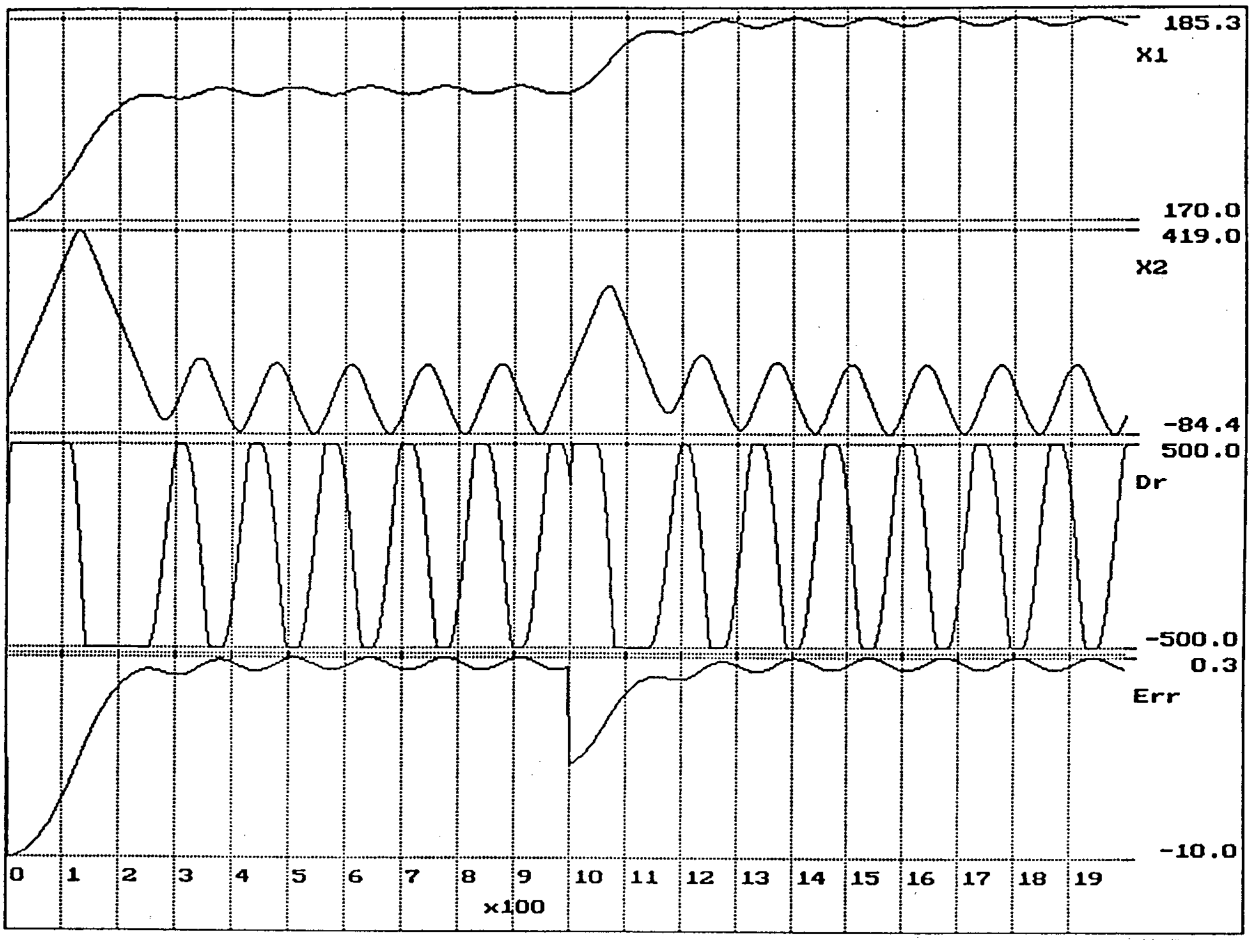
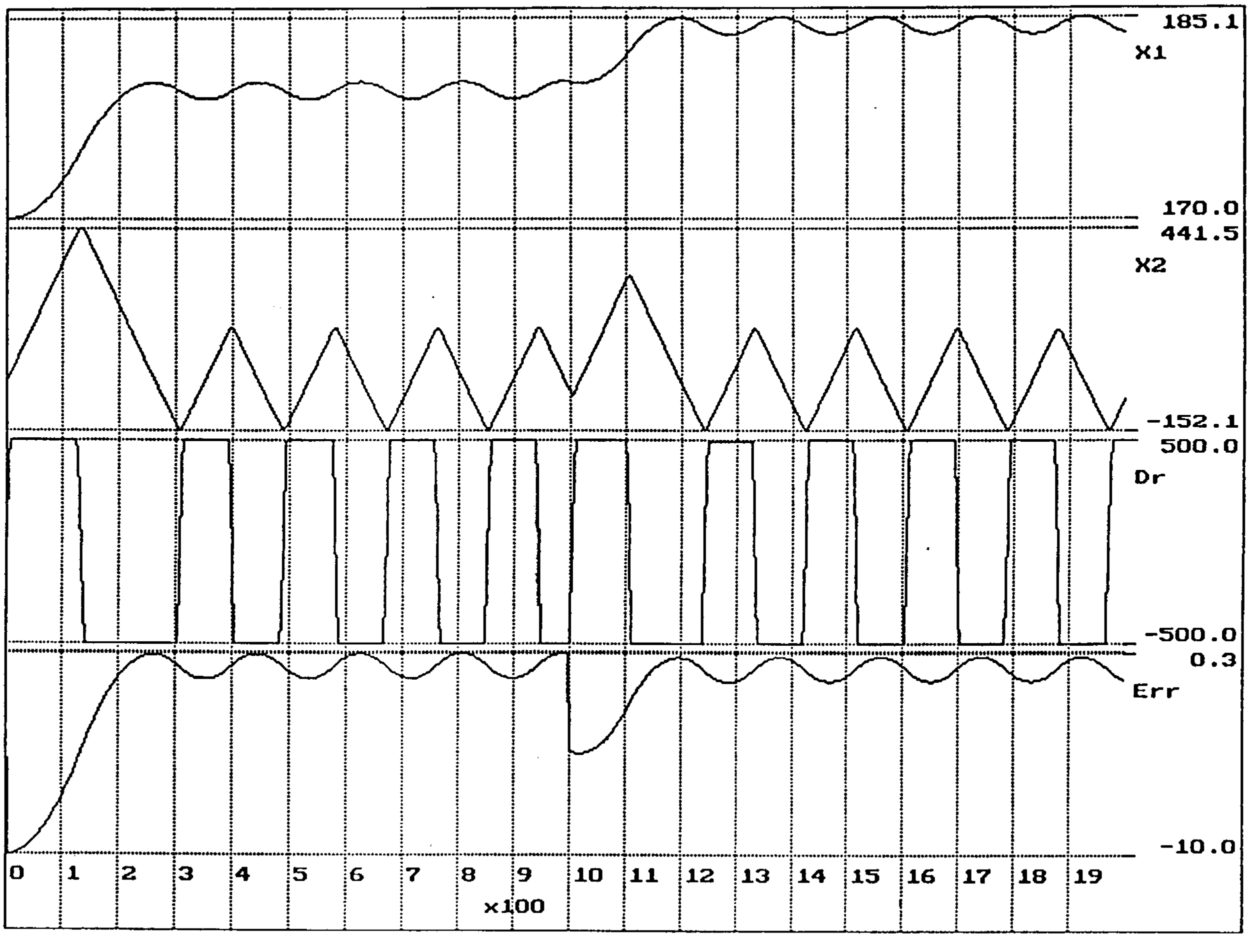


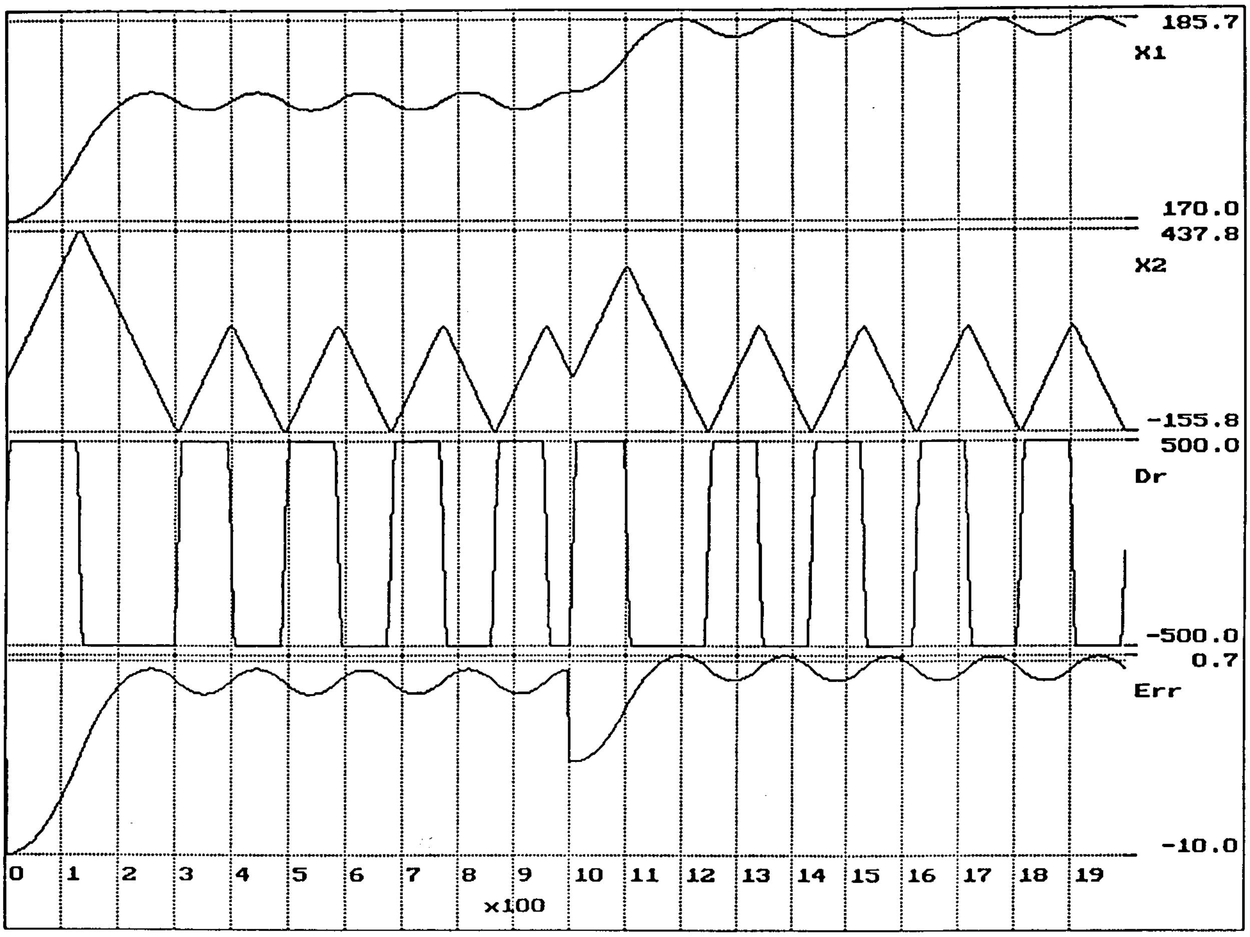
QI = 582.616107

Fig. 5. Results of control; fuzzy controller applied.
X1 = deflection angle, X2 = derivative of deflection angle, Dr = control value, Err - control error



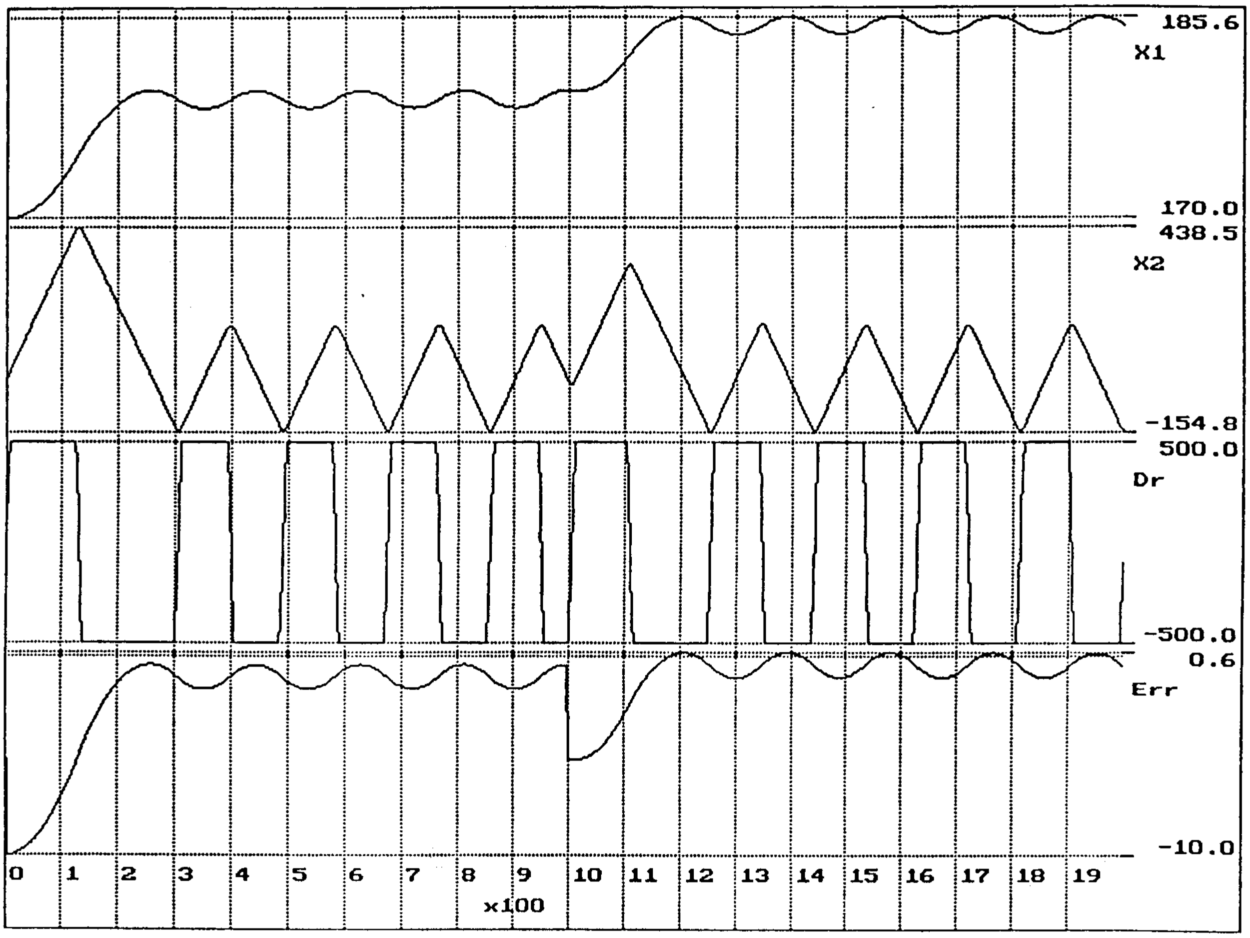
QI = 620.749039

Fig. 6. Results of control; rough fuzzy controller (using accuracy measure) applied.
X1 = deflection angle, X2 = derivative of deflection angle, Dr = control value, Err - control error



QI = 632.551801

Fig. 7. Results of control; rough fuzzy controller (using roughness) applied.
X1 = deflection angle, X2 = derivative of deflection angle, Dr = control value, Err - control error



QI = 636.343620

Fig. 8. Results of control; rough fuzzy controller (using rm-functions) applied.
X1 = deflection angle, X2 = derivative of deflection angle, Dr = control value, Err - control error

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