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Rough Sets Based Decision Algorithm for Treatment of Duodenal Ulcer by HSV

by

Jan FIBAK*, Zdzisław PAWLAK**, Krzysztof SŁOWIŃSKI*
and Roman SŁOWIŃSKI***

Summary. An information system which describes patients with duodenal ulcer treated by highly selective vagotomy (HSV) is analyzed with the concept of rough sets. The information system is a knowledge base in the form of a table composed of 122 rows corresponding to patients and 12 columns corresponding to attributes; the first 11 attributes concern anamnesis and preoperative gastric secretion examined with the histaminic test of Kay; the last attribute defines classification of patients according to long term results of operation evaluated by a surgeon in the modified Visick grading. Using manipulations based on the rough sets theory, the information system is reduced so as to get a minimum subset of attributes ensuring an acceptable quality of the classification. A "model" of patients in each class is constructed upon the analysis of values adopted by attributes from this subset. Then the reduced information system is identified with a decision table, assuming that the attributes in the minimum subset are condition attributes and that the 12-th attribute is a decision attribute. From this table, a decision algorithm is derived, composed of 39 decision rules. The algorithm and the models can be helpful in decision making concerning the treatment of duodenal ulcer by HSV.

1. Introduction. Highly selective vagotomy (HSV), also called proximal gastric vagotomy, is a method of treatment of duodenal ulcer which consists in vagal denervation of the stomach area secreting hydrochloric acid. Its theoretical basis, respecting the pathophysiology of the digestive system, was created in the 1950s [8], but first operations were performed in 1970 [2, 11]. Fifteen years of experience with highly selective vagotomy have proved it to be a safe method of treatment of duodenal ulcer which resulted in a small number of complications with very low mortality [4, 5, 9]. For this reason, highly selective vagotomy is still in the field of interest and under intensive study.

The concept of rough sets introduced by Pawlak [16] proved to be an effective tool for the analysis of information systems describing a set of objects by a set of multi-valued attributes. In particular, in the case where the set of objects is classified subject to an expert opinion, this approach enables one to deal with two basis problems of information systems — how to reduce the set of attributes to a subset ensuring an acceptable quality of the classification and how to derive decision rules from the information system.

In [20] we applied the rough set approach to a set of 77 patients with duodenal ulcer treated by highly selective vagotomy in the Department of Surgery at the F. Raszeja Memorial Hospital. Each patient has been described in terms of eleven attributes concerning anamnesis data and preoperative gastric secretion. A surgeon classified the patients from the point of view of long term results of the operation into four classes corresponding to the known Visick grading.

Using the method of rough classification, we have found the minimum subset of attributes significant for high quality classification. Basing upon analysis of the distribution of values adopted by attributes from the subset in particular classes, we have constructed "models" of patients for each class. The "models" corresponding to good results of the operation have determined indications for treatment of duodenal ulcer by HSV.

Between 1977 and 1985, out of 165 HSV patients of Surgical Department, 122 took part in the follow-up program. As the information system has grown, a natural question has arisen whether the results obtained in [20] will be consistent with results for a larger sample of patients. In [6] an answer to this question was given, showing a very high consistency of the obtained results.

In this paper, we want to make another step forward; using the rough sets approach we will derive the decision algorithm from the information system composed of the 122 HSV patients. This algorithm, together with the "models" of patients, can be helpful in decision making concerning the treatment of duodenal ulcer by HSV.

In the next section we present basic notions of the rough sets theory and in section **3** we give more information about HSV. Then, in section **4**, we describe the information system, and in section **5** we look for the minimum set of attributes ensuring an acceptable quality of classification. In section **6**, the "models" of patients in each class are defined, and in section **7**, the decision algorithm is derived from the information system. Indications for treatment by HSV, as well as a summary of results, are presented in the last two sections.

2. Basic concepts of rough sets theory. In this section we summarize the rough sets theory created by Pawlak in the 1980s (see the relevant references at the end of the paper). The summary is limited to the concepts used in the presented application. The formal presentation is preceded by an informal comment on the rough sets approach.

2.1. Introductory remarks. A rough set is a mathematical model used to deal with an approximate classification. The classification concerns a set of *objects* (patients) described by a set of *attributes*, i.e. properties of objects (anamnesis and names of medical tests), and the set of *descriptors* which is the set of values for each pair (object, attribute). Objects, attributes and descriptors are three basic components of an *information system* which can be presented as a table with rows corresponding to objects and columns corresponding to attributes. Each row of the table contains values of particular attributes and a *class number* ascribed by the expert to the corresponding object.

Given an ordinary equivalence relation, viewed as an indiscernibility relation between objects, which thus induces an approximation space made of equivalence classes, a rough set is a pair of a lower and of an upper approximation of a set in terms of these classes of indiscernible objects. In other words, a rough set is a collection of objects which cannot be precisely characterized in terms of the values of a set of attributes, while a lower and an upper approximation of the collection can be characterized in terms of these attributes.

Using the rough set approach we can deal with two major problems: (a) reduce redundant objects and attributes so as to get the minimum subset of attributes ensuring a good approximation of classes and an acceptable quality of classification, and (b) derive rules from the information systems, which relate the minimum subset of attributes (called conditions) to a particular class number (called decision).

An analysis of the significance of a subset of attributes for the quality of classification could be compared with a correlation analysis; the latter, however, examines the correlation between *only two* selected attributes, e.g. a medical test and the class numbers [22], so its results are less useful.

2.2. Information systems. By an information system we understand the 4-tuple $S=(U,Q,V,\varrho)$, where U is a finite set of *objects*, Q is a finite set of *attributes*, $V=\bigcup_{q\in Q} V_q$ and V_q is a *domain* of the attribute q, and $\varrho: U\times Q\to V$ is a total function such that $\varrho(x,q)\in V_q$ for every $q\in Q$, $x\in U$, called *information function*. Any pair (q,v), $q\in Q$, $v\in V_q$ is called *descriptor* in S.

Thus, an information system may be considered as a finite table in which columns are labeled by attributes, rows are labeled by objects, and the entry in column q and row x has the value $\varrho(x,q)$. Each row in the table represents the *information* about an object in S.

2.3. Indiscernibility relation. Let $S=(U,Q,V,\varrho)$ be an information system and let $P \subseteq Q$, $x, y \in U$. We say that x and y are *indiscernible* by set of attributes P in S (denotation $x\tilde{P}y$) iff $\varrho(x, q) = \varrho(y, q)$ for every $q \in P$. One can easily check that $\tilde{P}$ is an *equivalence* relation on U for every $P \subseteq Q$. Equivalence classes of relation $\tilde{P}$ are called *P-elementary sets* in S. Q-elementary sets are called *atoms* of S.

Any finite union of P-elementary sets will be called *P-definable* set in S. Information system S is *selective* iff all atoms in S are one-element sets, i.e. $\tilde{Q}$ is an identity relation.

2.4. Representation of an information system. A *P-representation* of S is an information system $S_P = (U/\tilde{P}, P, V_P, \varrho_P)$, where $U/\tilde{P}$ is the family of all equivalence classes of relation $\tilde{P}$, denoted by $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_k$; $V_P = \bigcup_{q \in P} V_q$; and $\varrho_P\colon U/\tilde{P} \times P \to V_P$ is the information function such that $\varrho_P(\underline{x}_j, q) = \varrho(x, q)$ for every $x \in \underline{x}_j$, $q \in P$ and $j = 1, \ldots, k$.

$\mathrm{Des}_P(\underline{x}_j)$ denotes the *description* of equivalence class (P-elementary set) $\underline{x}_j$, i.e.

$\mathrm{Des}_P(\underline{x}_j) = \{(q, v) | \varrho_P(\underline{x}_j, q) = V\}$, $j = 1, 2, \ldots, k$. Of course, any two objects $x_1, x_2 \in U$ have the same description: $\mathrm{Des}_P(x_1) = \mathrm{Des}_P(x_2) = \mathrm{Des}_P(\underline{x}_j)$, if they both belong to the same equivalence class $\underline{x}_j$.

2.5. Approximation of sets in an information system. By *P-lower* (*P-upper*) approximation of set X in S we understand the sets $\underline{P}X$ ($\bar{P}X$) defined as follows:

$$\underline{P}X = \{x \in U | x \in \underline{x}_j \text{ and } \underline{x}_j \subseteq X, \text{ for } j = 1, 2, \ldots, k\}$$
$$\bar{P}X = \{x \in U | x \in \underline{x}_j \text{ and } \underline{x}_j \cap X \neq \emptyset, \text{ for } j = 1, 2, \ldots, k\}$$

Set $Bn_P(X) = \bar{P}X - \underline{P}X$ is referred to as a *P-boundary* of X in S. It is also called the *P-doubtful region* of X, since it is not possible to determine whether an object in $Bn_P(X)$ belongs to X solely on the basis of descriptions of the P-elementary sets. Moreover, $\underline{P}X$ is called *P-positive region and* $U - \bar{P}X$, *P-negative region* of X in S.

Fig. 1 depicts the notions of lower and upper approximation of set X in a two-dimensional approximation space. The biggest rectangle represents set U partitioned into P-elementary regions corresponding to equivalence classes of $\tilde{P}$. The shaded area represents the doubtful region of X.

2.6. Accuracy of approximation. With every subset $X \subseteq U$ we associate a number $\mu_P(X)$ called an *accuracy of approximation* of set X by P in S, or, in short, accuracy od X, defined as

$$\mu_P(X) = \frac{\mathrm{card}(\underline{P}X)}{\mathrm{card}(\bar{P}X)}$$

Let us notice that $0 \leqslant \mu_P(X) \leqslant 1$, and $\mu_P(X) = 1$ if set X is *P-definable* in S.

$\alpha_P(X)$ is another useful index, called a *discriminant index* of $\tilde{P}$ with respect to $X \subseteq U$ [26], defined as

$$\alpha_P(X) = \frac{\text{card}(U) - \text{card}(Bn_P(X))}{\text{card}(U)}$$

$\alpha_P(X)$ provides a measure of the degree of certainty in determining whether objects in U are members of the subset X. Clearly, $\mu_P(X) = 1$ iff $\alpha_P(X) = 1$, but if $\mu_P(X) = 0$, then not necessarily $\alpha_P(X) = 0$.

2.7. Non-definable sets. Let us notice that set X is P-definable in S iff $\underline{P}X = \bar{P}X$; otherwise set X is non-definable in S and belongs to one of the following classes:

a) Set X is *roughly* P-definable in S iff $\underline{P}X \neq 0$ and $\bar{P}X \neq U$;
b) Set X is *internally* P-non-definable in S if $\underline{P}X \neq \emptyset$;
c) Set X is *externally* P-non-definable in S iff $\bar{P}X = U$;
d) Set X is *totally* P-non-definable in S iff $\underline{P}X = \emptyset$ and $\bar{P}X = U$.

An example of a roughly P-definable set X is presented in Fig. 1. Fig. 2 shows examples of non-definable sets in the same convention.

2.8. Approximation of families of sets. Let S be an information system, $P \subseteq Q$, and let $\mathfrak{X} = \{X_1, X_2, \ldots, X_n\}$, $X_i \subseteq U$, be a family of subsets of U.

By P-lower (P-upper) approximation of $\mathfrak{X}$ in S denoted $\underline{P}\mathfrak{X}$ ($\bar{P}\mathfrak{X}$), we mean sets $\underline{P}\mathfrak{X} = \{\underline{P}X_1, \underline{P}X_2, \ldots, \underline{P}X_n\}$ and $\bar{P}\mathfrak{X} = \{\bar{P}X_1, \bar{P}X_2, \ldots, \bar{P}X_n\}$, respectively. If $\mathfrak{X}$ is a *classification* of U, i.e. $X_i \cap X_j = \emptyset$ for every $i, j \leqslant n$, $i \neq j$ and $\bigcup_{i=1}^{n} X_i = U$, X_i are called *classes* of $\mathfrak{X}$. If every class of $\mathfrak{X}$ is P-definable then classification $\mathfrak{X}$ will be called *P-definable*.

$\text{Pos}_P(\mathfrak{X}) = \bigcup_{i=1}^{n} \underline{P}X_i$ will be called the *P-positive* region of classification $\mathfrak{X}$ in S. $Bn_P(\mathfrak{X}) = \bigcup_{i=1}^{n} Bn_P(X_i)$ will be called the P-doubtful region of classification $\mathfrak{X}$ in S. Since $U = \bigcup_{i=1}^{n} \bar{P}X_i$, there is no P-negative region for any P of classification $\mathfrak{X}$ in S.

If $\mathfrak{X}$ is a classification of U then

$$\beta_P(\mathfrak{X}) = \frac{\sum_{i=1}^{n} \text{card}(\underline{P}X_i)}{\sum_{i=1}^{n} \text{card}(\bar{P}X_i)}$$

is called the *accuracy of approximation of* $\mathfrak{X}$ by P in S, or simply accuracy of classification.

The coefficient

$$\gamma_P(\mathscr{X}) = \frac{\sum_{i=1}^{n} \operatorname{card}(\underline{P}X_i)}{\operatorname{card}(U)}$$

is called the *quality of approximation of classification* $\mathscr{X}$ by set P of attributes, or, shortly, quality of classification $\mathscr{X}$. It expresses the ratio of all P-correctly classified objects to all objects in the system. In this case, $\beta_P(\mathscr{X}) \leqslant \gamma_P(\mathscr{X})$ and $\beta_P(\mathscr{X}) = \gamma_P(\mathscr{X})$ iff $\mathscr{X}$ is P-definable.

2.9. Reduction of attributes. We say that the set of attributes $R \subseteq Q$ *depends* on set of attributes $P \subseteq Q$ in S (denotation $P \to R$) iff $\tilde{P} \subseteq \tilde{R}$. It can be shown that $P \to R$ iff P-representation of $S_{P \cup R}$ is selective.

In the sequel we are using the following definitions:

a) Set $P \subseteq Q$ is *independent* in S iff for every $P' \subset P$, $\tilde{P}' \supset \tilde{P}$.
b) Set $P \subseteq Q$ is *dependent* in S iff there exists $P' \subset P$, such that $\tilde{P}' = \tilde{P}$.
c) Set $P \subseteq Q$ is a *reduct* of Q in S iff P is the greatest independent set in Q.

If P is independent in S then for every $p, q \in P$ neither $p \to q$ nor $q \to p$, i.e. all attributes from P are pairwise independent. Also, if P is independent in S, then for every $P' \subset P$, card $(U/\tilde{P}') <$ card $(U/\tilde{P})$. Thus, in order to check if set $P \subseteq Q$ is independent in S it is sufficient so check for every attribute whether its removal increases the number of elementary sets in the system. This leads to a very simple algorithm. If P is dependent in S, then there exists $P' \subset P$, independent in S, such that $P' \to P - P'$; the greatest P' is of course a reduct of P in S. Let us notice that an information system may have more than one reduct.

2.10. Decision tables. The concept of decision tables can be precisely formulated in terms of the rough sets approach [18, 19]. We shall identify an information system $S = (U, Q, V, \varrho)$ with a decision table, assuming that $Q = C \cup D$ and $C \cap D = \emptyset$, where C are called *condition attributes*, and D, *decision attributes*. Attributes from D define some decisions which are made provided some conditions pointed out by conditions attributes are met.

A decision table $S = (U, C \cup D, V, \varrho)$ is *deterministic* iff $C \to D$; otherwise it is *non-deterministic*. Moreover, S is selective iff $\widetilde{C \cup D}$ is the identity relation.

The deterministic decision table describes uniquely decisions to be made when some conditions are satisfied. In the case of a non-deterministic table, decisions are not uniquely determined by the conditions. Instead, a subset of decisions is defined which could be taken under circumstances determined by the conditions. The latter case can be interpreted as a kind of inconsistency or uncertainty, i.e. the decisions determined by the decision tables are not well-defined.

The properties concerning dependence of attributes, given in section 2.9., can be used to check whether a given decision table is deterministic or non-deterministic. Similarly, the notion of reduct can be used in decision tables to optimize the table, i.e. to find the smallest set of condition attributes and/or decision attributes.

2.11. Decision rules and decision algorithm. For a given decision table $S = (U, C \cup D, V, \varrho)$, let $\{\underline{x}_1, \underline{x}_2, ..., \underline{x}_k\}$ be the C-definable classification of U, and $\{X_1, X_2, ..., X_n\}$, the D-definable classification of U. Class X_j can be identified with decision j $(j = 1, 2, ..., n)$. The expression $\mathrm{Des}_C(\underline{x}_i) \Rightarrow \mathrm{Des}_D(X_j)$ will be called *(C, D)-decision rule* in S, where $\mathrm{Des}_C(\underline{x}_i)$ and $\mathrm{Des}_D(X_j)$ are unique descriptions of the classes $\underline{x}_i$ and X_j, respectively $(i = 1, 2, ..., k;$ $j = 1, 2, ..., n)$. The set of decision rules $\{r_{ij}\}$ for each class (decision) X_j $(j = 1, 2, ..., n)$ can be defined as follows:

$$\{r_{ij}\} = \{\mathrm{Des}_C(\underline{x}_i) \Rightarrow \mathrm{Des}_D(X_j) | \underline{x}_i \cap X_j \neq \emptyset, i = 1, 2, ..., k\}.$$

A decision rule r_{ij} is *deterministic* iff $\underline{x}_i \cap X_j = \underline{x}_i$, and r_{ij} is *non-deterministic* otherwise.
In other words, if $\mathrm{Des}_C(\underline{x}_i)$ uniquely "implies" $\mathrm{Des}_D(X_j)$, then r_{ij} is deterministic; otherwise r_{ij} is non-deterministic.

The set of decision rules for all classes X_j $(j = 1, 2, ..., n)$ is called the *decision algorithm* resulting from S.

2.12. Rough sets approach on the microcomputer. We use a microcomputer program RoughDAS which:

(i) computes lower and upper approximations of sets;

(ii) checks whether a set of attributes is dependent or independent;

(iii) computes reducts of a set of attributes;

(iv) computes the accuracy of approximation and the quality of classification;

(v) derives the decision algorithm from a decision table.

3. More about HSV. Highly selective vagotomy consists in cutting of the gastric branches of vagal nerve (Latarjet nerves) which innervate the gastric area containing parietal cells [10].

This procedure results in diminishing secretion of parietal cells as well as their sensitivity to gastric and histamine [12, 13, 15, 24]. This, in turn, lowers the secretion of gastric juice and raises its pH. Due to this operation, small branches of the vagal nerve supplying the pyloric and duodenal area are spared. In result, the gastric emptying proceeds normally. Therefore, drainage procedures, such as pyloroplasty, gastroenterostomy and anterectomy, are not required [1, 9, 23, 25].

In the Department of Surgery of the F. Raszeja Memorial Hospital the indications for the operation were the following:

1. Ineffectiveness of the conservative treatment of uncomplicated duodenal ulcers (76 patients);
2. Duodenal ulcer bleeding in the past (33 patients);
3. Perforated and sutured duodenal ulcer (10 patients);
4. Minor pyloric stenosis with active pyloric ulcer (3 patients).

4. The POD information system. The information system called POD (PreOperating Data) is composed of 122 patients (objects) with duodenal ulcer treated by HSV, described in terms of 12 attributes (see Appendix). The first eleven attributes are the following:

1. Sex
2. Age
3. Duration of the disease
4. Complications of ulcer
5. HCl concentration
6. Basic volume of gastric juice per hour
7. Volume of residual gastric juice
8. Basic acid output (BAO)
9. HCl concentration under histamine
10. Volume of gastric juice per hour under histamine
11. Maximal acid output (MAO).

Attributes 1–4 concern anamnesis, and the remaining attributes relate to preoperative gastric secretion examined with the histaminic test of Kay [14]. For all attributes, except 1 and 4, we have established some norms resulting from clinical experience; these norms correspond to intervals of "low", "medium", or "high" values. Different norms are assumed for men (♂) and women (♀). A domain of the eleven attributes is shown in Table I.

The twelfth attribute defines a long-term result of HSV, evaluated by a surgeon in the modified Visick grading. The grading was derived from the following definition [7]:

1. *Excellent*: absolutely no symptoms, perfect result.
2. *Very good*: patient considers result perfect, but interrogation elicits mild occassional symptoms easily controlled by a minor adjustment of diet.
3. *Satisfactory*: mild or moderate symptoms easily controlled by care, which cause some discomfort, but patient and surgeon are satisfied with result which does not interfere seriously with life or work.
4. *Unsatisfactory*: moderate or severe symptoms or complications which interfere with work or normal life; patient or surgeon dissatisfied with result; includeds all cases with recurrent ulcer and those submitted to further operation, even though the latter may have been followed by considerable symptomatic improvement.

TABLE I

Domain of 11 atributes of the POD information system

No.	Attribute units	Values 0	1	2	3	4		Remarks
1.	Sex	♂		—	—	—		
2.	Age [years]	$\leqslant 35$	> 35	—	—	—		
3.	Duration of the disease [years]	$\leqslant 0.5$	$0.5 \div 3$	> 3	—	—		
4.	Complications of ulcer	none	acute haemorrhage	multiple haemorrhages	perforation in the past	pyloric stenosis		
5.	HCl concentration	$\leqslant 2$	$2 \div 4$	> 4	—	—	♂	
	[mmol HCl/100 ml]	$\leqslant 1$	$1 \div 3$	> 3	—	—	♀	basic gastric
6.	Volume of gastric juice/h [ml]	$\leqslant 70$	$70 \div 150$	> 150	—	—	♂♀	secretion
7.	Volume of residual gastric juice [ml]	$\leqslant 50$	$50 \div 100$	> 100	—	—	♂♀	
8.	Basic acid output (BAO)	$\leqslant 2$	$2 \div 3$	> 3	—	—	♂	
	[mmol HCl/h]	$\leqslant 1$	$1 \div 2$	> 2	—	—	♀	
9.	HCl concentration	$\leqslant 10$	$10 \div 15$	> 15	—	—	♂	
	[mmol HCl/100 ml]	$\leqslant 7$	$7 \div 10$	> 10	—	—	♀	
10.	Volume of gastric juice/h [ml]	$\leqslant 100$	$100 \div 250$	> 250	—	—	♂♀	gastric secretion
11.	Maximal acid output (MAO)	$\leqslant 15$	$15 \div 25$	$25 \div 40$	> 40	—	♂	stimulated by
	[mmol HCl/h]	$\leqslant 15$	$15 \div 18$	$18 \div 30$	> 30	—	♀	histamine

The Visick grading defines classification $\mathscr{X}$ of set U (122 patients) into four classes, i.e. $\mathscr{X} = \{X_1, X_2, X_3, X_4\}$, $X_i \cap X_j = \emptyset$ for every $i, j \in \{1, 2, ..., 4\}$, $i \neq j$ and $\bigcup_{i=1}^{4} X_i = U$; X_i are classes of $\mathscr{X}$.

5. Reduction of attributes. Let us consider classification $\mathscr{X}$ defined by the twelfth attribute. Table II shows the accuracy of approximation of each particular class X_i by the set of the first eleven attributes denoted by Q.

TABLE II

Accuracy of approximation of each class by Q

Class	Number of patients card (X_i)	Lower approx. card $(\underline{Q}X_i)$	Upper approx. card $(\bar{Q}X_i)$	Accuracy $\mu_Q(X_i)$
1	86	85	87	0.98
2	20	19	21	0,90
3	8	8	8	1.0
4	8	8	8	1.0

It can be seen that classes 3 and 4 are Q-definable and classes 1 and 2 are roughly Q-definable in POD, although the accuracy of their approximation is very high. The quality of classification by set Q equals 0.97. The POD information system is almost selective. The number of atoms is 116; 4 atoms are 2-element sets and 1 atom is a 3-element set. Moreover, only one 2-element atom is composed of patients belonging to different classes. This proves that the norms for attributes are well-defined. The norms would have been inappropriately defined if many atoms had been composed of patients belonging to different classes, i.e. if the quality of classification had been lower.

According to section 2.9, in order to check whether a set of attributes is dependent or not, we have to remove one attribute at a time and compute the number of elementary sets for each case. If the set, say P, is independent, then the reduction of one attribute, say r, results in the equality of at least two rows of the reduced information system. These equal rows (P-elementary sets) are clustered together forming a $(P\text{-}\{r\})$-elementary set, and thus we get a smaller number of elementary sets. Let us notice, however, that if the clustered rows belong to the same P-lower approximation of set X_i $(i = 1, ..., n)$, then $\mu_P(X_i) = \mu_{P-\{r\}}(X_i)$ for $i = 1, ..., n$, and $\beta_P(\mathscr{X}) = \beta_{P-\{r\}}(\mathscr{X})$, $\gamma_P(\mathscr{X}) = \gamma_{P-\{r\}}(\mathscr{X})$; otherwise, the signs of equality are replaced by $\geqslant$.

Proceeding in this way, we have found out that set Q of all eleven attributes is dependent and has one following reduct of ten attributes: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. The fact that the attribute 1 (sex) has turned out to be redundant is not a surprising result because the information about sex is included in values of attributes concerning gastric secretion (see Table I).

Then, we removed the particular attributes from set Q and observed the decrease in the accuracy of classes X_i $(i = 1, \ldots, 4)$ and the quality of classification $\mathcal{X}$. We did this to find the smallest set of relevant attributes which would give a relatively high quality of classification.

TABLE III

Accuracy of classes and quality of classification approximated by a set of five attributes

Sequence	Removed attributes	Accuracy of classes				Quality of classification	$\bigcup_{i=1}^{4} Bn(X_i)$
		1	2	3	4		
E	1 8 11 2 7 3	0.63	0.30	0.35	0.38	0.51	40
G	8 11 2 1 7 10	0.72	0.34	0.42	0.11	0.55	32
H	8 11 7 1 2 5	0.74	0.26	1.0	0.60	0.64	27

Let an ordered subset of attributes be called a *sequence of attributes*. The elimination of attributes from set Q, according to a given sequence, consists in removing the first attribute, then the first and the second, and then the first, the second and the third, and so on, until all the attributes in the sequence have been removed. In [6], using a trial-and-error procedure, we found three sequences of attributes, denoted by E, G and H, which are characterized by the least steep descent of the accuracy of classes and the quality of classification in the course of elimination. These sequences contain 6 attributes each. The accuracy of classes and the quality of classification approximated by the sets of 5 remaining attributes are given in Table III. The results of the elimination of attributes according to sequences E, G and H are presented graphically in Figs **3**, **4** and **5**, respectively, in the system of coordinates where the abcissa corresponds to the removed attributes and the ordinate to the accuracy of particular classes and the quality of

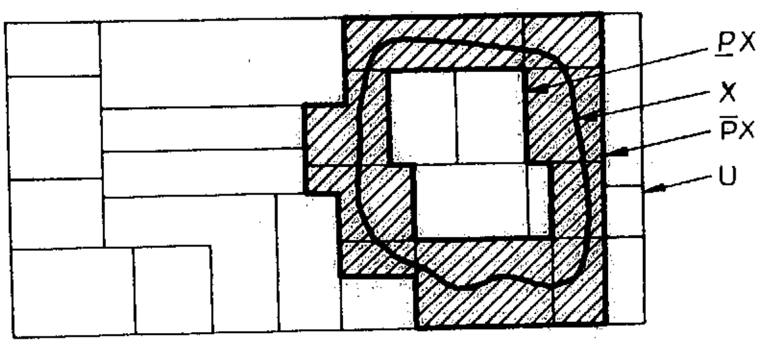

Fig. 1. Approximation of set X in A

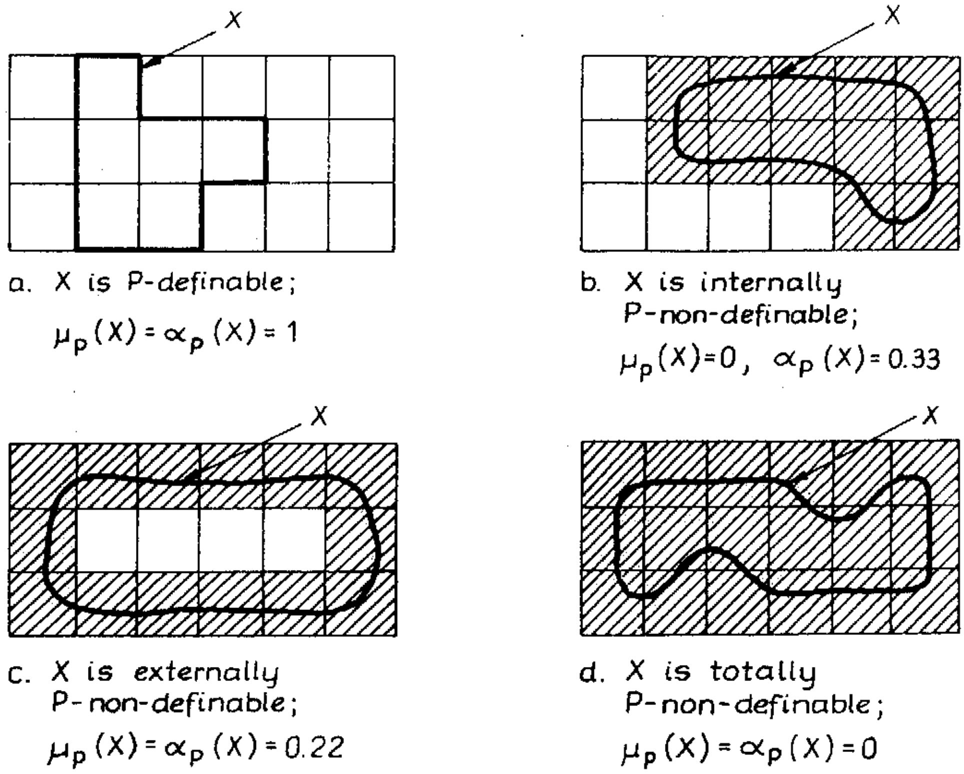

Fig. 2. P-definability of sets in S

classification. The list of signs given in Fig. 3 applies to all the three figures. Let us remark that the elimination of any attribute from outside the considered sequence causes a steep descent in the quality of classification (by at least 0.14 and by at most 0.38).

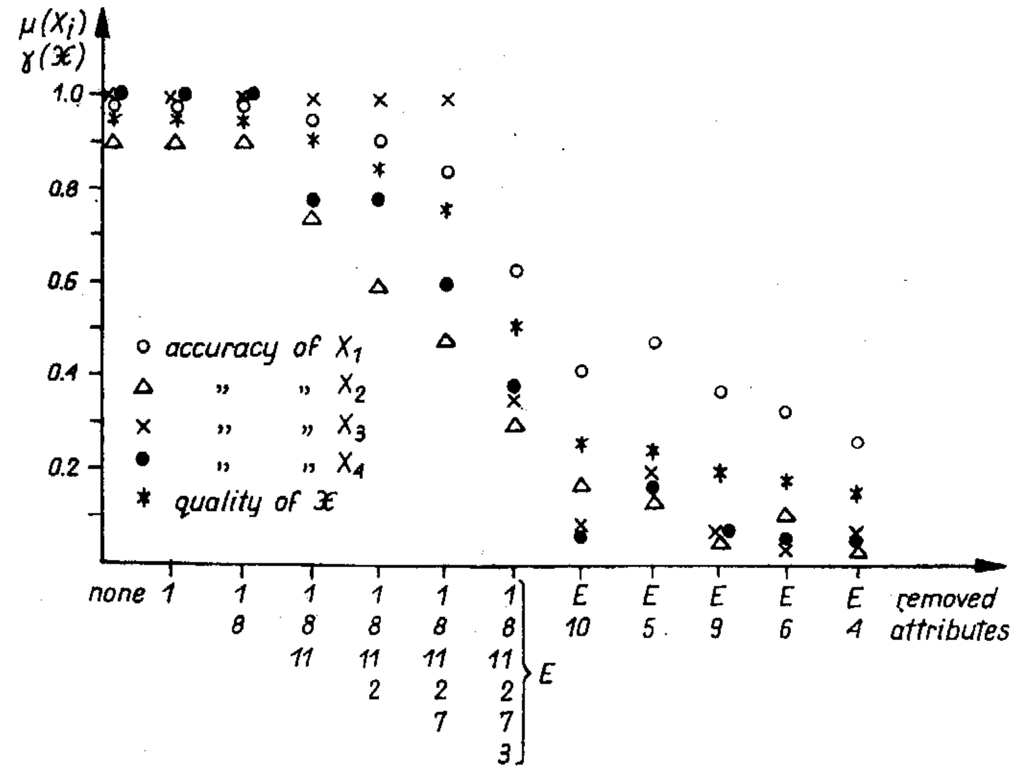

Fig. 3. Accuracy of classes and quality of classification *vs.* removed attributes

A careful analysis of Table III and Figs 3, 4 and 5 leads to the conclusion that *set R of the most significant attributes* ensuring a satisfactory quality of classification is composed of attributes from outside of the sequence H, i.e.

$$R = Q - \{H\} = \{3, 4, 6, 9, 10\}$$

where $\{H\}$ is the set of attributes in sequence H. The reasons for it could be summarized in the following points:

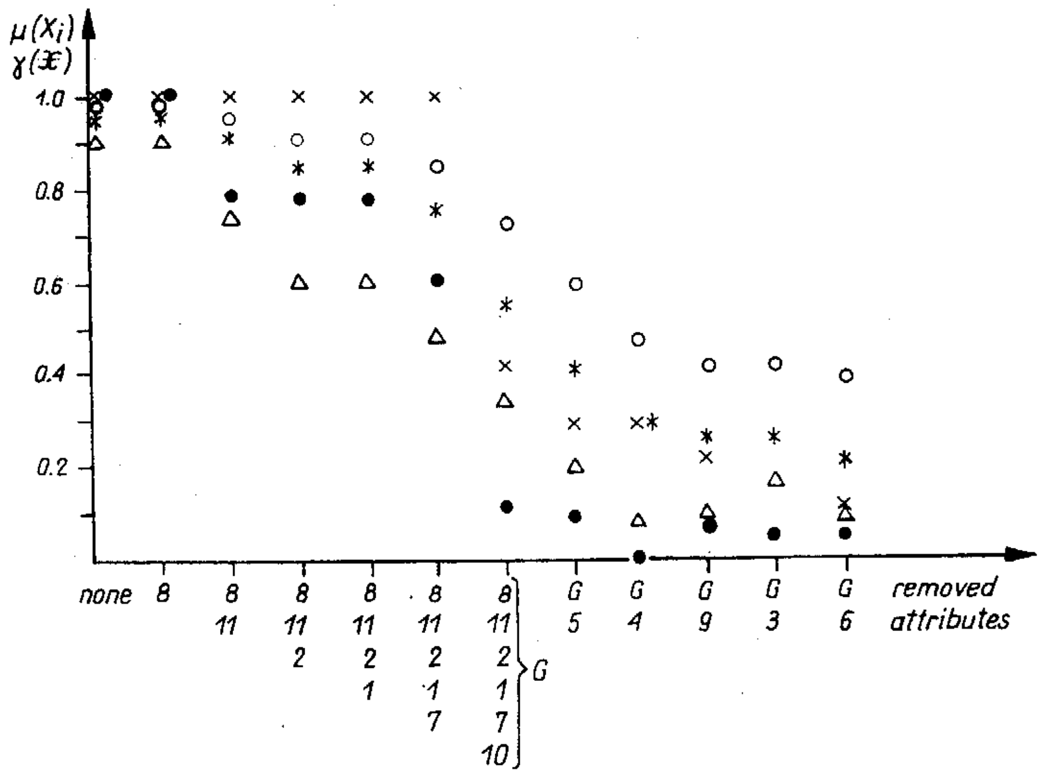

Fig. 4. Accuracy of classes and quality of classification *vs.* removed attributes

(i) The quality of classification for H (0.64) is higher than for E (0.51) or G (0.55).

(ii) For H, class 3 is R-definable because its accuracy is equal to 1; for E and G there is not a single class with such a high accuracy.

(iii) The R-doubtful region of classification $\mathfrak{X}$ is smallest (27) in comparison to the Q-$\{E\}$- and Q-$\{G\}$- doubtful region of $\mathfrak{X}$ (40 and 32, respectively).

(iv) The low accuracy of class 2 (0.26) for H is not disturbing in this case because in the R-boundary of class 1 there are almost the same patients as in the R-boundary of class 2, and both these classes correspond to positive results of the treatment by HSV. Indeed, in $Bn_R(X_1)$ there are 25 patients—23 of them also belong to $Bn_R(X_2)$ and 2 of them, to $Bn_R(X_4)$; in $Bn_R(X_2)$ there are 25 patients—23 of them also belong to $Bn_R(X_1)$ and 2 of them, to $Bn_R(X_4)$; $Bn_R(X_3)$ is empty and in $Bn_R(X_4)$ there are 4 patients.

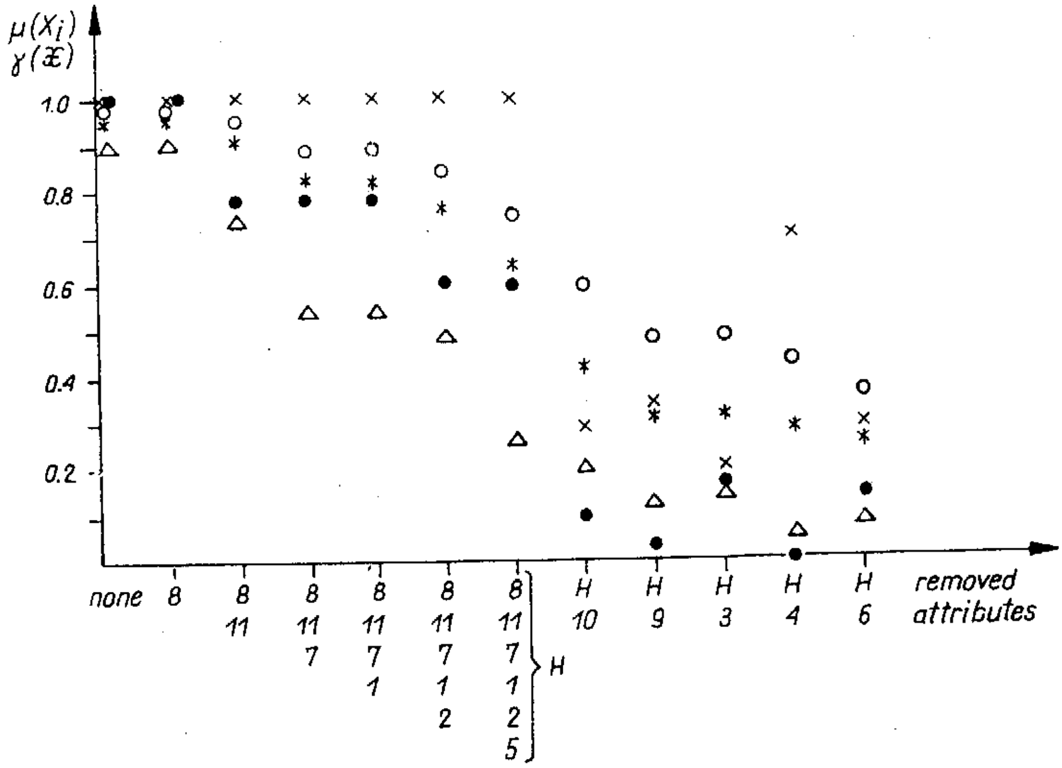

Fig. 5. Accuracy of classes and quality of classification *vs.* removed attributes

This means that if classes 1,2 and 3,4 were aggregated, so that $X_{\mathrm{I}} = X_1 \cup X_2$ and $X_{\mathrm{II}} = X_3 \cup X_4$, the accuracy of class I would be equal to 0.96, the accuracy of class II, to 0.77, and the quality of classification, to 0.97. We claim therefore that set R of attributes has a very high descriminating power in the classification into positive (class I) and negative results (class II) of the operation.

The list of attributes from set R, in the descending order of the influence on the quality of classification, is as follows:

— basic volume of gastric juice per hour (6);
— complications of ulcer (4);
— duration of the disease (3);
— HCl concentration under histamine (9);
— volume of gastric juice per hour under histamine (10).

6. "Models" of patients in each class. An analysis of the distribution of values adopted by attributes from set R in particular classes, carried out in [6], resulted in the definition of the most characteristic values of these attributes for each class. The set of characteristic values for a given class defines a "model" of patients belonging to this class in terms of attributes from set R. When analyzing the distribution, we took into account the patients belonging to R-lower approximations of classes only. We avoided defining any characteristic value for a class if the distribution of values in this class was almost uniform.

The "models" are given below:

Class 1

— medium or small basic volume of gastric juice per hour;
— no complications of ulcer or acute haemorrhage from ulcer;
— long or medium duration of the disease;
— high HCl concentration under histamine;
— medium volume of gastric juice per hour under histamine.

Class 2

— medium basic volume of gastric juice per hour;
— multiple haemorrhages;
— medium HCl concentration under histamine;
— medium volume of gastric juice per hour under histamine.

Class 3

— high and small basic volume of gastric juice per hour;
— perforation of ulcer in the past;
— short duration of the disease;
— low HCl concentration under histamine;
— low volume of gastric juice per hour under histamine.

Class 4

— high basic volume of gastric juice per hour;
— low HCl concentration under histamine;
— high volume of gastric juice per hour under histamine.

7. Decision algorithm. The POD information system can be identified with the decision table $S = (U, C \cup D, V, \varrho)$, where the set of condition attributes $C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ and the set of decision attributes $D = \{12\}$. We can see from section **5** that, since the POD information system is almost selective, decision table S is almost deterministic.

We shall derive the decision algorithm from the reduced decision table $S_R = (W, R \cup D, V, \varrho)$, where W is the R-positive region of classification $\mathcal{X}$ in R-representation of the POD information system, i.e. $W = \mathrm{Pos}_R(\mathfrak{X}) = \bigcup_{i=1}^{4} \underline{R}X_i$, and $R = \{3, 4, 6, 9, 10\}$; card $(W) = 95$. Let us observe that the decision table S_R is deterministic.

Given S_R and a subset of objects $X \subseteq W$, representing an equivalence class of $\tilde{D}$, we can compute the discriminant index $\alpha_B(X)$ for any subset of condition attributes $B \subseteq R$. Using the discriminant indices as attribute selection criteria, we can choose an appropriate subset of attributes in R to construct the decision rules for this particular X. The same procedure can be applied repeatedly to derive decision rules for every equivalence class of $\tilde{D}$. Below we present the steps of the whole procedure.

Step 0. Set the number of class $i = 1$.

Step 1. Set $W' = W$, $R' = R$, $B = \emptyset$ and $X = \underline{R}X_i$.

Step 2. Calculate the set of discriminant indices

$$\alpha_{B'}(X) = 1 - \frac{\operatorname{card}(\bar{B}'X) - \operatorname{card}(\underline{B}'X)}{\operatorname{card}(W')}$$

Set $B' = B \cup \{r\}$ for every $r \in R'$.
Select the set of attributes $B' = B \cup \{r\}$ with the highest value of $\alpha_{B'}(X)$. Set $B = B'$.

Step 3. If $\underline{B}X = \emptyset$ then go to step 4, otherwise identify B-elementary sets $\{x_1, x_2, \ldots, x_k\}$ contained in $\underline{B}X$ and generate deterministic decision rules:

$$\{\mathrm{Des}_B(x_j) \Rightarrow \mathrm{Des}_D(X) | j = 1, 2, \ldots, k\}.$$

Step 4. Set $W' = \bar{B}X - \underline{B}X$ and $X = X - \underline{B}X$.
If $W' = \emptyset$ then go to step 6, otherwise set $R' = R' - B$ and if $R' \neq \emptyset$ then go to step 2.

Step 5. Identify R-elementary sets $\{x'_1, x'_2, \ldots, x'_l\}$ on W' and generate non-deterministic decision rules:

$$\{\mathrm{Des}_R(x'_j) \Rightarrow \mathrm{Des}_D(X) | j = 1, 2, \ldots, l\}.$$

Step 6. Set $i = i+1$.
If $i \leqslant 4$ then go to step 1, otherwise end procedure.

Let us notice that in step 2 there can be several attributes giving the highest value of $\alpha_{B'}(X)$. In such case, depending on selection of one attribute, we can get slightly different decision algorithms. Thus, in general, we should check all the possibilities in order to find the decision algorithm with the smallest number of rules; if there were more than one such decision algorithm, it would be natural to keep one with the smaller number of descriptors appearing in the definition of all decision rules.

As a result of the application of the above procedure to decision table S_R equivalent to R-representation of the POD information system, we obtained four different decision algorithms. According to the previous remark, depending on the order of attributes appended to set B, the decision algorithms have from 39 to 46 decision rules. In Table IV we present the best decision algorithm for the following order of attributes:

class 1: 3—4—6—9—10
class 2: 3—4—6—9
class 3: 4—3—6—9—10
class 4: 4—6—9—10—3

8. Indications for treatment by HSV. The "models" of patients in each class as well as the decision algorithm need, however, an important comment. The POD information system applies to patients who *have been* operated with HSV, thus the surgeons expected good results of operations on these patients, taking into account clinical experience. Hence the distribution of patients in classes is unequal. For this reason, the results of our analysis may be useful in establishing indications rather than counterindications for HSV. In other words, the most representative "models" and decision rules are those concerning class 1 and class 2.

Let us explain how the decision algorithm from Table IV should be applied. The algorithm is derived from an information system representing an expert knowledge. When examining a new case of duodenal ulcer the clinicist should first define values of attributes 3 and 4. If their values were, for example, 2 and 3, respectively, then according to Table IV, regardless of other attributes, the patient would probably belong to class 1 when treated by HSV: so, in this case, the HSV is recommended. If, however, the values of attributes 3 and 4 were, for example, 2 and 0,

TABLE IV

Decision algorithm derived from R-representation of the POD information system

condition attributes					class	condition attributes					class
3	4	6	9	10		3	4	6	9	10	
2	3				⇒1	1	2				⇒2
1	1				⇒1	2	4	0			⇒2
0	3				⇒1	1	3	1			⇒2
2	0	1			⇒1	0	1	1			⇒2
2	1	0			⇒1	2	0	2	1		⇒2
2	1	2			⇒1	2	1	1	1		⇒2
2	2	0			⇒1	2	2	1	1		⇒2
2	4	2			⇒1	0	0				⇒3
1	0	1			⇒1	1	3	0			⇒3
2	0	0	1		⇒1	2	0	0	2		⇒3
2	0	2	0		⇒1	1	3	2	0		⇒3
2	1	1	2		⇒1	1	0	0	1	1	⇒3
2	2	1	2		⇒1	1	0	2	0	2	⇒3
1	0	0	2		⇒1	0	1	2	0	2	⇒3
1	0	0	0		⇒1		0	0	0	0	⇒4
1	0	2	2		⇒1		0	2	2	2	⇒4
2	0	0	0	1	⇒1		0	2	0	1	⇒4
2	0	2	2	0	⇒1	2	1	1	0	1	⇒4
1	0	0	1	0	⇒1	1	3	2	1	2	⇒4
0	1	2	0	1	⇒1						

respectively, then the clinicist should ask next for attribute 6; if its value is equal 1, the patient probably belongs to class 1. If it is 0, then attribute 9 should be asked for; if its value is 1, the patient probably still belongs to class 1. If it is 2, then he probably belongs to class 3. So, in the last case, HSV is not recommended. Let us make it clear that the word "probably" is used to stress that the expert knowledge, based on 122 cases where HSV was performed, is neither complete nor certain. It may happen that a new case will show different values of the five attributes than those in the rows of Table IV. Then, the best we can suggest is to fit this case into one of the "models" defined in section **6**.

In decision making concerning the treatment of duodenal ulcer by HSV, the decision algorithm should be considered along with "models" of patients. It is connected with the fact that decision rules may be presented by different numbers of patients from the reduced POD information system; the distribution of these numbers is just amalgamated in the "models".

In the literature concerning indications for treatment of duodenal ulcer by HSV, the anamnesis and secretion attributes are commonly taken into account. Some authors have considered more extensive anamnesis data [21] or other secretion tests [9]. The prevailing opinion is that the attributes have unequal influence on indication for treatment by HSV. It is generally agreed that the main indication is ineffectiveness of the conservative treatment of uncomplicated duodenal ulcer, while secretion criteria have not been generally agreed upon [9].

To date, retrospective studies based on traditional statistical methods, or only on clinical experience and intuition, have not proved, however, the attributes to be the most important in predicting the outcome of the operation.

9. Summary of results. The results following the application of the rough sets theory in analyzing the POD information system including 122 HSV patients can be summarized in the following points:

1. The proposed norms for attributes define the domain ensuring a good classification of patients.

2. The information system was reduced from eleven to five attributes $\{3, 4, 6, 9, 10\}$ ensuring a satisfactory quality of classification.

3. "Models" of patients in each class were constructed in terms of the most significant attributes (section 6).

4. A decision algorithm was derived from the reduced representation of the POD information system (Table IV).

* DEPARTMENT OF SURGERY, F. RASZEJA MEMORIAL HOSPITAL, 60-833 POZNAŃ
(ODDZIAŁ CHIRURGII, SZPITAL im. R. RASZEJI)
** DEPARTMENT OF COMPLEX CONTROL SYSTEMS, BAŁTYCKA 5, POLISH ACADEMY OF SCIENCES, 44-100 GLIWICE
(ZAKŁAD SYSTEMÓW AUTOMATYKI KOMPLEKSOWEJ, PAN)
*** INSTITUTE OF CONTROL ENGINEERING, TECHNICAL UNIVERSITY, 60-965 POZNAŃ
(INSTYTUT AUTOMATYKI, POLITECHNIKA POZNAŃSKA)

REFERENCES

[1] A. Alichniewicz, A. Sołtysiak, *Wagotomia wysoce wybiórcza*, Wydawnictwo Łódzkie, Łódź 1980.

[2] B. M. Amdrup, A. Jensen, *Selective wagotomy of the parietal cell mass preserving innervation of the undrained antrum*, Gastroenterology **59** (1970), 522.

[3] M. Boryczka, *Data analysis using rough sets description of program "KLAS"* [in Polish], Preliminary Report, Institute of Control Engineering, Technical University of Poznań, April 1986.

[4] N. J. Dorricot, A. R. McNeish, J. A. Williams, C. M. S. Royston, W. M. Cooke, J. Spencer, B. C. DeVries, H. Muller, *Prospective randomized multicentre trial of proximal gastric vagotomy or truncal vagotomy and antrectomy for chronic duodenal ulcer: interim results*, Br. J. Surg. **65** (1978), 152–154.

[5] D. C. Dunn, W. E. G. Thomas, J. O. Hunter, *An evaluation of highly selective vagotomy in the treatment of chronic duodenal ulcer*, Surg. Gynecol. Obstet., **150** (1980), 845–849.

[6] J. Fibak, K. Słowiński, R. Słowiński, *The application of rough sets theory to the verification of indications for treatment of duodenal by HSV*. Proc. 6th International Workshop on Expert Systems and their Applications, vol. 1, Avignon 1986, 587–599.

[7] J. C. Goligher, G. L. Hill, T. E. Kenny, E. Nutter, *Proximal gastric vagotomy without drainage for duodenal ulcer: results after 5–8 years*, Br. J. Surg., **65** (1978), 145–151.

[8] C. Gryffith, H. N. Harkins, *Parietal gastric vagotomy: an experimental study*, Gastroenterology, **32**, (1957), 96.

[9] P. Hautefeuille, R. Picaud, *Les récidives après vagotomies pour ulcère duodénal*, Masson, Paris, 1983.

[10] F. Holle, S. Anderson, *Vagotomy*, Springer-Verlag, Berlin–Heidelberg–New York, 1974.

[11] D. Johnston, A. R. Wilkinson, *Highly selective vagotomy without a drainage procedure in the treatment of duodenal ulcer*, Br. J. Surg., **57** (1970), 289.

[12] D. Johnston, *Highly selective vagotomy*, Progr. Surg. **14** (1975), 1.

[13] P. Jordan, *Current state of parietal cell vagotomy*, Ann. Surg., **184** (1976), 659.

[14] A. Kay, *Memorisl lecture. An evaluation of gastric acid secretion tests*, Gastroenterology, **53** (1967), 834–852.

[15] S. Knight, R. McIsaac, L. Fielding, *The effect of highly selective vagotomy on the relationship between gastric mucosal blood flow and acid secretion in man*, Br. J. Surg., **65** (1978), 721.

[16] Z. Pawlak, *Rough sets*, Int. J. Information and Computer Sci., **11** (1982), 341–356.

[17] Z. Pawlak, *Rough classification*, Int. J. Man-Machine Studies **20** (1984), 469–483.

[18] Z. Pawlak, *Decision tables and decision algorithms*, Bull. Pol. Ac. Pol. Tech.:, **33**, (1985), 478–494.

[19] Z. Pawlak, *Rough Sets and Decision Tables*, Lecture Notes on Computer Sci. vol. 208, Springer–Verlag, Berlin 1986, 186–196.

[20] Z. Pawlak, K. Słowiński, R. Słowiński, *Rough classification of patients after highly selective vagotomy for duodenal ulcer*, Int. J. Man-Machine Studies, **24** (1986), 413–433.

[21] T. Popiela, Z. Szafran, *Leczenie chirurgiczne choroby wrzodowej żołądka i dwunastnicy. Diagnostyka przedoperacyjna. Zasady kwalifikacji i doboru metody zabiegu operacyjnego oraz badania kontrolne*. Materiały z konferencji i kursów AM w Krakowie, Kraków, 1978.

[22] K. Słowiński, *Gastric secretion and clinical results of highly selective vagotomy for duodenal ulcer*, [in Polish] Ph. D. dissertation, Medical Academy, Poznań, 1983.

[23] A. Sołtysiak, D. Brykalski, M. Zalech, Cz. Laśkiewicz, *Opróżnianie żołądka u chorych na wrzód dwunastnicy po wagotomii wysoce wybiórczej*, Badania izotopowe, Pol. Tyg. Lek., **35** (1980), 342.

[24] H. Troidl, W. Lorenz, H. Rohde, G. Häfner, H. Hamelmann, *Effect of selective gastric vagotomy on histamine concentration in gastric mucosa of patients with duodenal ulcer*. Br. J. Surg., **65** (1978), 10.

[25] R. Waluk, R. S owiński, S. Goryń, H. Markiewicz, *Wplyw wysoce wybiórczej wagotomii na czynność ruchową żołądka, jelit i pęcherzyka żółciowego*, Pol. Tyg. Lel., **36** (1981), 361.

[26] S. K. M. Wong, W. Ziarko, R. Li Ye, *Comparison of rough set and statistical methods in inductive learning*, Int. J. Man-Machine Studies.

Я. Фибак, З. Павляк, К. Словиньски, Р. Словиньски, **Алгоритм принтия решений, основанный на приближенных множествах, для лечения язвы двенадпатиперстной кишки с помощью высокоселективной ваготомии (HSV)**

С помощью концепции приближенных множеств анализируется информационная система, описывающая пацентов, страдающих язвой двенадцатиперстной кишки, котрых лечат высокрселективной ваготомией (HSV). Информационная система представляет собой таблицу, составленную из 122 рядов, соответствующих отдельным пациентам, и 12 столбцов, отвечающих аттрибутам. Первые 11 аттрибутов касаются анамнеза и доопе-рационной гастральной секреции, иследуемых гистаминическим тестом Кэя. Последний аттрибут определяет классификацию пациентов относительно долгосрочных результатов операции, оцениваемых хирургом по модифицированным степеням Висика. Минимальное подмножество аттрибутов, значимых для качественной классификации, получается при помощи метода приближенной классификации. „Модель" пациентов в каждом классе строится на основе анализа величин, принимаемых аттрибутами из данного подмножества. Затем восстановленная информационная система определяется по таблице принятия решений, предполагая, что аттрибуты в минимальном подмножестве являются аттрибутами условий, в двенадцатый аттрибут — аттрибутом принятия решений. Из этой таблицы выводится алгоритм принятия решений, состоящий из 39 решительных правил. Данный алгоритм и модели могут быть пригодными для принятия решений по лечению язвы двенадцатиперстной кишки с помощью HSV.

APPENDIX

The POD information system

No.	Sex	Age	Duration of the disease	Compli-cations of ulcer	Basic gastric secretion: HCl concen-tration	Basic gastric secretion: Vol. of gastric juice	Basic gastric secretion: Vol of gastric juice on an empty stomach	Basic gastric secretion: BAO	Gastric secretion stimulated by Histamine: HCl concen-tration	Gastric secretion stimulated by Histamine: Vol. of gastric juice	Gastric secretion stimulated by Histamine: MAO	Visick grading
1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	2	0	2	1	0	2	2	1	2	1
2	0	0	1	1	2	0	0	2	2	1	2	1
3	0	0	2	0	2	1	0	2	2	0	0	1
4	0	1	1	0	2	0	0	2	2	1	3	1
5	1	0	0	0	2	1	0	2	2	1	1	3
6	0	0	2	1	2	0	1	2	1	1	1	1
7	0	0	1	1	2	0	0	0	1	0	0	1
8	1	0	1	1	2	0	0	2	2	0	0	1
9	0	1	2	0	2	1	0	2	2	1	3	1
10	0	0	2	3	2	1	0	2	2	2	3	2
11	0	1	1	0	2	2	0	2	2	1	3	1
12.	0	1	2	2	1	1	0	0	0	1	0	2
13	0	1	2	3	2	1	0	2	1	1	0	2
14	0	0	1	0	2	0	0	0	2	1	1	1
15	0	1	2	2	2	1	0	2	2	2	3	1
16	0	0	1	1	2	2	0	2	1	1	0	1
17	0	0	2	0	2	2	0	2	2	1	3	1
18	1	1	2	0	2	0	0	2	0	0	0	4
19	0	1	2	0	2	1	0	2	2	2	3	1
20	0	0	2	0	1	2	0	2	0	1	0	2
21	0	0	2	0	0	2	1	0	0	1	0	1
22	0	0	2	4	1	2	2	2	0	1	0	1
23	0	0	0	1	2	2	0	2	0	1	1	1
24	0	0	1	3	0	0	0	0	1	1	0	3
25	0	1	1	0	2	1	0	2	2	1	2	1
26	0	0	1	0	2	1	0	2	2	1	2	1
27	0	1	2	0	0	2	1	2	2	0	0	1
28	1	1	1	0	2	0	0	1	1	0	0	1
29	0	0	1	0	1	2	0	2	1	1	1	1
30	0	0	2	0	2	1	0	2	0	1	0	1
31	0	1	1	0	2	1	0	2	1	1	2	1
32	0	1	2	0	1	0	0	0	0	1	0	1
33	1	1	2	1	1	0	2	1	0	0	0	1
34	0	0	1	0	2	2	0	2	2	1	3	1
35	0	0	1	0	2	1	0	2	0	1	2	1
36	0	0	2	0	2	1	0	2	0	2	0	1
37	0	1	1	0	2	0	0	2	0	1	0	1

33	1	1	2	1	1	0	2	1	0	0	0	1
34	0	0	1	0	2	2	0	2	2	1	3	1
35	0	0	1	0	2	1	0	2	0	1	2	1
36	0	0	2	0	2	1	0	2	0	2	0	1
37	0	1	1	0	2	0	0	2	0	1	0	1
38	0	0	1	1	0	2	1	1	0	1	0	1
39	0	0	2	0	2	2	0	2	2	2	3	4
40	0	1	2	0	2	2	2	2	0	2	2	1
41	0	0	2	2	0	0	0	0	0	1	0	1
42	0	0	1	0	2	2	0	2	0	1	0	4
43	0	1	1	3	2	2	2	2	0	1	0	3
44	0	1	0	1	2	1	0	2	0	1	0	2
45	0	0	1	2	2	1	1	2	2	1	3	2
46	0	1	2	1	2	1	1	2	2	1	2	1
47	1	0	2	1	2	1	0	2	2	1	1	1
48	0	1	1	0	2	1	0	2	1	1	2	1
49	0	0	2	3	2	0	0	2	1	1	0	1
50	0	0	2	3	1	0	0	2	0	2	1	1
51	1	0	1	3	2	1	1	2	0	1	0	2
52	0	1	2	1	2	0	0	2	1	0	0	1
53	0	0	1	0	2	1	0	2	1	1	0	1
54	0	0	2	0	1	2	0	2	2	1	1	2
55	0	1	2	3	2	1	0	2	1	1	0	1
56	1	0	2	1	2	2	2	2	2	2	3	1
57	0	1	1	1	1	1	0	2	2	1	2	1
58	0	0	0	3	2	2	2	2	1	2	3	1
59	0	1	2	4	2	2	0	2	2	1	3	4
60	1	0	2	0	1	0	1	0	1	1	0	1
61	1	1	1	0	2	0	0	2	2	2	3	1
62	0	0	2	2	2	1	0	2	1	1	1	2
63	0	1	1	3	1	2	1	2	1	2	2	4
64	0	0	1	2	1	1	0	2	1	0	0	2
65	1	0	0	0	2	0	1	2	2	0	0	3
66	1	0	1	0	1	0	0	0	1	0	0	1
67	0	1	2	0	1	1	0	1	0	1	0	1
68	0	0	1	0	2	1	1	2	1	1	0	1
69	0	0	2	0	1	2	1	2	0	1	0	1
70	1	0	2	0	2	0	1	2	2	2	3	1
71	1	1	2	2	1	1	0	2	2	0	0	1
72	0	0	2	0	2	1	0	2	0	1	0	1
73	0	0	2	4	2	0	2	0	0	1	1	2
74	0	1	2	0	0	0	0	0	0	1	0	1
75	1	0	2	0	2	2	2	2	2	2	3	4
76	0	0	2	0	2	1	0	2	1	2	3	1
77	0	0	1	0	1	2	0	2	1	2	2	1
78	0	0	2	0	2	1	0	2	0	1	0	1
79	0	0	0	1	0	2	0	1	0	2	2	3
80	0	1	2	0	2	2	0	2	0	1	0	2
81	0	0	1	0	2	1	1	2	0	1	0	1
82	0	0	2	0	2	2	0	2	2	1	1	2
83	0	1	2	0	2	1	0	2	2	1	1	1
84	0	0	2	0	2	2	1	2	2	1	2	1
85	1	0	2	0	2	1	1	2	2	1	3	1
86	0	0	1	0	2	2	0	2	1	1	1	2

72	0	0	2	0	2	1	0	2	0	1	0	1
73	0	0	2	4	2	0	2	0	0	1	1	2
74	0	1	2	0	0	0	0	0	0	1	0	1
75	1	0	2	0	2	2	2	2	2	2	3	4
76	0	0	2	0	2	1	0	2	1	2	3	1
77	0	0	1	0	1	2	0	2	1	2	2	1
78	0	0	2	0	2	1	0	2	0	1	0	1
79	0	0	0	1	0	2	0	1	0	2	2	3
80	0	1	2	0	2	2	0	2	0	1	0	2
81	0	0	1	0	2	1	1	2	0	1	0	1
82	0	0	2	0	2	2	0	2	2	1	1	2
83	0	1	2	0	2	1	0	2	2	1	1	1
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85	1	0	2	0	2	1	1	2	2	1	3	1
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87	0	0	2	0	2	2	0	2	1	2	2	2
88	0	1	1	1	2	1	0	2	2	1	2	1
89	1	0	2	0	2	1	1	2	2	1	2	1
90	0	0	2	0	2	2	1	2	1	2	2	2
91	0	0	2	0	1	0	2	1	1	1	0	1
92	0	0	2	0	2	2	0	2	1	1	1	4
93	0	1	2	0	1	1	0	2	0	1	0	1
94	0	1	2	0	2	2	2	2	2	1	1	1
95	1	1	2	0	2	0	0	2	2	2	3	4
96	0	0	2	1	2	1	1	2	1	2	3	2
97	0	1	2	0	2	1	0	2	1	2	2	1
98	0	0	2	0	2	1	1	2	1	2	2	1
99	0	0	2	2	1	0	2	0	0	0	0	1
100	0	0	2	0	2	1	1	2	1	1	1	1
101	0	0	2	1	0	1	1	0	0	1	0	4
102	0	0	2	2	2	1	0	2	0	1	0	1
103	0	0	1	0	2	2	1	2	1	2	2	2
104	0	1	2	2	2	1	1	2	2	2	3	1
105	0	1	1	0	2	0	0	2	0	1	0	1
106	1	1	1	0	2	1	0	2	2	0	0	1
107	1	1	2	0	2	0	2	2	2	0	0	3
108	0	0	1	1	2	1	1	2	1	2	2	1
109	0	1	1	0	2	2	0	2	0	2	0	3
110	1	1	2	2	1	1	0	2	1	1	0	2
111	0	1	2	0	2	2	0	2	2	1	1	1
112	1	0	2	0	2	1	1	2	2	1	2	1
113	0	0	1	0	2	1	1	2	2	2	3	1
114	0	1	2	0	2	1	0	2	1	1	0	1
115	0	1	2	0	1	1	0	2	0	2	0	1
116	0	0	1	1	2	1	0	2	2	1	3	1
117	0	1	2	1	2	0	0	1	2	1	2	1
118	0	1	1	0	2	0	2	2	1	1	2	3
119	0	1	2	0	2	0	0	2	0	1	0	1
120	0	0	2	0	2	1	0	2	1	1	2	2
121	0	0	2	0	2	1	0	2	1	1	1	2
122	0	1	2	2	0	1	0	0	0	1	0	1