

## New Method of Parenthesis-free Notations of Formulae

by

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*Presented by P. SZULKIN on March 16, 1960*

The author presented in the paper [1] an organization of the computer, where the programme of operation is noted in the form of a series of ordered pairs of arguments and the result of each operation is noted in the "nearest blank space". This principle of programming results in a new simple method of parenthesis-free notations of mathematical formulae. The notation in question resembles the well known parenthesis-free notation of Łukasiewicz [2]. If the symbol of operation in the programme is not located near the corresponding pair of arguments, but at the site of the nearest left hand letter B, the obtained expression may be considered as a new kind of parenthesis-free notation of the formulae. It can be shown that this notation is unique.

Hereunder follow several examples of formulae in parenthesis notation, in the parenthesis-free notation of Łukasiewicz and in the proposed notation:

$$1. (p \rightarrow q) \rightarrow ((p \rightarrow r) \rightarrow (p \rightarrow (q+r))),$$

$$\rightarrow \rightarrow pq \rightarrow \rightarrow pr \rightarrow p+qr,$$

$$\rightarrow \rightarrow \rightarrow pq \rightarrow \rightarrow prp^+q.$$

$$2. (((a+b) \cdot c) - e)/f,$$

$$/ - \cdot + abcef,$$

$$/ - f \cdot ec + ab.$$

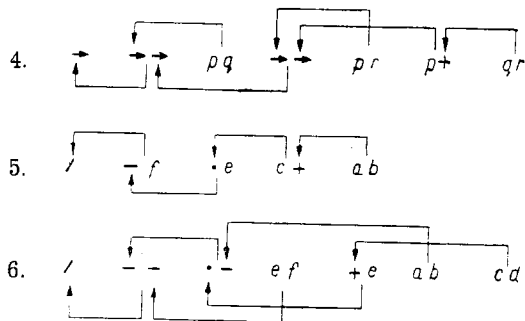
$$3. ((c+d) \cdot e) - (a-b)/(e-f),$$

$$/ - \cdot + ade - ab - ef,$$

$$/ - - \cdot - ef + abcd.$$

The proposed parenthesis-free notation reads as follows: All symbols — starting from the right — are grouped into pairs; operations determined

by the nearest symbol on the left are next carried out on each pair, while the result replaces the symbol. The diagrams show the order in which computations are carried out for the aforesaid schemes.



This notation may be considered as a programme for the address-free computer.

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#### REFERENCES

- [1] Z. Pawlak, *Organization of the address-free digital computer for calculating simple arithmetical expressions*, Bull. Acad. Polon. Sci., Sér. sci. techn., 8 (1960), this issue, p. 193.
- [2] J. Łukasiewicz, *Elementy logiki matematycznej*, Warsaw, 1929.