

The Application of Systematic Binary Expansions to Decimal Codes

by

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The systematic binary expansions of a real number X will be called as follows

$$(1) \quad X = \sum_{i=-\infty}^m 2^i \cdot x_i (-1)^{F(i)} + c,$$

where m, x_i, c are integers ($0 \leq |x_i| \leq 1$), and $F(i)$ is a function defined on integers (see, e.g. [1]).

In the present paper the systematic binary expansions of the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 such that $x_i = 1 - y_i$ for every i , $0 \leq i \leq 3$, if $X + Y = 9$ are given. These expansions are listed in Table I. For the sake of simplicity, the sign of each power of 2 is given in each column of the Table instead of the function $F(i)$. For instance, the functions $F(i)$ have in the first three columns the form:

$$F(i) = i, \quad F(i) = E\left(\frac{i+2}{2}\right), \quad F(i) = E\left(\frac{i+1}{2}\right),$$

where $E(x)$ denotes integral part of x . The last column contains the known "excess three code".

The expansion

$$(2) \quad X = \sum_{i=0}^3 2^i y_i (-1)^{F_1(i)} + c_1,$$

will be called inverse to the expansion

$$(3) \quad Y = \sum_{i=0}^3 2^i y_i (-1)^{F_2(i)} + c_2,$$

if for every i the following equalities: $x_i = y_i$, $F_1(i) = -F_2(i)$ and $c_1 + c_2 = 9$ are valid.

TABLE I

X	c=2	c=0	c=3	c=1	c=-1	c=-2	c=4	c=-3
	+--+	++--	+--+	+--+	+--+	++--	+--+	++++
0	0 1 1 0	0 0 0 0	0 1 0 1	0 1 1 1	0 0 0 1	0 0 1 0	0 1 0 0	0 0 1 1
1	0 0 0 1	0 1 1 1	0 0 1 0	0 0 0 0	0 1 1 0	0 1 0 1	0 0 1 1	0 1 0 0
2	0 0 0 0	0 1 1 0	0 0 1 1	0 0 0 1	0 1 1 1	0 1 0 0	0 0 1 0	0 1 0 1
3	0 0 1 1	0 1 0 1	0 0 0 0	0 0 1 0	0 1 0 0	0 1 1 1	0 0 0 1	0 1 1 0
4	0 0 1 0	0 1 0 0	0 0 0 1	0 0 1 1	0 1 0 1	0 1 1 0	0 0 0 0	0 1 1 1
5	1 1 0 1	1 0 1 1	1 1 1 0	1 1 0 0	1 0 1 0	1 0 0 1	1 1 1 1	1 0 0 0
6	1 1 0 0	1 0 1 0	1 1 1 1	1 1 0 1	1 0 1 1	1 0 0 0	1 1 1 0	1 0 0 1
7	1 1 1 1	1 0 0 1	1 1 0 0	1 1 1 0	1 0 0 0	1 0 1 1	1 1 0 1	1 0 1 0
8	1 1 1 0	1 0 0 0	1 1 0 1	1 1 1 1	1 0 0 1	1 0 1 0	1 1 0 0	1 0 1 1
9	1 0 0 1	1 1 1 1	1 0 1 0	1 0 0 0	1 1 1 0	1 1 0 1	1 0 1 1	1 1 0 0

It is easily seen that by replacing in the Table I X by $9 - X$, c — by $9 - c$, and all the plus signs by the minus signs and *vice versa* we shall obtain a Table (which is not given in the present paper) of codes inverse in relation to the codes given in the present Table.

It may be shown that both Tables exhaust all the possible binary systematic expansions of the numbers 0, ..., 9.

The expansions presented may find application in computing devices working in decimal system.

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