BULLETIN DE L'ACADÉMIE
POLONAISE DES SCIENCES
Série des sciences techniques
Volume VIII, No. 3 — 1960

COMPUTERS

The Application of Negative Base Number System to Digital Differential Analyser

by

Z. PAWLAK

Presented by P. SZULKIN on February 2, 1960

The basic element of digital differential analyser is a digital integrator. In the present paper an application of negative base number system [1], [2] to a digital integrating device is described. This system seems to have certain advantages in view, among others, of the facility of obtaining three-valued increments. The application of this system seems to be particularly useful for the conception of digital analyser, due to Kosharskiï [4].

Now, we shell give some properties of negative base system useful for the description of the integrating device.

Every real number X

(1) $$\frac{2^{2E\left(\frac{m+2}{2}\right)}}{3} \leqslant X \leqslant \frac{2^{2E\left(\frac{m+1}{2}\right)+1}}{3},$$

where m (an integer) has an expansion of the form

(2) $$X = \sum_{i=-\infty}^{m} x_i (-2)^i,$$

where $-1 \leqslant x_i \leqslant 0$ is an integer. Numbers X, $-1/3 \leqslant X \leqslant 2/3$ will be denoted by $\overline{x}$.

Let

(3) $$x = \sum_{i=-\infty}^{-1} x_i (-2)^i,$$

(4) $$y = \sum_{i=-\infty}^{-1} y_i (-2)^i.$$

Since $-1/3 \leqslant \overline{x} \leqslant 2/3$, $-1/3 \leqslant \overline{y} \leqslant 2/3$, therefore $-2/3 \leqslant \overline{x} + \overline{y} \leqslant 4/3$, or $-1 + 1/3 \leqslant \overline{x} + \overline{y} \leqslant 1 + 1/3$.

Hence, $\bar{x}+\bar{y}=p_0+\bar{r}$, where

$$p_0=P(\bar{x},\bar{y})=\begin{cases}1, & \text{if} \quad \bar{x}+\bar{y}>2/3,\\ 0, & \text{if} \quad -1/3\leqslant \bar{x}+\bar{y}\leqslant 2/3,\\ -1, & \text{if} \quad \bar{x}+\bar{y}<-1/3,\end{cases} \tag{5}$$

but $-1/3\leqslant r\leqslant 2/3$, and

$$\bar{r}=R(\bar{x},\bar{y})=\begin{cases}\bar{x}+\bar{y}, & \text{if} \quad -1/3\leqslant \bar{x}+\bar{y}\leqslant 2/3,\\ -1+\bar{x}+\bar{y}, & \text{if} \quad \bar{x}+\bar{y}>2/3,\\ 1+\bar{x}+\bar{y}, & \text{if} \quad \bar{x}+\bar{y}<-1/3.\end{cases} \tag{6}$$

It can be easily observed that p_0 is a carry obtained by adding of digits x_{-1} and y_{-1}.

A digital integrator realizes the relations:

$$\begin{aligned}&\text{(i)} \quad r_{i+1}=R(\bar{r}_i,\bar{y}_i),\\ &\text{(ii)} \quad \Delta r_i=p_0=P(r_i,\bar{y}_i),\end{aligned} \tag{7}$$

where

$$\bar{y}_{i+1}=\bar{y}_i+\Delta y_i(-2)^{-h}\cdot(-1)^{-h}, \tag{8}$$

and $\Delta\bar{r}_i$, Δy_i may take one of the three values 0, 1, — 1; h denotes the so called scale factor.

The conception just presented has been proved by M. J. Kolupa, T. Kulikowski and J. Sadzikowski on the "—2" digital computer [3].

The author is indebted to Mr. A. Wakulicz for help in preparing the present paper.

INSTITUTE OF MATHEMATICS, POLISH ACADEMY OF SCIENCES
(INSTYTUT MATEMATYCZNY, PAN)
DEPARTMENT OF TELE- AND RADIOPHONIC CONSTRUCTIONS, WARSAW TECHNICAL UNIVERSITY
(ZAKŁAD KONSTRUKCJI TELE- I RADIOFONII POLITECHNIKI WARSZAWSKIEJ)

REFERENCES

[1] Louis B. Wadel, *Negative base number systems*, IRE. Trans. Électronic Comput., 6 (1957), 123.

[2] Z. Pawlak, A. Wakulicz, *Use of expansions with a negative basis in the arithmometer of a digital computer*, Bull. Acad. Polon. Sci., Sér. sci. math., astr. et phys., 5 (1957), 233.

[3] Z. Pawlak, *An electronic computer based on the "—2" system*, Bull. Acad. Polon. Sci., Sér. sci. techn., 7 (1959), 713.

[4] L. A. Kosharskiĭ, *On a certain method of constructing digital differential analyzer. Problems of the theory of mathematical machines* (in Russian), Moscow, 1958, pp. 219—230.