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Concept Forming and Black Boxes

by

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Summary. Black boxes introduced in the article [1] are used to solve the problem whether two sets of features generate the same concept or not. This problem has a simple solution if the concepts are formed on the basis of the so called good context. Algorithms are given for recognizing good contexts and for deciding whether two sets of features in a good context generate the same concept or not. Contexts are proved to correspond to the so called admissible boxes.

1. Introduction. Let (G, M, r) be an ordered triple where G, M are finite nonempty sets and r is a correspondence from G to M. Then the triple is said to be a *context* and the elements in G are interpreted to be objects, the elements in M features; $(g, m) \in r$ means that the object $g \in G$ has the feature $m \in M$; cf. [3].

For any $X \subseteq G$ and any $Y \subseteq M$, we set (cf. [2] § 27, 28)

$$s(X) = \{m \in M;\ (g, m) \in r \text{ for any } g \in X\},$$

$$t(Y) = \{g \in G;\ (g, m) \in r \text{ for any } m \in Y\},$$

$$p = t \circ s,\quad q = s \circ t.$$

Then the ordered pair of mappings (s, t) establishes a Galois connection and the mappings p, q are closure operators; p is a closure operator on G, q is a closure operator on M.

It is well-known that the set $\mathbf{C}$ of all p-closed sets is a complete lattice if ordered by inclusion; similarly, the set $\mathbf{D}$ of all q-closed sets is a complete lattice that is antiisomorphic to $\mathbf{C}$.

An ordered pair $(C, D) \in \mathbf{C} \times \mathbf{D}$ is said to be a concept if $D = s(C)$ or, equivalently, $C = t(D)$ (cf. [3]). Then C is said to be the *extent* of the concept (C, D), D is called its *intent*.

If $N \subseteq M$ is a set of features, then $(t(N), q(N))$ is a concept that is said to be the *concept generated by the set* N. Two sets of features $N_1 \subseteq M$ and $N_2 \subseteq M$ are said to *generate the same concept* if $(t(N_1), q(N_1)) = (t(N_2), q(N_2))$. It follows from the general properties of Galois connections that N_1 and N_2 generate the same concept if and only if $t(N_1) = t(N_2)$ or, equivalently, $q(N_1) = q(N_2)$.

We shall deal with the following problems.

Problem 1. Find an algorithm for recognition whether two sets of features generate the same concept or not.

Problem 2. Find a class of contexts with a simple algorithm solving Problem 1.

The following will be useful in what follows.

1. Lemma. *If* (G, M, r) *is a context and* $Y \subseteq M$, *then* $t(Y) = \bigcap \{t(\{m\});\ m \in Y\}$.

Indeed, $t(Y) = \{g \in G;\ (g, m) \in r \text{ for any } m \in Y\} = \bigcap \{\{g \in G;\ (g, m) \in r\};\ m \in Y\} = \bigcap \{t(\{m\});\ m \in Y\}$. □

Let (G, M, r) be a context; an object $g \in G$ is said to be *parasitic* if $(g, m) \notin r$ for any $m \in M$. A context (G, M, r) is said to be a *context without parasitic objects* if no element in G is parasitic. To any context (G, M, r), there exists a context (H, M, r) without parasitic objects such that $t(N) = t'(N)$ for any $N \subseteq M$ where $t'(N) = \{g \in H;\ (g, m) \in r \text{ for any } m \in N\}$; we take H to be the set of all nonparasitic objects in G.

2. Examples. We may suppose that a context is given by means of its incidence table. Let $G = \{g_1, \ldots, g_m\}$, $M = \{m_1, \ldots, m_n\}$ where $m \geq 1$, $n \geq 1$ are integers; we may suppose that $g_i \neq g_j$ for $1 \leq i < j \leq m$ and $m_h \neq m_k$ for $1 \leq h < k \leq n$. We define

$$a_{ij} = \begin{cases} 1 & \text{if } (g_i, m_j) \in r \\ 0 & \text{if } (g_i, m_j) \notin r, \end{cases}$$

for any i, j with $1 \leq i \leq m$, $1 \leq j \leq n$. The incidence matrix (a_{ij}) defines the correspondence r. If we add the entries formed of elements in G and of elements in M, we obtain the *incidence table* of the context. In what follows we suppose that any context is given by its incidence table.

We now extend the definition of a_{ij}. If $\emptyset \neq N \subseteq M$, $N = \{m_{j(1)}, m_{j(2)}, \ldots, m_{j(k)}\}$ where $1 \leq k \leq n$, $1 \leq j(l) \leq n$ for every l with $1 \leq l \leq k$, we put $a_{iN} = \inf \{a_{ij(l)};\ 1 \leq l \leq k\}$ for any i with $1 \leq i \leq m$. Furthermore, we set $a_{i\emptyset} = 1$ for any i with $1 \leq i \leq m$. Hence, a_{iN} is defined for any i with $1 \leq i \leq m$ and for any $N \subseteq M$.

Directly from definitions we obtain

1. LEMMA. *If (G, M, r) is a context and $N \subseteq M$, then $t(N) = \{g_i;\ a_{iN} = 1\}$.* □

From this Lemma an *algorithm* can be deduced *for recognizing whether* $t(N_1) = t(N_2)$ or not; it suffices to test whether $a_{iN_1} = a_{iN_2}$ for any $i = 1, 2, \ldots, m$ or not.

2. EXAMPLE. Let us have $G = \{g_1, \ldots, g_4\}$, $M = \{m_1, \ldots, m_5\}$; suppose that the incidence table of (G, M, r) is as follows

r	m_1	m_2	m_3	m_4	m_5
g_1	1	1	0	0	1
g_2	0	0	1	1	1
g_3	1	1	0	0	0
g_4	0	0	1	1	0

Then, e.g., $t(\{m_1, m_3\}) = \emptyset = t(\{m_1, m_4, m_5\})$. □

3. EXAMPLE. Let G, M be the same as in 2; let the incidence table be as follows.

r	m_1	m_2	m_3	m_4	m_5
g_1	1	1	0	0	1
g_2	0	0	1	1	1
g_3	1	1	1	1	0
g_4	0	0	1	1	0

Then, e.g., $t(\{m_1, m_3\}) = \{g_3\}$, $t(\{m_1, m_4, m_5\}) = \emptyset$. □

3. Black boxes. An ordered triple $\langle I, O, f\rangle$ where I, O are finite nonempty sets and f is a mapping of the set $\mathbf{B}(I)$ of all subsets of I into O is said to be a *(black) box*. The elements in I are said to be inputs, the elements in O are called outputs. The mapping f assigns an output to any set of inputs. An output o is called *parasitic*, if $f(X) \neq o$ for any $X \in \mathbf{B}(I)$. A box $\langle I, O, f\rangle$ is said to be a *box without parasitic outputs* if no element in O is parasitic. If $\langle I, O, f\rangle$ is a box, then $\langle I, P, f\rangle$ is a box without parasitic outputs where P is the set of all nonparasitic outputs in O.

Let $\langle I, O, f\rangle$ be a box and suppose that the following condition is satisfied.

(i) For any X, $Y \in \mathbf{B}(I)$ and any $x \in I$ the condition $f(X) = f(Y)$ implies that $f(X \cup \{x\}) = f(Y \cup \{x\})$.

Then the box is said to be *admissible*.

Let $\langle I, O, f\rangle$ be an admissible box satisfying the following condition.

(ii) For any $X \in \mathbf{B}(I)$ and any $x \in I$ with $f(X \cup \{x\}) = f(X)$, there exists $y \in X$ such that $f(\{x\}) = f(\{y\})$.
Then the box is said to be **good**.

A box $\langle I, O, f \rangle$ is called *very good* if the mapping f is injective.

There is a natural relationship between contexts and boxes. For any context $T = (G, M, r)$, the ordered triple $S = \langle M, C, t \rangle$ is a box where $\mathbf{C}$ and t have the same meaning as in **1**. We set $S = \mathscr{C}(T)$ and the box S is said to be *assigned* to T.

1. THEOREM. *For any context T the box $\mathscr{C}(T)$ is admissible and has no parasitic outputs.*

Proof. By 1.3, we have $t(X \cup \{x\}) = \bigcap \{t(\{m\}); m \in X \cup \{x\}\} = \bigcap \{t(\{m\}); m \in X\} \cap t(\{x\}) = t(X) \cap t(\{x\}) = t(Y) \cap t(\{x\}) = t(Y \cup \{x\})$ if $t(X) = t(Y)$; thus, (i) holds.

Let $C \in \mathbf{C}$ be arbitrary. We put $D = s(C)$. Then $C = t(D)$ where $D \in \mathbf{B}(M)$. It follows that C is not parasitic. □

Hence, admissible boxes are interesting in connection with our problems. In the remaining part of this paragraph, we prove that any admissible box without parasitic outputs is assigned to a suitable context.

2. LEMMA. *A box $\langle I, O, f \rangle$ is admissible if and only if $f^{-1} \circ f$ is a congruence on the semilattice $(\mathbf{B}(I), \cup)$.*

For the proof see [1] 2.1. □

3. COROLLARY. *If $\langle I, O, f \rangle$ is an admissible box, then for any $X \in \mathbf{B}(I)$ the set of all elements $Z \in \mathbf{B}(I)$ with $f(X) = f(Z)$ has a greatest element (with respect to inclusion).* □

This element will be denoted by $u(X)$. We set $\mathbf{Z} = \{u(X); X \in \mathbf{B}(I)\}$.

4. COROLLARY. *The mapping u of $\mathbf{B}(I)$ into $\mathbf{B}(I)$ is a closure operator.*

Indeed, $X \subseteq u(X)$ and $u(u(X)) = u(X)$ obviously hold for any $X \in \mathbf{B}(I)$. If $X \subseteq Y$, then $(u(X), X) \in f^{-1} \circ f$, $(u(Y), Y) \in f^{-1} \circ f$ imply that $(u(X) \cup u(Y), Y) = (u(X) \cup u(Y), X \cup Y) \in f^{-1} \circ f$ by 2; since $u(Y)$ is the greatest element in the block of $f^{-1} \circ f$ that contains Y, we have $u(X) \subseteq u(Y) \cup u(Y) \subseteq u(Y)$. □

5. COROLLARY. *For any $X \in \mathbf{B}(I)$, we have $\{Z \in \mathbf{Z}; X \subseteq Z\} = \{Z \in \mathbf{Z}; u(X) \subseteq Z\}$.*

Indeed, if $Z \in \mathbf{Z}$, $X \subseteq Z$, then $u(X) \subseteq u(Z) = Z$. □

Let $S = \langle I, O, f \rangle$ be an admissible box without parasitic outputs. We set
(*) $r = \{(y, x) \in O \times I$; there exists $Z \in \mathbf{Z}$ with $x \in Z, y = f(Z)\}$. Then (O, I, r) is a context; we put $(O, I, r) = \mathscr{E}(S)$; the context $\mathscr{E}(S)$ is said to be *associated* with S.

We construct $\mathscr{C}(\mathscr{E}(S)) = \mathscr{C}((O, I, r)) = \langle I, \mathbf{C}, t\rangle$ where r is defined by (*) and $\mathbf{C}$, t are defined according to **1** taking O for G and I for M.

6. LEMMA. *For any* $X \in \mathbf{B}(I)$, *we have* $t(X) = \{f(Z);\ X \subseteq Z \in \mathbf{Z}\}$.

Clearly, for any $x \in I$, we have $t(\{x\}) = \{y \in O;\ (y, x) \in r\} = \{y \in O$; there exists $Z \in \mathbf{Z}$ with $x \in Z$, $y = f(Z)\} = \{f(Z);\ x \in Z \in \mathbf{Z}\}$. Therefore, for any $X \in \mathbf{B}(I)$, we obtain $t(X) = \bigcap\{t(\{x\});\ x \in X\} = \bigcap\{\{f(Z);\ x \in Z \in \mathbf{Z}\};\ x \in X\} = \{f(Z);\ X \subseteq Z \in \mathbf{Z}\}$ by 1.3. □

Let $\langle I, O, f\rangle$, $\langle I, P, g\rangle$ be boxes, h a mapping of O into P such that $h(f(X)) = g(X)$ for any $X \in \mathbf{B}(I)$. Then h is said to be an *o-homomorphism* of $\langle I, O, f\rangle$ into $\langle I, P, g\rangle$. A bijective o-homomorphism is said to be an *o-isomorphism*. Two boxes are said to be *o-isomorphic* if there exists an o-isomorphism of the first onto the second.

7. THEOREM. *If* $\langle I, O, f\rangle$ *is an admissible box without parasitic outputs, then* $\langle I, O, f\rangle$ *and* $\mathscr{C}(\mathscr{E}(\langle I, O, f\rangle))$ *are o-isomorphic.*

Proof. Put $\mathscr{E}(\langle I, O, f\rangle) = (O, I, r)$, $\mathscr{C}((O, I, r)) = \langle I, \mathbf{C}, t\rangle$. For any $o \in O$, we set $h(o) = t(u(X))$ where $X \in \mathbf{B}(I)$ is an arbitrary element with $f(X) = o$ and u has been defined above. This definition is correct because $f(X) = o = f(Y)$ implies $u(X) = u(Y)$ and, hence, $t(u(X)) = t(u(Y))$.

Furthermore, if $o_1 \in O$, $o_2 \in O$ are such that $h(o_1) = h(o_2)$, there are $X_1 \in \mathbf{B}(I)$, $X_2 \in \mathbf{B}(I)$ with $f(X_1) = o_1$, $f(X_2) = o_2$, $h(o_1) = t(u(X_1))$, $h(o_2) = t(u(X_2))$. Thus, $t(u(X_1)) = t(u(X_2))$ which implies that $\{f(Z);\ u(X_1) \subseteq Z \in \mathbf{Z}\} = \{f(Z);\ u(X_2) \subseteq Z \in \mathbf{Z}\}$ by 6. Hence, there exists $Y \in \mathbf{Z}$ with $u(X_2) \subseteq Y$, $f(Y) = f(u(X_1))$, i.e., $(Y, u(X_1)) \in f^{-1} \circ f$. This implies that $Y \subseteq u(u(X_1)) = u(X_1)$ by 4 and, hence, $u(X_2) \subseteq u(X_1)$. Similarly, we obtain $u(X_1) \subseteq u(X_2)$ and, therefore, $u(X_1) = u(X_2)$. This entails $h(o_1) = t(u(X_1)) = t(u(X_2)) = h(o_2)$ and the mapping h is injective.

Clearly, $h(f(X)) = t(u(X)) = t(X)$ for any $X \in \mathbf{B}(I)$ by 5 and 6 and h is an o-homomorphism of $\langle I, O, f\rangle$ into $\mathscr{C}(\mathscr{E}(\langle I, O, f\rangle))$.

Finally, $\{h(o);\ o \in O\} = \{t(X);\ X \in \mathbf{B}(I)\} = \mathbf{C}$ and h is surjective. □

8. EXAMPLE. Let $\langle I, O, f\rangle$ be a box such that $I = \{a, b, c\}$, $O = \{o_1, o_2, o_3, o_4\}$, and f be given by the following table

$\emptyset$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$	I
o_1	o_1	o_2	o_3	o_2	o_3	o_4	o_4

It is easy to see that $f^{-1} \circ f$ is a congruence on $(\mathbf{B}(I), \cup)$ so that $\langle I, O, f\rangle$ is an admissible box without parasitic outputs by 2. Clearly, $\mathbf{Z} = \{\{a\}, \{a, b\}, \{a, c\}, I\}$. If putting $\mathscr{E}(\langle I, O, f\rangle) = (O, I, r)$, the table of r is as follows

r	a	b	c
o_1	1	0	0
o_2	1	1	0
o_3	1	0	1
o_4	1	1	1

We now construct the mapping t of the context (O, I, r). Clearly, $t(\emptyset) = O = t(\{a\})$, $t(\{b\}) = \{o_2, o_4\} = t(\{a, b\})$, $t(\{c\}) = \{o_3, o_4\} = t(\{a, c\})$, $t(\{b, c\}) = \{o_4\} = t(I)$. Hence, $\mathbf{C} = \{\{o_4\}, \{o_3, o_4\}, \{o_2, o_4\}, O\}$. Furthermore, $h(o_1) = O$, $h(o_2) = \{o_2, o_4\}$, $h(o_3) = \{o_3, o_4\}$, $h(o_4) = \{o_4\}$ and h is an o-isomorphism of $\langle I, O, f\rangle$ onto $\langle I, \mathbf{C}, t\rangle$. □

4. Good boxes. We now introduce some algebraic concepts for boxes. If $S = \langle I, O, f\rangle$ is a box and $J \subseteq I$, $J \neq \emptyset$, is a set, then $S' = \langle J, O, f \upharpoonright \mathbf{B}(J)\rangle$ is a box that is said to be a *subbox* of S; the symbol $f \upharpoonright \mathbf{B}(J)$ means the restriction of f to the set $\mathbf{B}(J)$. Clearly, for any $X \in \mathbf{B}(J)$, S and S' produce the same output; but, for $X \in \mathbf{B}(I) - \mathbf{B}(J)$, the box S' provides no output. This leads to the following definition.

Let $S = \langle I, O, f\rangle$, $S' = \langle J, O, g\rangle$ be boxes, h a mapping of I into J such that $f(X) = g(h[X])$ for any $X \in \mathbf{B}(I)$ where $h[X] = \{h(x);\ x \in X\}$. Then the mapping h is said to be an *i-homomorphism* of S into S'. Clearly, any output of S can be obtained by means of S'.

A subbox $S' = \langle J, O, g\rangle$ of a box $S = \langle I, O, f\rangle$ is said to *simulate* the activity of the box S if there exists an i-homomorphism h of S into S' such that $h \upharpoonright J = id_J$; this i-homomorphism will be said to be a *simulation* of S in S'.

1. LEMMA. *Let $\langle I, O, f\rangle$ be a box and h its simulation in its very good subbox $\langle J, O, g\rangle$. Then $\langle I, O, f\rangle$ is good.*

Proof. Suppose $f(X) = f(Y)$ and $x \in I$. Then $g(h[X]) = f(X) = f(Y) = g(h[Y])$ and the injectivity of g implies that $h[X] = h[Y]$ which entails $h[X \cup \{x\}] = h[X] \cup \{h(x)\} = h[Y] \cup \{h(x)\} = h[Y \cup \{x\}]$ and, thus, $f(X \cup \{x\}) = g(h[X \cup \{x\}]) = g(h[Y \cup \{x\}]) = f(X \cup \{x\})$. Therefore, $\langle I, O, f\rangle$ is admissible. Furthermore, if $f(X) = f(X \cup \{x\})$, then, similarly, $h[X] = h[X \cup \{x\}] = h[X] \cup \{h(x)\}$ which implies the existence of $y \in X$ with $h(y) = h(x)$. Thus, $f(\{y\}) = g(\{h(x)\}) = f(\{x\})$. Hence, the box is good. □

If I is a set, R an equivalence on I and J a subset of I such that $I \cap C$ has exactly one element for any $C \in I/R$, then J is said to be a set *of representatives of R-blocks.*

2. LEMMA. *Let $\langle I, O, f\rangle$ be an admissible box. We put $R = \{(x, y) \in I \times I;$*

$f(\{x\})=f(\{y\})\}$. Let J be a set of representatives of R-blocks and, for any $x\in I$, let $h(x)$ be such that $h(x)\in J\cap C$ where $x\in C\in I/R$.

Then h is a simulation of $\langle I,O,f\rangle$ in $\langle J,O,f\upharpoonright \mathbf{B}(J)\rangle$.

Proof. Clearly, $h\upharpoonright J = id_J$.

Let $X\in\mathbf{B}(I)$ be arbitrary. Then $h[X]\in\mathbf{B}(J)$ and, for any $x\in X$, we have $(x,h(x))\in R$, i.e., $f(\{x\})=f(\{h(x)\})$. By 2.4 (e) of [1]. we obtain $f(X)=f(h[X])$ which means that h is an i-homomorphism in of $\langle I,O,f\rangle$ into its subbox $\langle J,O,f\upharpoonright\mathbf{B}(J)\rangle$. □

We now deduce a criterion for recognizing good boxes among admissible ones.

3. THEOREM. *Let $\langle I,O,f\rangle$ be an admissible box. Let us put $R= =\{(x,y)\in I\times I;\ f(\{x\})=f(\{y\})\}$. Let J be a set of representatives of R-blocks and, for any $x\in I$, let $h(x)$ be such that $h(x)\in J\cap C$ where $x\in C\in I/R$.*

Then $\langle I,O,f\rangle$ is good if and only if the mapping $f\upharpoonright\mathbf{B}(J)$ is injective.

Proof. If $\langle I,O,f\rangle$ is good and $X\in\mathbf{B}(J)$, $Y\in\mathbf{B}(J)$ are such that $f(X)=f(Y)$, then, for any $x\in X$, we have $f(Y\cup\{x\})=f(X\cup\{x\})=f(X)= =f(Y)$. Thus, there exists $y\in Y$ with the property $f(\{y\})=f(\{x\})$, i.e., $(x,y)\in R$. Hence, there exists $C\in I/R$ such that $x\in C\cap J$, $y\in C\cap J$. By definition of J, we obtain $x=y$. Thus $X\subseteq Y$ and, similarly, $Y\subseteq X$. Hence, $X=Y$ and $f\upharpoonright\mathbf{B}(J)$ is an injective mapping.

If $f\upharpoonright\mathbf{B}(J)$ is injective, then $\langle J,O,f\upharpoonright\mathbf{B}(J)\rangle$ is a very good box and $\langle I,O,f\rangle$ is good by 1 and 2. □

A context T is said to be *good* if the box $\mathcal{C}(T)$ is good.

From 3, we deduce an *algorithm for recognizing good contexts.*

Let a context (G,M,r) be given. Construct $R=\{(m_i,m_j)\in M\times M;$ $a_{ki}=a_{kj}$ for any $k\in\{1,\dots,m\}\}$, $M/R=\{C_1,\dots,C_p\}$, $J=\{m_{l_1},\dots,m_{l_p}\}$ where $m_{l_t}\in C_t$ for any $t\in\{1,\dots,p\}$ (we may choose l_t to be the least index $h\in\{1,\dots,n\}$ such that $m_h\in C_t$). For any $N\subseteq J$ and any $i\in\{1,\dots,m\}$ define a_{iN} by 2.1. For any $N\in\mathbf{B}(J)$, $N'\in\mathbf{B}(J)$, $N\neq N'$ test whether $a_{iN}=a_{iN'}$ for any $i\in\{1,\dots,m\}$; if such sets N,N' exist, the context is not good; in the opposite case it is good.

4. EXAMPLE. Let us consider the context of 1.2. Then the blocks of R are $\{m_1,m_2\}$, $\{m_3,m_4\}$, $\{m_5\}$. We may take $J=\{m_1,m_3,m_5\}$ and, clearly, $a_{i\{m_1,m_3\}}=0=a_{i\{m_1,m_3,m_5\}}$ for any $i\in\{1,\dots,4\}$. Thus, the context is not good. □

5. EXAMPLE. Let us consider the context of 1.3. Then the blocks of R are $\{m_1,m_2\}$, $\{m_3,m_4\}$, $\{m_5\}$. We may take $J=\{m_1,m_3,m_5\}$. We form the table of values a_{iN} for all subsets N of J.

	$\emptyset$	$\{m_1\}$	$\{m_3\}$	$\{m_5\}$	$\{m_1,m_3\}$	$\{m_1,m_5\}$	$\{m_3,m_5\}$	J
g_1	1	1	0	1	0	1	0	0
g_2	1	0	1	1	0	0	1	0
g_3	1	1	1	0	1	0	0	0
g_4	1	0	1	0	0	0	0	0

Clearly, all columns are different and the context is good. □

6. THEOREM. *Let $T=(G,M,r)$ be a good context. We put $R=\{(x,y)\in M\times M;\ (g,x)\in r$ and only if $(g,y)\in r$ for any $g\in G\}$. Let us have $N_1\subseteq M$, $N_2\subseteq M$. Then N_1 and N_2 generate the same concept if and only if $N_1\cap C\neq\emptyset$, $N_2\cap C\neq\emptyset$ are equivalent for any $C\in M/R$.*

Proof. We put $\mathcal{C}(T)=\langle M,\mathbf{C},t\rangle$. Then $(x,y)\in\mathbf{R}$ means $t(\{x\})=t(\{y\})$. Let J be an arbitrary set of representatives of R-blocks and, for any $x\in M$, let $h(x)$ be such that $h(x)\in J\cap C$ where $x\in C\in M/R$.

By 2, h is a simulation of $\langle M,\mathbf{C},t\rangle$ in $\langle J,\mathbf{C},t\restriction\mathbf{B}(J)\rangle$ and $t\restriction\mathbf{B}(J)$ is injective by 3. Thus, $t(N_i)=t(h[N_i])$ for $i\in\{1,2\}$.

Suppose $N_1\cap C\neq\emptyset$ if and only if $N_2\cap C\neq\emptyset$ for any $C\in M/R$. If $x\in N_1\cap C$, then there exists $x'\in N_2\cap C$ and $h(x)=h(x')$ which means that $h[N_1]\subseteq h[N_2]$; similarly $h[N_2]\subseteq h[N_1]$ holds. Thus, $h[N_1]=h[N_2]$ and, thus $t(N_1)=t(N_2)$ holds.

If, conversely, $t(N_1)=t(N_2)$ holds, we have $t(h[N_1])=t(h[N_2])$. Since $h[N_1]\in\mathbf{B}(J)$, $h[N_2]\in\mathbf{B}(J)$ and $t\restriction\mathbf{B}(J)$ is injective, we have $h[N_1]=h[N_2]$. If $x\in C\cap N_1$ where $C\in M/R$, then $h(x)\in h[N_1]$ and there exists $y\in N_2$ such that $h(y)=h(x)$ which implies that $t(\{x\})=t(\{h(x)\})=t(\{h(y)\})=t(\{y\})$ and, hence, $(x,y)\in R$ which entails $y\in C$. Thus, $C\cap N_1\neq\emptyset$ implies that $C\cap N_2\neq\emptyset$ and, similarly, the reverse implication holds. □

Let a good context (G,M,r) be given. We construct $R=\{(m_i,m_j)\in M\times M;\ a_{ki}=a_{kj}$ for any $k\in\{1,\dots,m\}\}$, $M/R=\{C_1,\dots,C_p\}$.

Then the *algorithm for recognizing whether $t(N_1)=t(N_2)$* or not is as follows.

For any $h\in\{1,\dots,p\}$ test whether $N_1\cap C_h=\emptyset$, $N_2\cap C_h\neq\emptyset$ or $N_1\cap C_h\neq=\emptyset$, $N_2\cap C_h=\emptyset$. If such h exists, then $t(N_1)\neq t(N_2)$; if not, then $t(N_1)=t(N_2)$.

Clearly, this algorithm is simpler than the algorithm deduced in **2** for a general context.

7. EXAMPLE. Let (G,M,r) be the context from 5. Let us have $N_1=\{m_1,m_2,m_3\}$, $N_2=\{m_2,m_4\}$. Then $N_1\cap\{m_1,m_2\}\neq\emptyset\neq N_2\cap\{m_1,m_2\}$, $N_1\cap\{m_3,m_4\}\neq\emptyset\neq N_2\cap\{m_3,m_4\}$, $N_1\cap\{m_5\}=\emptyset=N_2\cap\{m_5\}$. Thus, N_1, N_2 generate the same concept. □

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М. Новотны, З. Павляк, **Образование понятий и "черные ящики"**

"Черные ящики", введеные в работе [1] используются для решения задачи, когда два множества свойств порождают то же самое понятие (=концепцию). Эта задача имеет простое решение, когда концепции образуются на основе, так называемых хороших контекстов. Дается описание алгоритма распознавания хороших контекстов и решения задачи, когда два множества свойств в хорошем контексте порождают или нет ту же самую концепцию. Выяснено отношение контекстов и, так называемых, „допустимых ящиков".