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Jones, Lowell

The converse to the fixed point theorem of P. A. Smith. I.

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Disks are of dimension at least 6. The working category is the PL category. If K is a finite CW complex of codimension ≥ 3 and if $H_*(K, Z_n) = 0$ there exists a contractible finite CW complex X containing K as a subcomplex and a cellular periodic homeomorphism $\beta: X \to X$ of period n with K as the fixed point set and the orbit of x, $x \in K$, contains n points. The idea is to add cells equivariantly to kill off the fundamental group and all homology groups. Here β is determined by the equivariant construction. The success of the program is based on the fact that the desired construction is effected after a finite number of steps. The technical argument involves a notion of resolution. Thus X_j, α_j is a j-resolution if X_j is a finite CW complex containing K as a subcomplex with trivial homotopy below $j+1$ and α_j on $X_j \to X_j$ has K as its fixed point set, where the integer homology of K and X_j are the same in dimensions above j. An induction argument is used to construct these resolutions successively till $j = l$, for which $H_i(K, Z) = 0$, $i \geq l$. For K a finite simplicial complex, X is a disk. Two other papers are planned continuing the general program of classifying β type homeomorphisms induced by cyclic groups on D^r and S^r, assuming a nonempty fixed point set.

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