Multiwinner Election Rules: Beyond Scoring Protocols

Research Outline

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1 Scientific Goal of the Project

The project is set in the field of computational social choice and its main topic is a formal (axiomatic and algorithmic) analysis of interesting and advanced multiwinner voting systems which are not extensions of single-winner scoring protocols. Our focus will be on rules based on the classic idea of Condorcet-consistency, and on several modern approaches, such as priceability, that only very recently have been suggested in the literature [63].

A *multiwinner election rule* is a function (an algorithm) that takes as input a collection of voters' preferences over a predefined set of candidates and an integer k specifying the desired size of the committee to be elected. It outputs a k-element subset of the set of candidates—the returned subset is called the *winning committee*. Multiwinner election rules are useful tools that can be applied to problems from and beyond the political domain. Example applications outside of the political domain include preparing a list of items a search engine should display in response to a query [31, 75] (where the profiles of potential users of a search engine correspond to the voters, and the items to be displayed—to candidates), solving a wide range of resource allocation problems [72, 61], and improving genetic algorithms [37]. We will discuss the applications of multiwinner rules in more detail in Section 2.3.

A special case of multiwinner elections is when the size of the committee to be elected equals $k = 1$; in such a case we speak of *single-winner elections*. Historically, the social choice theory focused on the analysis of single-winner voting rules, and this area of the field is nowadays well explored. Currently, our understanding of multiwinner rules is far less advanced than that of single-winner methods, but the situation is changing rapidly. The model of multiwinner elections attracted researchers from many research groups,[1] and a great progress in our understanding of the model has been made in recent years [40, 3]. Nevertheless, there are still many basic and fundamental questions concerning multiwinner voting rules, that have not yet been addressed in the literature. In particular, nowadays, only a very specific class of multiwinner rules—the class of committee scoring rules—is well understood. Indeed, this class is fairly well explored both axiomatically [33, 74, 30, 41, 55, 54, 53, 39, 18, 38, 4, 66] and algorithmically [39, 18, 73, 6, 16, 76, 17, 72, 71, 62, 80, 69, 21, 36, 24, 7]. On the other hand, the ideas that played an instrumental role in the development of the single-winner voting theory, such as the Condorcet principle, have not yet been properly analyzed in the context of multiwinner elections.

Consider the model when the voters rank the candidates from the most to the least preferred one. For single-winner elections the two main classes of voting rules are (1) scoring methods, and (2) Condorcet rules. According to scoring rules, each voter awards each candidate with a certain number of points that depends on the position that the candidate occupies in the voter's preference ranking. The candidate with the highest number of points garnered from all the voters is the winner. Various scoring rules differ in a way in which the points are awarded to the candidates depending on their positions in the voters' preference rankings (we will formalize this concept in Section 2.2.1). The definitions of Condorcet rules are, on the other hand, based on the (very different) idea of pairwise comparisons. If there exists a candidate c such that c is preferred to any other candidate by a majority of voters, then we call c the *Condorcet winner*. We say that a single-winner rule is Condorcet-consistent if it always selects the Condorcet winner whenever such exists. Since the Condorcet winner might not exist, a voting rule must also specify the outcome of an election in the absence of the Condorcet winner—various rules handle

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[1]The institutes were multiwinner election rules are actively studied include: University of Oxford, Carnegie Mellon university, the University of Auckland, TU Berlin, Paris School of Economics, Université Paris-IX Dauphine, UNSW Sydney, Universität Düsseldorf, Weizmann Institute of Science, The Hebrew University of Jerusalem, Rensselaer Polytechnic Institute, Athens University of Economics and Business, IIT Gandhinagar, AGH University Kraków, etc.

these cases differently, which makes the class of Condorcet-consistent rules broad (we discuss Condorcet rules in Section 2.2.2). The two classes of rules are inherently very different, which we will explain in Section 2.2.3.

In the model of single-winner elections, the Condorcet's idea appeared very influential, and the class of Condorcet rules received perhaps the greatest attention from the scientific community (see the discussion in Section 2.2.3). Yet, quite surprisingly, in the world of multiwinner elections, research on rules based on the Condorcet principle is still scarce (we review the related literature in Section 2.4.2). The same comment applies to other interesting rules, such as rules based on iterative eliminations, and—in a somehow different model, where the voters express their preferences through approving candidates rather than by ranking them—to those based on the market approach (cf., the definition of the Phragmén's rule in Section 2.4.2 and the discussion in Section 3.2).

Similarly, as in the single-winner setting the multiwinner rules based on the Condorcet principle are very different from multiwinner scoring rules. We anticipate that they satisfy very different appealing properties from those satisfied by multiwinner scoring protocols. Further, we believe that our research on non-scoring multiwinner rules will give new tools useful in a broad class of scenarios where multiwinner rules are applicable. On the other hand, the rules based on the Condorcet principle are also mathematically more involved, which makes their analysis more challenging. This justifies why we plan to devote a considerable attention to them and this serves as a primary motivation for our research proposal.

Our goal is to algorithmically and axiomatically explore the classes of non-scoring multiwinner rules, in general, and multiwinner rules based on the Condorcet principle, in particular.

2 Significance of the Project

In this section we formalize our model, motivate our research tasks, provide an overview of the state of the art, and discuss the impact of the expected results on the development of the field of computational social choice.

2.1 The Model

A multiwinner election is a triple (C, V, k), where $C = \{c_1, c_2, \ldots, c_m\}$ is a set of candidates, $V = \{v_1, v_2, \ldots, v_n\}$ is a set of voters, and $k \in \mathbb{N}$ is the intended size of the committee to be elected (throughout the proposal, we will use the convention that n and m denote the number of voters, and the number of candidates, respectively). The voters are represented through their preferences. In perhaps the most classic *ranking-based model* the voters express their preferences by ranking the candidates from the most to the least preferred ones. For example, when we write $v_i \colon c_1 \succ c_3 \succ c_2$, we mean that c_1 is the most preferred candidate from the perspective of voter v_i, c_3 is v_i's second most preferred candidate and c_2 is the least preferred among the three candidates. In the *approval based-model* the voters do not provide rankings but rather subsets of candidates—such subsets are called approval sets and, intuitively, the approval set of a voter v_i consists of those candidates that v_i finds acceptable.

Given an election $E = (V, C, k)$ we call size-k subsets of C *committees*. A multiwinner election rule $\mathcal{R}$ is a function (an algorithm) that takes as input an election $E = (V, C, k)$ and returns a set of committees. Given an election E, we call the elements of $\mathcal{R}(E)$ *winning committees*. Typically, one would like to select a single winning committee, but sometimes it is more convenient to allow for ties and not to assume that there is a tie-breaking mechanism that handles the cases where e.g., the input election is symmetric. Intuitively, the elected committee should represent the opinions of the voters, yet the way in which this process is exactly handled may vary between various multiwinner rules, and might strongly depend on the context (we will elaborate on that in Section 2.3).

When the committee size k equals one, we speak of single-winner elections, and analogously, about single-winner election (voting) rules. The analysis of single-winner rules is not the primary purpose of this proposal, but we will provide a brief discussion on single-winner elections in order to give the reader the necessary context.

2.2 Single-Winner Elections: Scoring Methods versus Condorcet Rules

In this section we define and briefly discuss two classes of single-winner voting methods: (1) scoring rules, and (2) rules based on the Condorcet principle. We explain that these two classes of rules are fundamentally different, and justify why the Condorcet rules are important. These claims will demonstrate why it is critical to study Con-

dorcet rules, and since the class of Condorcet multiwinner rules is not yet well understood, they will motivate our objectives.

In this section we focus solely on the ranking-based model. However, our preliminary results [63] suggest that the main conceptual difference between single-winner scoring rules and single-winner Condorcet rules also appears among approval-based multiwinner rules—we will explain this difference in more detail in Section 3.2.

2.2.1 Scoring Rules

For a natural number $p \in \mathbb{N}$, by $[p]$ we denote the set $\{1, 2, \ldots, p\}$.

Given a voter v_i, and a candidate c, by $\mathrm{pos}_i(c)$ we denote the position of c in v_i's preference ranking: If c is the most preferred candidate from v_i's point of view, then $\mathrm{pos}_i(c) = 1$, if c is her second most preferred choice, then $\mathrm{pos}_i(c) = 2$, whereas if c is the most disliked candidate by v_i, then $\mathrm{pos}_i(c) = m$ (recall that m denotes the number of candidates). Consider a function $\lambda \colon [m] \to \mathbb{R}$; a scoring rule based on positional scoring function λ is defined as follows. For each candidate c we compute the λ-score of c as $\lambda\text{-sc}(c) = \sum_{v_i \in V} \lambda(\mathrm{pos}_i(c))$, and the candidates with the highest scores are announced winners. Several important rules used in real-life are scoring rules:

Borda. This is the rule based on a linear positional scoring function $\beta(p) = m - p$.

t-Approval. According to this rule a voter assigns one point to a candidate c if she ranks c among the top t positions. Formally, it is a scoring rule based on the function α_t such that $\alpha_t(p) = 1$ if $p \leq t$ and $\alpha_t(p) = 0$, otherwise. A special case of t-Approval, for $t = 1$ is called Plurality, and for $t = m - 1$ it is called Veto.

2.2.2 Condorcet Based Rules

Another class of rules is inspired by the idea of pairwise comparisons. A candidate c who is preferred to any other candidate by a majority of voters, is called the *Condorcet winner*. Such candidate, if exists, is always unique. E.g., in election (a), below, c_1 is the Condorcet winner, as a majority of voters prefers c_1 to c_2 (voters v_1 and v_2) and a majority of voters prefers c_1 to c_3 (v_1 and v_3). On the other hand, in election (b) there is no Condorcet winner.

$$(a)$$
$$v_1 \colon c_1 \succ c_2 \succ c_3$$
$$v_2 \colon c_3 \succ c_1 \succ c_2$$
$$v_3 \colon c_2 \succ c_1 \succ c_3$$

$$(b)$$
$$v_1 \colon c_1 \succ c_2 \succ c_3$$
$$v_2 \colon c_2 \succ c_3 \succ c_1$$
$$v_3 \colon c_3 \succ c_1 \succ c_2$$

We say that a voting rule is Condorcet-consistent if it always selects the Condorcet winner whenever such exists.

The Condorcet's idea was very influential. Nowadays many Condorcet-consistent rules are known and they are well understood from the theoretical perspective (examples include, Kemeny's rule, Schulze's method, Copeland's rule, Ranked Pairs, Minimax, Baldwin's method, or Tideman's alternative method). Many of these rules are used in practice (for example, the list of users of the Schulze's method includes over 60 organizations and institutions).

2.2.3 Comparison of the Two Classes

The difference between the scoring rules and the Condorcet-consistent rules is inherent. It is known that no scoring rule is Condorcet-consistent, and vice-versa—some properties which are typical for scoring rules (such as consistency [79] or participation [1]) cannot be satisfied by Condorcet-consistent rules. The two classes of rules are very different in nature and they exhibit very different properties. For more discussion on the differences between single-winner Condorcet-consistent and scoring rules we refer to the book chapter by Zwicker [81].

Historically, the analysis of differences between single-winner scoring methods, and Condorcet-consistent rules has played a major role on the development of the field of social choice. It is not an exaggeration to say that the whole field started from the scientific dispute in the 18th century between Jean-Charles de Borda and Nicolas de Condorcet [81] regarding single-winner voting rules. Many years later the most influential works in the field asked the very same question, trying to establish which of the two classes of rules is better. The Condorcet's idea was considered fundamental by some of the most prominent figures in the area. For example, Kenneth Arrow wrote that "(...) one of the consequences of the assumptions of rational choice is that the choice in any environment

can be determined by a knowledge of the choices in two-element environments (...)", and the concept of pairwise comparisons gave rise to the subfield of social choice that studied various tournament solution concepts [57].

While the dispute on which single-winner rule is the best is still active, nowadays some experts agree that using Condorcet-based rules is a better choice than using scoring protocols. Just to give an example, during 2010 VPP Workshop on Assessing Alternative Voting Procedures, the participants (who were mostly experts in the field) used approval voting to vote on the following question "What is the best voting rule that the city council of your town should use to elect the mayor?". Among the four most-approved choices there were two Condorcet-consistent rules, and not a single scoring method. The best rule according to the respondents was Approval Voting—a rule which takes a different kind of input—we will elaborate on this in the further part of the proposal.

Indeed, Condorcet-consistent rules have several advantages over the scoring rules. The Condorcet-consistency on its own is a very desired property. Further, Condorcet rules are usually much more resilient to cloning (some of them, like the Schulze's method or Ranked Pairs are provably clone-proof)—this property is very important especially in the political context. Finally, the Condorcet-consistent rules are much better in terms of the distortion [2], i.e., they find much better candidates when voters' preferences are induced by spatial models of opinions [29, 67].

Given the above arguments, we anticipate that studies on Condorcet rules will have a similar effect on the area of multiwinner voting as they had on the area of single-winner social choice. We further believe that these studies will give rise to new multiwinner rules that would exhibit better properties than the known scoring methods.

2.3 Multiwinner Elections: Motivation

In this section we discuss three main classes of potential application domains of multiwinner rules [40]. Following the literature, we refer to these classes using the names: (1) proportionality, (2) diversity, and (3) individual excellence. Intuitively, these names describe high-level principles that rules suitable for respective classes of applications should follow. We start by examining two simple examples that explain different natures of multiwinner rules in the light of the three principles. Then we describe the three categories in more detail.

Example 1. Consider the following approval-based election. We have 9 candidates $C = C_1 \cup C_2 \cup C_3$, with $|C_1| = |C_2| = |C_3| = 3$, and the voters' preferences are as follows: 70 voters approve C_1, 29 voters approve C_2 and a single voter approves C_3. The goal is to select $k = 3$ committee members. Which committee should be chosen for this particular election? Here, the answer strongly depends on the context. If our goal is to select finalists in a competition (in such a case, the judges/experts act as the voters and the contestants act as candidates) it seems natural to consider the candidates separately, and to select those candidates that are the best as judged by the experts—in this example, this would correspond to choosing the candidates from C_1, since those candidates are approved by most voters. If a rule selects this committee, we say that it follows the principle of individual excellence. On the other hand, if our goal is to select a representative body for a group of voters, the selected committee should be proportional. Intuitively, this would be a committee that consists of two candidates from C_1 and one candidate from C_2—indeed according to this choice the number of committee members selected from each group C_i is roughly proportional to the number of voters who approve this group. Finally, if our goal is to represent the widest possible spectrum of opinions present in the society (e.g., this is arguably the case in the deliberative democracy [25, 61]) then we should perhaps choose one candidate from each of the three sets. □

The above example is very simple, due to the very specific structure of the preferences—each two approval sets are either the same or disjoint. Defining desired outcomes for such elections is intuitively easy. The challenge is to design rules that behave well also for more complex elections, when the approval sets can arbitrarily overlap.

The second example is similar in nature, but it describes a ranking-based election; this example only illustrates the difference between individual excellence and diversity. For ranked preferences it is harder to intuitively reason about proportionality. Our preliminary results suggest that there are at least two distinct ways in which the concept of proportionality can be understood [42]; as a part of this project we plan to formalize these two interpretations.

Example 2. Consider the following election with 5 voters and 20 candidates. The committee size is $k = 5$:

$$v_1 : c_1 \succ c_{10} \succ c_{11} \succ \ldots \succ c_{20} \succ c_2 \succ c_3 \ldots \succ c_9$$
$$v_2 : c_2 \succ c_{10} \succ c_{11} \succ \ldots \succ c_{20} \succ c_1 \succ c_3 \ldots \succ c_9$$
$$v_3 : c_3 \succ c_{10} \succ c_{11} \succ \ldots \succ c_{20} \succ c_1 \succ c_2 \succ c_4 \succ \ldots \succ c_9$$
$$v_4 : c_4 \succ c_{10} \succ c_{11} \succ \ldots \succ c_{20} \succ c_1 \succ \ldots \succ c_3 \succ c_5 \succ \ldots \succ c_9$$
$$v_5 : c_5 \succ c_{10} \succ c_{11} \succ \ldots \succ c_{20} \succ c_1 \succ \ldots \succ c_4 \succ c_6 \succ \ldots \succ c_9$$

Which committee should be elected here? Is it better to pick a consensus committee $W_1 = \{c_{10}, c_{11}, c_{12}, c_{13}, c_{14}\}$ or to aim for a greater diversity by choosing $W_2 = \{c_1, c_2, c_3, c_4, c_5\}$? There are arguments for and against each of these two possible choices. For example, each voter is reasonably pleased with all the candidates from W_1, but no voter feels really represented by any member of this committee. On the other hand, no member of W_2 is particularly liked by the voters (in fact, for each candidate $c \in W_2$ as much as 80% of voters prefer all members of W_1 to c), but each voter considers *some* member of W_2 best, and could feel represented by such a candidate. In this example, the choice between W_1 and W_2 should again depend on the context. If the above preferences describe opinions of the judges over contestants, then W_1 seems like a better choice. If the candidates correspond to potential locations of public facilities of a certain type, and if we know that each voter will most likely use only one (the most preferred) facility, then selecting W_2 would be a better choice. This would also be the case for deliberative democracy. □

Examples 1 and 2 illustrate how important it is to have good insights into the nature and properties of multiwinner voting rules. Such rules might have very different objectives, and understanding which rule is suitable for which application is of the critical importance; especially, when one wants to decide in a principled way which rule should be used given a particular application domain. For instance, a rule that is particularly good for selecting finalists in a competition could perform rather poorly in other contexts, e.g., when the goal is to elect a representative body such as a parliament, or a faculty board. These differences and a variety of applications of multiwinner rules have been recognized in the literature, and the three principles—proportionality, diversity, and individual excellence—have been identified. Below we explain these three principles in more detail, and we recall a few example scenarios that are often discussed in the literature in the context of the respective principles.

Proportionality. When the goal is to elect a representative body such as a supervisory board, a trade union, or a parliament, the rule to be chosen should be proportional. Typically, in high-stake domains, such as parliamentary elections, proportionality is achieved by using party-list elections and by employing one of the proportional apportionment methods [9, 64]. This solution has certain drawbacks, e.g., the elected candidates feel more accountable to their parties than to the voters, especially in the presence of a voting discipline. An alternative solution is to allow the voters to vote for individual candidates rather than for parties, and to use a proportional multiwinner election rule. In Example 1 a proportional outcome would be to select two candidates from C_1 and one candidate from C_2.

Diversity. Different multiwinner rules are suitable when the goal is to select a diverse committee. Here, example applications include deciding which set of products a company should offer to its customers [58, 59][2] or deciding where to locate public facilities. For such applications we are less interested in giving more representatives to large groups of voters with coherent opinions; the primary goal is rather to maximize the number of voters (customers/citizens) that can find a good candidate (product/facility) among those selected. In Example 1 a diverse rule would select one candidate from each of the three sets, C_1, C_2, and C_3. In Example 2, W_2 is a diverse committee.

Individual Excellence. Yet another application domain is when the goal is to prepare a shortlist of candidates to be invited for an interview, or to choose a set of finalists of a competition. In such cases our election methods should focus on the merits of individual candidates as judged by the voters. In particular, except for some borderline cases, two similar candidates (i.e., those judged similarly by the voters) should either be both selected to the winning committee or both should be omitted. This is very different from how diverse or proportional rules operate. A rule oriented towards individual excellence would pick in Example 1 committee C_1, and in Example 2, committee W_1.

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[2]As a concrete example, consider a company producing mobile phones, and imagine the company faces the following decision: "Which mobile phones should be pushed to production?". This is a multiwinner election problem, where the prototypes of mobile phones correspond to the candidates, and the potential customers—to the voters. In this scenario it is reasonable to assume that most customers will buy only a single phone from those available on the market. Thus the company should release a possibly diverse spectrum of products.

Tradeoffs Between the Principles. In many applications one needs a voting rule that achieves a certain degree of diversity, individual excellence and proportionality. For example, when judging project proposals we are primarily concerned about selecting proposals with the high overall support from the experts. Yet, we would also like to treat fairly proposals from less popular subfields (that can be underrepresented among the voters). Interestingly, there are works that analyze the spectrum of scoring rules between diversity, individual excellence and proportionality [39, 55, 54, 52], but to the best of our knowledge such an analysis does not yet exist for non-scoring methods.

2.4 Multiwinner Rules: State of the Art

In this section we first define and briefly discuss multiwinner scoring rules; for a comprehensive overview of the literature on this class of rules we refer to the two recent surveys [40, 3]. Next, we review the much more modest literature on non-scoring multiwinner protocols.

2.4.1 Overview of Committee Scoring Rules

For single-winner rules, the definition of scoring methods bases on the concept of "position in a ranking". This concept can be extended to apply to committees, yet in this setting the definition becomes a bit more involved. For a committee W by position of W in v_i's preference order we mean the set of positions that members of W occupy in v_i's ranking. Formally, $\text{pos}_i(W) = \{\text{pos}_i(c) : c \in W\}$. Given a function $\lambda : S_k([m]) \to \mathbb{R}$ we define the committee scoring rule based on λ analogously to the single-winner case. For each committee W we compute the score of W as $\lambda\text{-sc}(W) = \sum_{v_i \in V} \lambda(\text{pos}_i(W))$, and pick the committees with the highest score. Interestingly, this class of multiwinner voting rules is very broad and includes rules aimed at achieving quite different goals. We demonstrate this giving the following two examples:

k-Borda. This rule picks k candidates with the highest Borda scores. Formally, it is implemented by the function $\lambda_{\text{Borda}}(S) = \sum_{p \in S} \beta(p)$. It is an example of perhaps the simplest subclass of *separable* committee scoring rules [41], and it is oriented towards individual excellence. For example, in Example 2 it would pick W_1.

Chamberlin–Courant rule [25]. The idea behind the Chamberlin–Courant system is that each voter v_i judges the quality of a committee W based on her most preferred member of W, called *the representative* of v_i in W. Formally, this is a committee scoring rule based on function $\lambda_{\text{CC}}(S) = \max_{p \in S} \beta(p)$. In their paper, Chamberlin and Courant argued that their system is particularly suitable for *deliberative democracies*, i.e., for selecting committees whose main task is to discuss issues, argue and deliberate, rather than to make the decisions via majority voting. A reader might also notice a resemblance between finding winners according to the Chamberlin–Courant rule and the known facility location problem [44]. Indeed, this is a prime example of diverse-oriented rules; in particular, in Example 2 the Chamberlin–Courant rule would pick W_2.

We note that in the approval model an analogous class to committee scoring rules exists [54]. We will not describe this class in detail, but we only mention that there the prime examples of rules aimed at proportionality, diversity and individual excellence, are respectively, Proportional Approval Voting (PAV), Approval Chamberlin–Courant rule (Approval CC), and Multiwinner Approval Voting (MAV).

2.4.2 Beyond Scoring Rules

We start by reviewing the literature on multiwinner rules that are inspired by the Condorcet principle. Here, most of the proposed axioms and rules are suitable for scenarios where the goal is to select a committee of high-quality individuals. One approach to defining a Condorcet committee W is to require that each member of W should be preferred by a majority of voters to each candidate outside of W [50, 65, 10]. An alternative approach is to define the Condorcet committee as such that is preferred to any other committee by a majority of voters [46, 45, 28, 68] (in order to apply this approach in our setting one needs to specify how the preferences over the individual candidates are lifted to preferences over committees). Two papers that apply the diversity-oriented approach to the idea of Condorcet consistency stand out of this literature [35, 5]. However, the voting rules suggested there are NP-hard to compute. For the time being no approximation nor parameterized algorithms for computing those rules are known.

Other multiwinner ranking-based rules that do not fall into the category of committee scoring rules are, for example, the Monroe's rule and STV (for the sake of brevity, we do not define these rules in the proposal; their definitions—which can be found in the book chapter [40]—are not critical for formulating our research goals). While STV is a fairly well understood rule [78, 77, 32, 33], to the best of our knowledge no other multiwinner rules based on iterative eliminations have been considered in the literature. For the case of Monroe's rule the axiomatic studies are less advanced [32, 33]. Most of current research focused on overcoming its computational complexity: the problem has been studied from the perspective of approximation [73], parameterized complexity [14], and under assumptions that voters' preferences have a certain structure [76]. Very recently Brill at al. [19] analyzed the rules similar to Monroe but with relaxed quota constraints. What is not yet fully understood is how to compare the types of proportionality offered by STV, the Monroe's rule and other rules based on scoring protocols. Only preliminary studies have been initiated [42]; our plan is to continue this line of research.

Let us now move to the approval model. Among others, we plan to analyze rules inspired by the ideas suggested already in the 19th century by a Swedish mathematician Lars Edvard Phragmén. In one of several equivalent formulations of the Phragmén's rule, voters continuously earn money with the speed of $1 per one time unit. In the first time moment when there is a group of voters S satisfying the following two conditions (1) there exists a not-yet selected candidate c who is approved by each member of S, and (2) the members of S have in total at least (by continuity, exactly) n dollars, the rule pauses, adds c to the committee, and asks the voters from S to pay for c (resetting their budgets to zero); then the rule resumes. It stops when k candidates are selected.

For a summarized overview of the initial works of Phragmén we refer the reader to the recent survey [51]. Recent research on the subject had been initiated by Brill at al. [20], but their findings were inconclusive: they showed that the Phragmén's rule does not satisfy a rather strong property of Extended Justified Representation, and that the rule satisfies its much weaker variant—Proportional Justified Representation (we do not define these properties formally in order not to disturb the reader; the properties have been defined in the following two papers [4, 66]). More recently Skowron at al. [75] and Skowron [70] have strengthen these results, proving that under some interpretation the Phragmén's rule can be considered twice less proportional than PAV. Peters and Skowron [63] tackled the question of comparison of the Phragmén's rule and PAV with some generality—they introduced several axioms, including priceability. A further exploration of priceable rules is one of the primary goals of this proposal.

Finally, let us mention that there exist other approval based voting rules, such as Minimax Approval Voting [22, 27, 11, 12], but we will not be focusing on them within this proposal.

2.5 Expected Impact

As we have argued in Section 2.3 multiwinner elections have vast applications in and beyond the political domain. Having good rules with desired properties is thus very important. The existence of such good rules is also one of the primary postulates of the recent move for interactive democracy. Further, since committee elections are a special case of participatory budgeting (PB) [23], good understanding of multiwinner rules is a prerequisite for designing effective PB methods. Finally, our research can contribute to the literature on market design as the analysis of priceable rules is a first step towards analysis of more general markets for public (shared) indivisible goods.

We plan to publish our results in conferences and journals in the following three areas:

1. Artificial Intelligence and Multiagent Systems. Here the three most recognized journals are Artificial Intelligence, Journal of Artificial Intelligence Research, and Journal of Autonomous and Multi Agent Systems. The top-tier conferences include IJCAI, AAAI, and AAMAS.

2. Theoretical Computer Science with the focus on Algorithmic Game Theory. The main conferences in this field include ACM EC, WINE, and SAGT. The main field journal here is ACM Transactions on Economics and Computation, but the extended versions of articles presented at EC, WINE, and SAGT are also sometimes published in classic theoretical computer science journals.

3. Theoretical Economics, with the following example journals: Journal of Economic Theory, Theoretical Economics, Economic Theory, Games and Economic Behavior, Social Choice and Welfare.

3 Work Plan

In this section we discuss our research plan and specify concrete tasks that we plan to consider.

3.1 Exploring Multiwinner Condorcet Rules

Our first goal is to find a spectrum of Condorcet-based multiwinner rules that spreads between proportionality, individual excellence, and diversity. Such a spectrum can be defined in multiple ways for committee scoring rules [39], but an analogous class of the Condorcet based rules is unknown. So far, most of the literature focused on extending the Condorcet principle in the spirit of individual excellence. For example, in the approach suggested by Gehrlein [50] the candidates are considered independently, and it is required that each of them is preferred by a majority of voters to each candidate outside of the committee. The examples of such rules include:

Minimal Number of External Defeats (NED) [26]. The NED score of a committee W is the number of pairs (c, c'), $c \in W, c' \notin W$, such that c is preferred by a majority of voters to c'. Committee with the highest NED score win the election.

Minimal Size of External Opposition (SEO) [26]. According to SEO, the score of a committee W is defined as $\min_{c \in W, c' \in C \setminus W} |\{v \in V : c \succ c'\}|$. Similarly, as before the committee with the highest score is winning.

While the definitions of NED and SEO are rather complex, and (in order to keep our description concise) we omit their intuitive explanations, we will highlight their main distinctive property: The two rules defined above are very majoritarian. If 51% of voters rank some k candidates on top of their preference lists, then these candidates will be selected by NED and SEO, independently of the preferences of the remaining voters. In particular, in Example 2 both rules would select W_1. Only a few recent works propose alternative approaches to extending the Condorcet principle [35, 5] in a way that would protect the minorities. Those approaches feel diversity-oriented:

Locally Stable Extension of Copeland (LSE-Copeland) [5]. Here the score of a committee W is the number of candidates $c' \notin W$ such that at least $n - \left\lfloor \frac{n}{k+1} \right\rfloor$ voters prefer some member of W to c'.

Locally Stable Extension of Maximin (LSE-Maximin) [35]. A committee W is a θ-winning set if for each candidate $c' \notin W$ there are more than θn voters who prefer some member of W to c'. LSE-Maximin returns committees which are θ-winning sets for the greatest possible value of θ.

Note that in the definitions of LSE-Copeland and LSE-Maximin we compare candidates outside of the committee not with all candidates from the committee W (as it was the case for NED and SEO), but with *some* candidates from W. Here, the idea is that a voter will be satisfied even if the committee includes one candidate that she likes very much. Indeed, in Example 2 both rules would select W_2.

So far "proportional" extensions of single-winner Condorcet-consistent rules have not yet been proposed. Further, our initial results suggest that there are various ways in which proportionality can be formalized (and even interpreted) in the ranking model [42]. Finally, we do not know yet a class of natural rules that would implement a continuous spectrum between rules oriented towards diversity, proportionality and individual excellence.

Objective 1. *Define new multiwinner proportional Condorcet-consistent rules. Analyze the spectrum of Condorcet-inspired multiwinner rules that implement certain tradeoffs between diversity, proportionality and individual excellence. Suggest axioms or quantitative measures that would allow to reason whether and how well a given Condorcet-based rule is suited for applications requiring diversity, individual excellence, or proportionality.*

Another type of a research analysis that we plan to undertake is more algorithmic in its nature. Indeed, the two known diversity-oriented extensions of the Condorcet principle lead inevitably to computational hardness [35, 5]. The only known positive result related to the problem of computing locally stable extension (LSE) of single winner Condorcet-based rules are the following: It is known that locally stable committees can be computed in polynomial time when the voters' preferences have specific structures [5] (e.g., when they are single-peaked, single-crossing or when their structure resembles party-list elections—we omit the exact definitions of these domain restrictions; they can be found in the recent book chapter [34]). However, we do not yet have tools that would allow to deal with the high computational complexity of the problem when the preferences of the voters are arbitrary.

Objective 2. *Design approximation algorithms or prove inapproximability results for the problem of finding a locally stable committee. Analyze the problem from the perspective of parameterized complexity theory. Perform an analogous algorithmic analysis for other multiwinner Condorcet rules whose definitions will emerge as a result of works on Objective 1.*

3.2 An Analysis of Priceable Rules

In Section 2.4.2 we provided the definition of the Phragmén's rule. Let us recall that this is an approval-based rule, where the voters gradually earn money with the constant speed, and they spend their money (in a rigorous way, specified by the rule) on buying the candidates that they approve of. The Phragmén's rule is an example of priceable rules, the class of rules that has been only very recently defined in the literature [63].

A price system is a pair $(p, \{p_i\}_{v_i \in V})$, where $p \in \mathbb{R}_+$ is called a price, and $p_i \colon C \to [0,1]$ is called a payment function of voter v_i. The payment functions must satisfy the following conditions: (1) each voter has an initial budget of one dollar to spend: for each $v_i \in V$ we have $\sum_{c \in C} p_i(c) \leq 1$, and (2) each voter pays only for candidates she approves of: $p_i(c) > 0 \implies c \in A_i$. We say that a committee W is supported by a price system $(p, \{p_i\}_{v_i \in V})$ if: (1) each member $c \in W$ of a committee gets the total payment of p, i.e., $\sum_{v_i \in V} p_i(c) = p$, (2) unelected candidates are not paid for, i.e., for each $c \notin W$ we have that $\sum_{v_i \in V} p_i(c) = 0$, and (3) for each not-elected candidate $c \notin W$ the voters who approve c have in total no more than p dollars left: $\sum_{v_i \colon c \in A_i} (1 - \sum_{c' \in W} p_i(c')) \leq p$. We say that a committee W is priceable if there exists a price system supporting W. We say that a voting rule is priceable if it always returns only priceable committees. For example, the Phragmén's rule is priceable.

Let us now explain the relation between priceable rules (which are defined for approval ballots) and Condorcet rules (which are defined for ranking-based preferences). Consider the following example.

Example 3. Consider the following approval-based election with 6 voters and 15 candidates, and where the goal is to select the committee of size $k = 12$. In the figure below the candidates are represented by boxes, and each candidate is approved by the voters below the corresponding box. For example, voter v_1 approves c_1, c_2, c_3, and c_4.

c_4	c_5	c_6			
c_3			c_9	c_{12}	c_{15}
c_2			c_8	c_{11}	c_{14}
c_1			c_7	c_{10}	c_{13}
v_1	v_2	v_3	v_4	v_5	v_6

(a) Phragmén's rule

c_4	c_5	c_6			
c_3			c_9	c_{12}	c_{15}
c_2			c_8	c_{11}	c_{14}
c_1			c_7	c_{10}	c_{13}
v_1	v_2	v_3	v_4	v_5	v_6

(b) PAV

The example committees chosen for this election by the Phragmén's rule and by Proportional Approval Voting (PAV)—an approval-based multiwinner rule that is considered the most proportional among committee scoring rules [54, 4, 6]—are shaded. The output returned by the Phragmén's rule seems more proportional: The first half of the voters have disjoint preferences from the second half, so they should be able to elect half of the committee. Indeed, this intuition is captured by the concept of priceability: the first half of the voters have half of money and so they are able to decide about half of the committee; the other half of the society decides about another 6 committee members, choosing, e.g., $\{c_7, c_8, c_{10}, c_{11}, c_{13}, c_{14}\}$. In this example PAV prefers to reduce the societal inequality, giving each voter 3 representatives in the committee, even at the cost of treating the first half of voters unfairly. □

We can see that priceability implies the following interpretation of the proportionality—the power (here, measured as the amount of money) is equally distributed among the voters. This is similar to the idea behind the Condorcet consistency: if candidate c is preferred by majority of voters to any other candidate, then this means that when we compare c with a candidate $c' \neq c$, the voters who prefer c over c' will have more "power", and so they should be able to dictate their will. On contrary, the committee scoring rules that are proportional aim at the fair distribution of welfare (not power) and thus they are more similar to the Borda rule.

No scoring rules can implement the equal distribution of power, as formalized, e.g., by priceability. For example, in our preliminary studies [63] we have considered the well-known class of welfarist rules—these are rules which, roughly speaking, make decision based on arbitrarily aggregated satisfactions of voters; this class broadly generalizes the class of approval-based scoring rules. We proved that no welfarist rule is priceable.

The current understanding of the class of priceable rules is very limited. For the time being the only priceable rules known are two variants of the Phragmén's rule [63]. These rules, on the other hand are suboptimal with respect to several criteria. For example, they are not Pareto optimal—they can select a committee W which is weakly worse than some other committee W' for every voter and strictly worse for at least one of them. One can expect that the fact that the Phragmén's rules violate Pareto optimality is due to their sequential nature. It is natural then to ask what are other interesting examples of priceable rules, in particular whether there exist "optimal" variants of such rules.

Objective 3. *Explore the class of priceable rules. Find optimal variants of the Phragmén's rules or prove the non-existence of priceable Pareto-optimal rules. Analyze the tradeoffs between priceability and other properties of multiwinner rules, specifically the ones that pertain to proportionality.*

The next question that we intend to ask is whether the concept of priceability can be extended to the ranking-based model. A similar idea has been recently suggested in the divisible setting, where the goal is not to select a fixed size subset of candidates but rather to return a probability distribution over the candidates [49] (see also the related works from classic economics [48]). The main idea is that the voters can distribute their initial budgets among the candidates putting higher bids on those candidates that they prefer more. The rule returns a lottery based on the total bids put by the voters on the candidates. An alternative solution is to allow the voters to bid on pairwise contests between the candidates, an approach that has been also considered by Garg at al. [49]. However, the concepts from the divisible model do not easily extend to our setting. Coming up with an appropriate notion of priceability in the ranking-based model seems challenging.

Objective 4. *Extend the notion of priceability to the ranking-based model. Design and analyze (axiomatically and algorithmically) priceable ranking-based rules.*

The concept of priceability is inherently related to fairness (the fact that each voter has initially the same amount of money, or earns money with the same speed can be thought of as the equality of opportunity) and so priceable rules seem most suited for applications requiring proportionality. We ask whether there exist variants of the Phragmén's rule that are more directed towards diversity or individual excellence. One high-level idea that we want to explore is the following. Recall that in the definition of the Phragmén's rule the voters gradually earn money with an equal speed. One can assume that when the voters get their subsequent representatives, the speed with which they earn money changes. If it decreases, the rule becomes more oriented towards diversity (indeed the voters with less representatives earn money faster, and so they have greater voting power). If it increases the rule moves closer towards individual excellence.

Objective 5. *Find a spectrum of Phragmén-like rules that implement various tradeoffs between proportionality, diversity, and individual excellence. Suggest qualitative or quantitative measures for estimating how well various rules perform with respect to each of the three criteria. Analyze the aforementioned Phragmén-like rules against these measures and against other properties of multiwinner rules considered in the literature. If the rules occur to be hard to compute, find good approximation, parameterized or heuristic algorithms for computing them.*

3.3 Extensions of the Standard Model for Multiwinner Elections

Finally our plan is to demonstrate that the ideas and tools from the multiwinner voting theory can be adapted to other closely related models. We will do that by considering two related models that are recently gaining an increasing attention in the (computational) social choice literature.

In our first approach we extend the classic model for multiwinner elections by assuming that the candidates are associated with costs. Instead of the desired size of the committee, k, we are now given a global budget constraint B, and the goal is to select a subset of candidates whose total cost does not exceed the budget limit. This is the model for participatory budgeting. Several works have asked about what are suitable voting mechanisms for participatory budgeting [23, 13, 43, 15, 8, 47] but to the best of our knowledge, none of this works proposes mechanisms that would be inspired by the idea of priceability, Condorcet-consistency, or iterative eliminations.

Introducing costs complicates the model in numerous ways. First, the problem becomes algorithmically harder—indeed, even if we have only a single voter, finding an optimal set of candidates can boil down to the

knapsack problem, which is NP-hard. When we have multiple voters, the problem for most committee scoring rules becomes even harder; it is hard from the perspective of parameterized complexity, and even when the voters' preferences are nicely structured [47]. Thus, we anticipate that the algorithmic problem for most natural Condorcet-based knapsack rules will be also hard, and that tackling this problem will be challenging. Second, the problem becomes conceptually harder. If the costs of the candidates can differ it might no longer make sense to use equal quotas when comparing the candidates. For example, consider two candidates, c and c' such that 60% voters prefer c to c'. However, assume additionally that c is a very expensive candidate, while the cost of c' is negligible. It seems reasonable to include candidate c' (possibly with some other cheap candidates) instead of c in the elected committee. The notion of majority, predominantly used by Condorcet-based rules, needs to be revised.

Objective 6. *Design and analyze, axiomatically and algorithmically, voting rules for participatory budgeting that are based on the ideas of priceability, Condorcet-consistency, and iterative eliminations.*

The second extension that we plan to consider is when the candidates have attributes such as gender, age, profession, etc., and there are additional external constraints imposed on the structure of the committee to be elected. Such constraints can have the following forms: "the committee should contain at least 40% of people with age below 40 and at least 30% of man". The problem of electing committees with such diversity constraints has been considered in the literature, but again, only for committee scoring rules [16, 24, 56]. There, the main idea is to select the committee with the highest score among those that satisfy the diversity constraints. Designing non-scoring protocols with diversity constraints is much harder, since very often we do not have a concept of score that would allow to express the problem in the language of constrained optimization.

Objective 7. *Design and analyze, axiomatically and algorithmically, voting rules for electing committees with diversity constraints.*

4   Research Methodology

In our research we will apply standard methods from mathematics and computer since. Our axiomatic analysis will be mostly based on combinatorial arguments. For the algorithmic analysis we will be using standard tools from theoretical computer science, such as (parameterized) reductions, approximation algorithms, randomized algorithms and fixed-parameter-tractable algorithms.

Parts of our analysis will be experimental—we will verify how the considered rules behave and how frequently certain paradoxes appear when the preferences of voters come from certain predefined distributions, such as the Impartial Culture model, the Polya-Eggenberger urn model, the Mallows model, multi-dimensional Euclidean models and various single-peaked and single-crossing models. Some of our experiments will be performed on real data describing human's preferences, such as those collected in the PrefLib dataset [60] and we will also use new visualization techniques developed by Elkind at al. [32].

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