Fixed point characterization of infinite behavior of finite state systems

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Abstract. Infinite behavior of nondeterministic finite state automata running over infinite trees or more generally over elements of an arbitrary algebraic structure is characterized by a calculus of *fixed point terms* interpreted in powerset algebras. These terms involve the least and greatest fixed point operators and disjunction as the only logical operation. A tight correspondence is established between a hierarchy of Rabin indices of automata and a hierarchy induced by alternation of the least and greatest fixed point operators. It is shown that, in the powerset algebra of trees constructed from a set of functional symbols, the fixed point hierarchy is infinite unless all the symbols are unary (i.e. trees are words). It is also shown that an interpretation of a closed fixed point term in any powerset algebra can be factorized through the interpretation of this term in the powerset algebra of trees, from which it is deduced that the question whether a term denotes always $\emptyset$ can be answered in polynomial time.

Key words: fixed points, Rabin automata, powerset algebras.

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