Exam in information theory 8.02.2024. Theoretical part

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Please mark the answers that you consider right. Note that there may be more than one or none. You need not justify your choices. You **may optionally** add an argument for the point 5d (on the reverse side), which will be graded as *bonus*.

1. Let w_n, for $n \in \mathbb{N}$, be a sequence of bit words, such that $w_n \in \{0,1\}^n$, and $C_U(w_n) \geq n$, for some fixed universal Turing machine U; thus each w_n is Kolmogorov random w.r..t.. U. Which of the properties below can hold for **infinitely many** words w_n ?
   (a) w_n belongs to some Hamming code $(2^m - 1, 2^m - m - 1)$,
   (b) w_n belongs to some Huffman code (for some random variable),
   (c) w_n contains a subword 1^k, where $k \geq \sqrt{n}$,
   (d) w_n is a binary representation (possibly with leading zeros) of a number $\lfloor \sqrt{k} \rfloor \cdot \lceil \log_2 m \rceil$, for some integers $k, m \geq 5$.

2. Consider random variables A, B, C with some joint distribution. Suppose that $I(A;B|C) = I(A;B)$ and $I(A;C|B) = I(A;C)$. Which of the options below are consistent with this assumption (i.e., can happen)? Note that we ask about each option separately, not all of them together.
   (a) $I(B;C|A) \neq I(B;C)$,
   (b) $I(B;C|A) < I(B;C)$,
   (c) $I(B;C) = 0$,
   (d) $I(A;B|C) \neq I(A;C|B)$.

3. Suppose that, for a channel Γ with matrix $\begin{pmatrix} p & q \\ q & p \end{pmatrix}$, there exists a sequence of codes $C_n \subseteq \{0,1\}^n$, such that $|C_n| \to \infty$, $R(C_n) \to C_\Gamma$ and $\Pr_E(\Delta_o, A_n) \to 0$ (for $n \to \infty$), where A is a random variable taking values in C with uniform distribution, and Δ_o is (attention!) the **ideal observer rule**. Then we can claim **for sure** that
   (a) $p = 0$ or $p = 1$,
   (b) $p \neq \frac{1}{2}$,
   (c) $p > \frac{1}{2}$,
   (d) such a sequence cannot exist.

4. A matrix of a channel Γ has dimension 8×8, but only 8 of its values is different from zero. Then the capacity C_Γ
   (a) can be an arbitrary real number in the set $[0,3]$,
   (b) $0 < C_\Gamma < 3$,
   (c) can only assume a value from some **finite** set,
   (d) $C_\Gamma = 3$.

5. Three friends A, B, C decided to eat together **20** donuts[1] but some randomness enters in their feast. Let the random variables A, B, C take values in the set $\{0, 1, \ldots, 20\}$ and represent how many donuts each person eats; we assume that **all donuts** will be eaten. Then we can claim for sure that
   (a) $H(A,B,C) \leq \log 231$,
   (b) $H(A,B,C) = H(A+B, B+C, C+A)$,
   (c) $H(A|B+C) \leq I(A;B|C)$,
   (d) a disjunction holds: some pair of variables is dependent (i.e., $I(A;B) + I(B;C) + I(A;C) > 0$) or $H(A) = H(B) = H(C) = 0$.

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[1] Donut (or doughnut, in Polish: pączek) is a bakery that in Poland is traditionally eaten (in great quantities) on Fat Thursday (Tłusty Czwartek), which happens just **today**.