Exam in information theory 31.01.2023. Theoretical part

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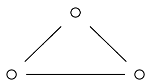
In case of the test questions (a), (b), (c), please mark the answers that you consider right. In the fist question you should additionally provide a numerical value.

You need not justify your choices.

1. For a fixed triangle



we choose one of its edges with the uniform probability; this makes a random variable A. Next, again with the uniform probability, we choose one of the ends of the previously chosen edge, this makes a random variable B.

(a) $H(A|B) < H(B|A)$ **(b)** $H(A|B) = H(B|A)$ **(c)** $H(A|B) > H(B|A)$

Value: $I(A;B) =$

2. Consider the set of binary representations of all prime numbers smaller than 2^{100} written with exactly 100 (one hundred) binary digits, with leading zeros added as needed. This code

(a) detects 4 errors, and corrects 2

(b) detects 0 errors

(c) it is not a code at all.

3. Suppose that, for a channel Γ with matrix $\begin{pmatrix} P & Q \\ Q & P \end{pmatrix}$, for all $\forall \varepsilon, \delta > 0$, there are infinitely many numbers $n \in \mathbb{N}$ and codes $\exists C \subseteq \{0,1\}^n$, such that $C_\Gamma - \varepsilon \leq R(C) \leq C_\Gamma$ and, for a random variable A uniformly distributed over C, it holds $\Pr_E(\Delta_o, A) \leq \delta$, where Δ_o is (attention!) the ideal observer rule. What can we claim **for sure** about the value P in the channel matrix (perhaps nothing of these)?

(a) $P = 0$ or $P = 1$ **(b)** $P \neq \frac{1}{2}$ **(c)** $P > \frac{1}{2}$.

4. Two channels act over the alphabet $\{0, 1, \ldots, 8, 9\}$. The first one swaps 6 and 9 with probability $\frac{1}{2}$ (i.e., sends 6 to 9 or *vice-versa*), and sends the remaining digits correctly. The second channel sends 0 to some even digit (0,2,4,6 lub 8) with probability $\frac{1}{5}$, and sends the remaining digits correctly.

(a) the capacity of both channels is at least $\log 9$

(b) the capacity of exactly one of these channels is exactly $\log 10$

(c) the capacity of at least one of these channels is strictly smaller than $\log 9$.

5. Recall that the Chaitin constant Ω_U depends in general on a prefix-free universal Turing machine U. Does there exist a universal machine U, such that

(a) $\Omega_U = \frac{1}{4}$

(b) $\Omega_U = \frac{1}{\pi}$

(c) every second digit in the binary representation of the fraction Ω_U is 1.