Information theory. Tutorials 27.10.2020

Infinite codes. The definition of a *code* applies without changes to an infinite subset of Σ^*, for a finite alphabet Σ. Show that an infinite code also satisfies the Kraft inequality, i.e., with $r = |\Sigma|$,

$$\sum_{w \in C} \frac{1}{r^{|w|}} \quad \leq \quad 1.$$

A code C is maximal if no extension $C \cup \{v\}$, with $v \notin C$, is a code.

Show that if a finite code is maximal, the Kraft inequality becomes equality.

Is it also true for infinite maximal codes?

Recognizing codes. Design an algorithm to decide whether a finite set C is a code. Estimate its complexity.

Optimal code. In the lecture, we have stated the question: Among all tuples $\ell_1, \ldots, \ell_m$, satisfying Kraft's inequality find a one with minimal $\sum_i p_i \cdot \ell_i$. But how do we know that such a code exists ?

Huffman codes. I assume that the construction is known (we will recall it at the session). The goal is to show that the Huffman code is indeed optimal.