Computational Complexity — tutorial 13

FPT 2

An **FPT reduction** from a parameterized problem L to a parameterized problem M is an algorithm mapping the inputs (w, k) of the problem L to the inputs (w', k') of the problem M such that:

- $(w, k) \in L \Leftrightarrow (w', k') \in M$,
- the reduction works in $f(k) \cdot \text{poly}(n)$ time for some computable function f,
- $k' \leqslant g(k)$ for some computable function g.

There are some problems predicted to **not** be in FPT, e.g. Clique, Dominating Set.

1. Take any two problems L, M such that there is an FPT reduction from L to M. Prove that if $M \in \mathsf{FPT}$, then also $L \in \mathsf{FPT}$.
(Conversely, if $L \notin \mathsf{FPT}$, then $M \notin \mathsf{FPT}$.)

2. Find an FPT reduction from Clique (parameterized by the size k of the clique) to Independent Set (parameterized by the size k of the independent set). Note that it follows that we don't expect Independent Set to be FPT.

3. Propose a reduction from Independent Set to Vertex Cover. Why doesn't it work as an FPT reduction?

4. Find an FPT reduction from from Clique to Clique on Regular Graphs (given a graph G such that each vertex has the same degree, and an integer k, decide if G has any k-clique).

5. Find an FPT reduction from Clique on Regular Graphs to Independent Set on Regular Graphs.

6. Find an FPT reduction from Independent Set on Regular Graphs to Partial Vertex Cover defined below:

> Partial Vertex Cover
> Input: graph G, two integers k, s
> Parameter: k (the size of the solution)
> Output: is there a subset of k vertices which is incident to at least s edges?

Note that it follows that we don't expect Partial Vertex Cover to have an FPT algorithm (even though Vertex Cover has many such algorithms!)

7. Find an FPT reduction from Dominating Set to Bipartite Dominating Set defined below:

> Bipartite Dominating Set
> Input: a bipartite graph G ($V(G) = A \cup B$ and there are only edges between A and B), integer k
> Parameter: k (the size of the solution)
> Output: is there a subset of k vertices of A such that every vertex of B is adjacent to some selected vertex of A?