Computational Complexity — tutorial 2

Turing Machines, contd.

Lecture: Space and time complexities.

1. A *two-way automaton* (2DFA) is a single tape TM whose only tape is **read-only** and contains the input.

(a) Prove that 2DFAs are equivalent to finite automatons (DFAs)—that is, they both recognize exactly regular languages.

(b) What if 2DFA also contained a work (writable) tape of constant size?

2. Consider a model where TMs which can only write on blanks.

(a) Prove that two tape TMs can recognize any semi-decidable language.

(b) What about single tape TMs?

(c) What if we still have one tape, but we can change the character at each cell (including the input) at most once?

3. Consider the following language: $L_A = \{(M, w) \mid M \text{ is a TM accepting } w\}$.

(a) Is L_A decidable? Is the complement of L_A decidable?

(b) Is L_A semi-decidable? Is the complement of L_A semi-decidable?

4. Does there exist a language L such that neither L nor the complement or L is semi-decidable? Can we show an explicit example of such L?

5. A language L is *recursively enumerable* (RE) if there exists a Turing machine enumerating all strings of L (e.g. writing them to some output tape).

(a) Prove that L is recursively enumerable iff it is semi-decidable.

(b) Let L be a language over unary alphabet, i.e. $L \subseteq \{1\}^\star$. Show that if L is infinite and semi-decidable, then there exists an infinite decidable subset of L.

6. Consider the language L containing words $\#000\ldots00\#000\ldots01\#\ldots\#111\ldots10\#111\ldots11\#$ (for every k, the numbers from 0 to $2^k - 1$ in binary, each number taking k bits). So we have $L = \{\#0\#1\#, \#00\#01\#10\#11\#, \ldots\}$.

(a) Prove that L is not regular.

(b) Prove that L can be recognized using a TM with additional $O(\log \log n)$ space (n = size of the input).