Computational Complexity — tutorial 1

Turing Machines

Lecture: Turing Machines, computability.
New homework: implement a TM simulator and converter.

1. We write some simple TMs.

(a) Write a single tape Turing Machine recognizing words over $\{0, 1\}$ consisting of an even number of 0s and an odd number of 1s.

(b) Write a single tape TM recognizing palindromes consisting of 0s and 1s. How many operations (asymptotically) it needs in the worst case on words of length n?

(c) Solve (b) using a two tape TM. It should only require a linear number of operations now.

(d) (If you want) Write a single tape TM which takes an input a string consisting of n ones (1^n) and outputs a string consisting of 2^n ones (1^{2^n}).

2. Prove that single tape TMs with the tape infinite in one direction is equivalent to TMs with the tape infinite in both directions.

3. Describe a single tape TM recognizing the language $\{0^n 1^n \mid n \in \mathbb{N}\}$ in $O(n \log n)$ time.

4. A *two-way automaton* (2DFA) is a single tape TM whose only tape is **read-only** and contains the input.

(a) Prove that 2DFAs are equivalent to finite automatons (DFAs)—that is, they both recognize exactly regular languages.

(b) What if 2DFA also contained a work (writable) tape of constant size?

5. Consider a model where TMs which can only write on blanks.

(a) Prove that two tape TMs can recognize any semi-decidable language.

(b) What about single tape TMs?

(c) What if we still have one tape, but we can change the character at each cell (including the input) at most once?