Online Multi-Level Aggregation with Delays and Stochastic Arrival Times

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Plan

online multi-level aggregation with delays

Plan

adversarial model

stochastic model

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1

motivation example

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online multi-level aggregation with delays

1 motivation example

2 problem statement

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3 previous results

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1. motivation example
2. problem statement
3. previous results
4. simple cases

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5. general instance

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Supply chain management

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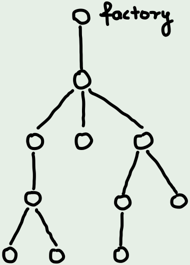

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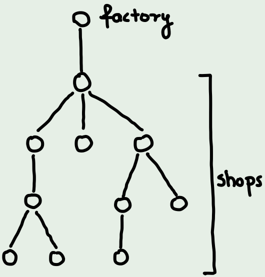

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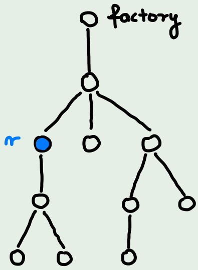

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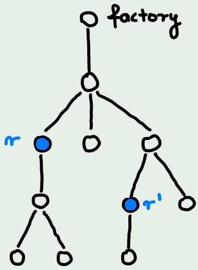

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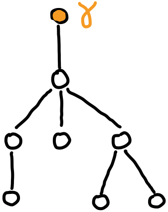
- **tree** T **rooted** at some node γ and a **weight** function $w : E(T) \to \mathbb{R}_+$



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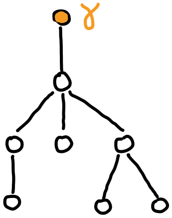

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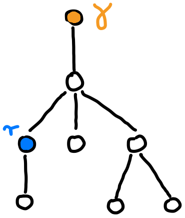

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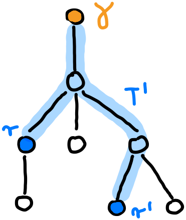

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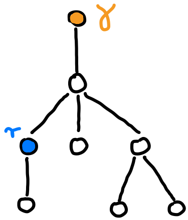

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- the target is to **minimize the total cost** produced by the online algorithm for serving all the requests

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- **2-approximation** algorithm was proposed by Becchetti et al.

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Poisson arrival process

- **waiting time** between any two consecutive requests arriving at any node v follows an **exponential distribution** $\mathrm{Exp}(\lambda_v)$

How to compare performance?

Ratio of expectations

Algorithm ALG for MLA has a **ratio of expectations** $C \geq 1$, if

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Main theorem

For MLA with linear delays in the Poisson arrival model, there exists a deterministic algorithm that achieves a **constant ratio of expectations**.

Exponential variables recap

Memoryless property

If X is an exponential variable with parameter λ, then for all $s, t \geq 0$, we have

$$\mathbb{P}(X > s + t \mid X > s) = \mathbb{P}(X > t) = e^{-\lambda t}.$$

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Minimum of exponential variables

Given n independent exponential variables $X_i \sim \mathrm{Exp}(\lambda_i)$ for $1 \leq i \leq n$, let $Z := \min\{X_1, X_2, \ldots, X_n\}$ and let $\lambda := \sum_{i=1}^{n} \lambda_i$. It holds that
$$Z \sim \mathrm{Exp}(\lambda), \qquad \mathbb{P}(Z = X_i) = \lambda_i / \lambda, \qquad Z \perp \{Z = X_i\},$$
where $\perp$ denotes independence.

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centralized version

- waiting time between any two consecutive requests in the given **tree** T follows an exponential distribution with parameter $\lambda(V(T)) := \sum_{v \in V(T)} \lambda_v$
- each time a request arrives, the probability of it appearing at node v equals $\lambda_v / \lambda(V(T))$

One edge — two approaches

edge $e = (\gamma, u)$: **weight** w, **arrival rate** of node u equal λ

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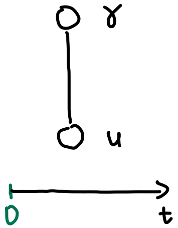

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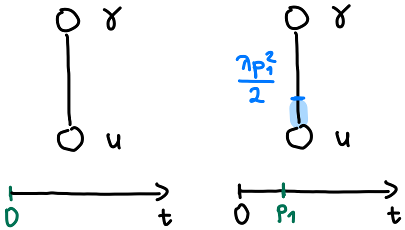

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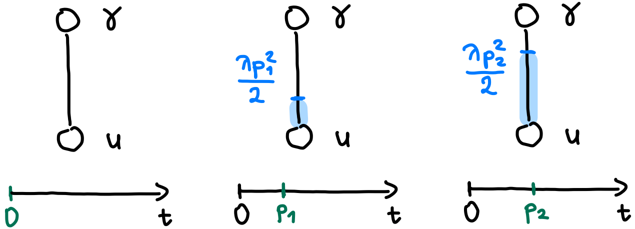

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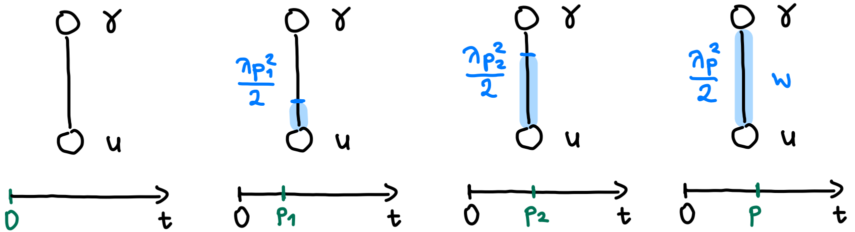
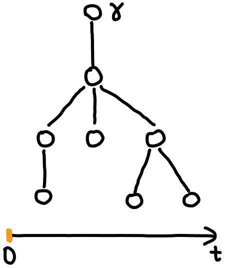


Heavy instances — clusters



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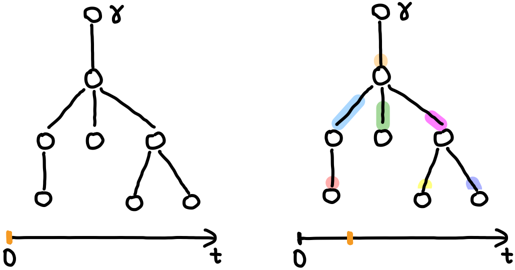

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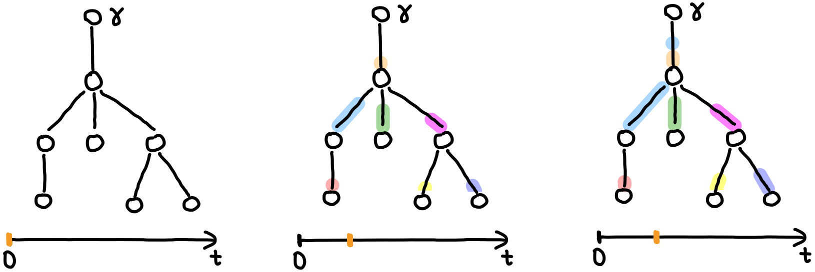

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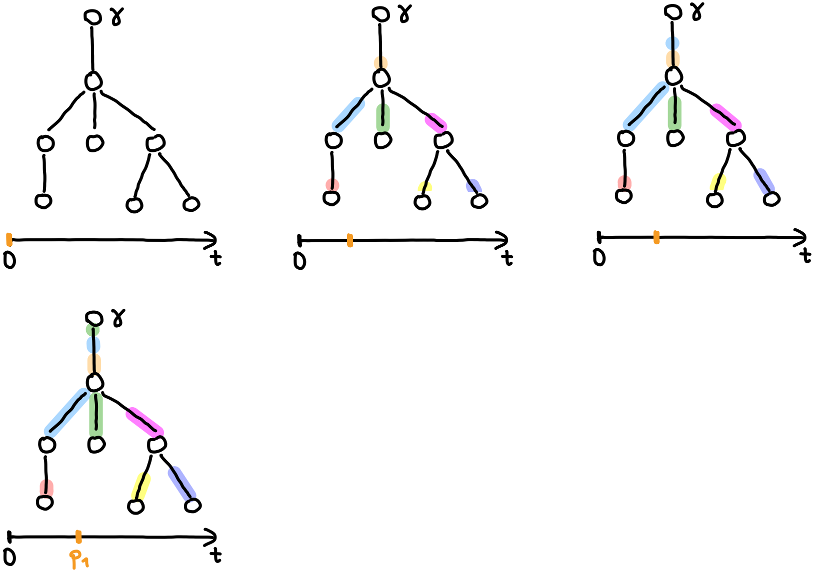

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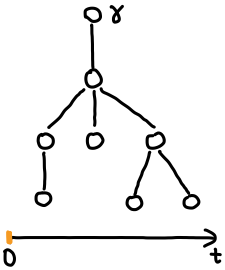

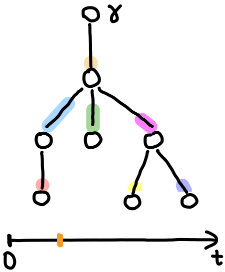

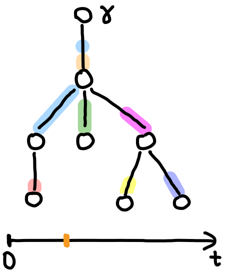

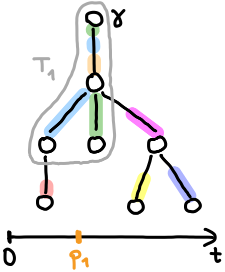

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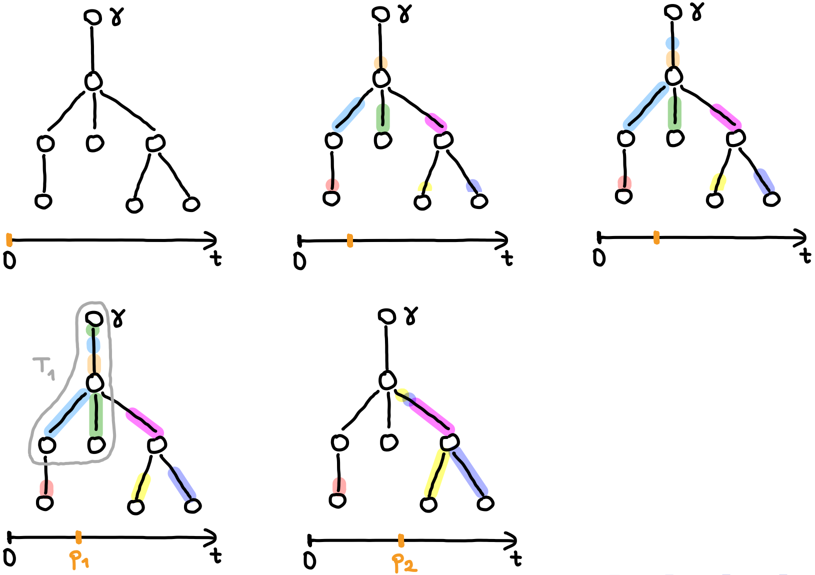

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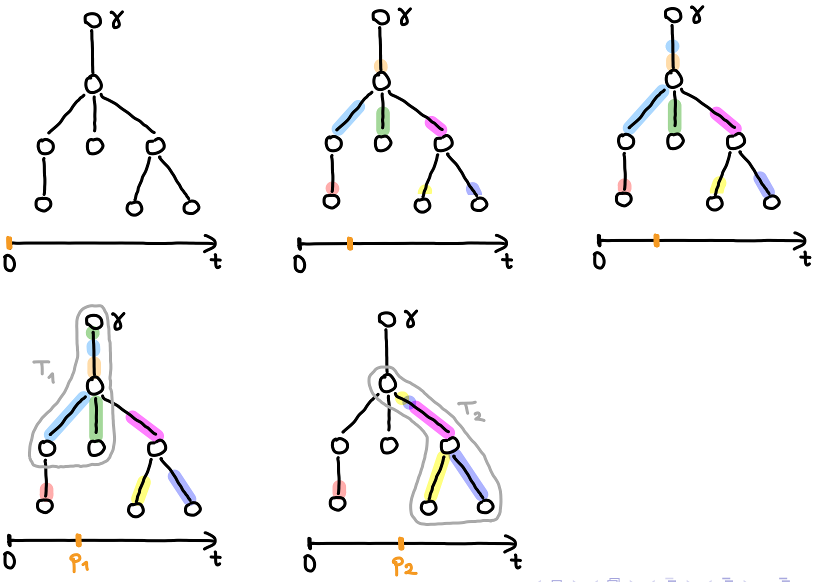

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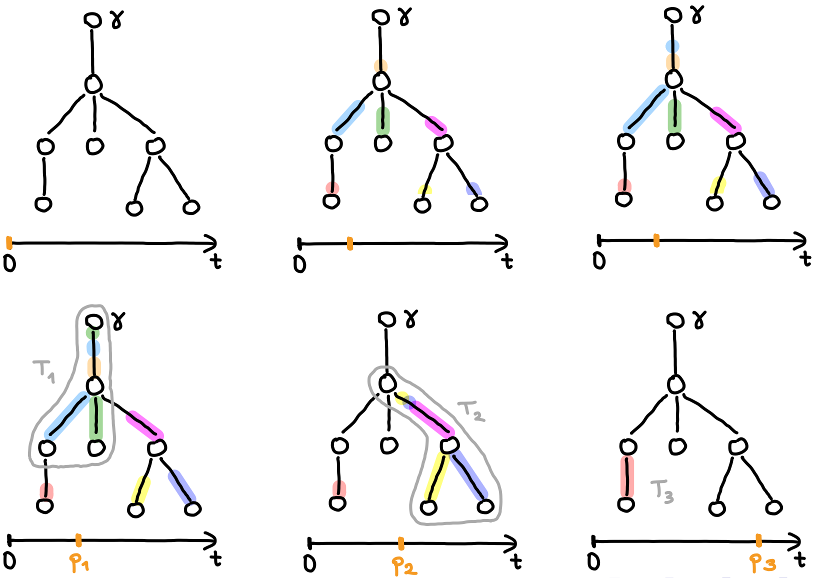

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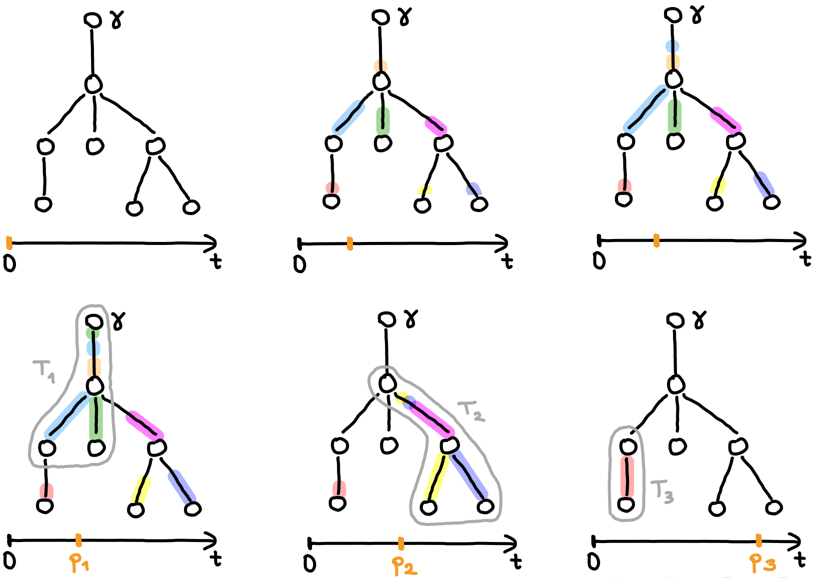

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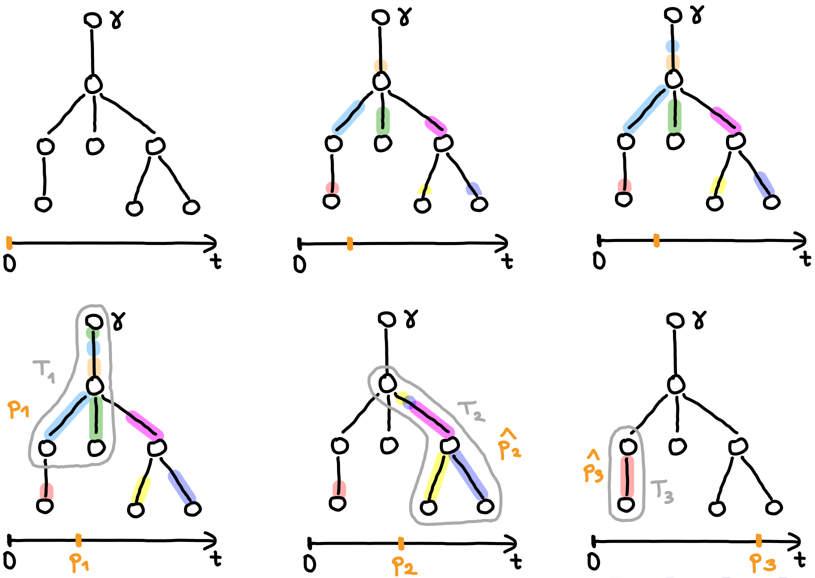

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$$\begin{aligned}\sum_{v \in L} \hat{w}_v &= \sum_{v \in L} \frac{\lambda(v) \cdot p^2}{2} \leq \frac{1}{2} \sum_{v \in L} \frac{\lambda(v)}{\lambda^2(v)} \\ &= \frac{1}{2} \sum_{v \in L} \frac{1}{\lambda(v)} \leq \frac{1}{2} \sum_{v \in T} \frac{1}{\lambda(v)} \leq \frac{1}{2} \sum_{v \in T} w_v\end{aligned}$$

Light instances

Definition

A stochastic MLA instance (T, w, λ) is called light if

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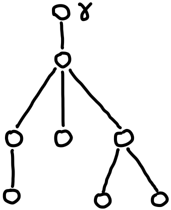

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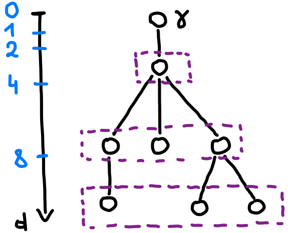

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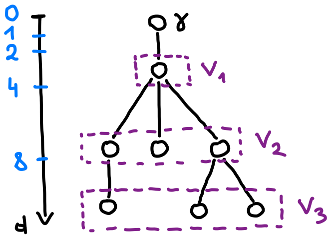

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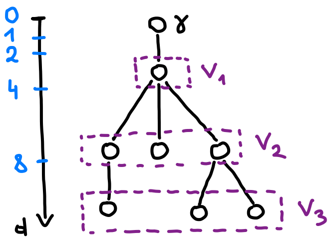

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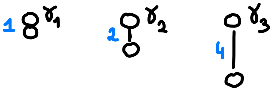

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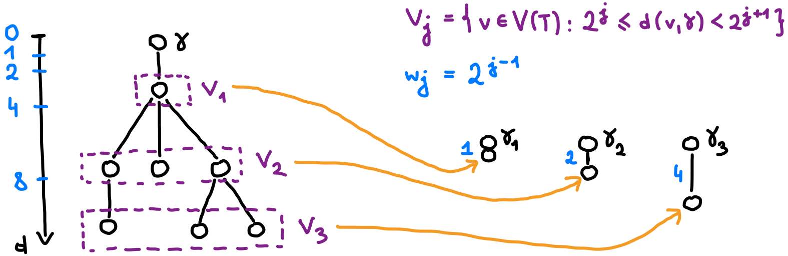
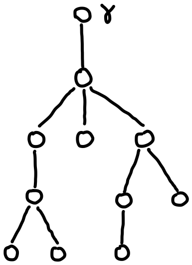


General case

given tree



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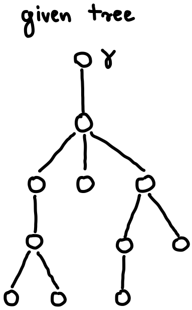

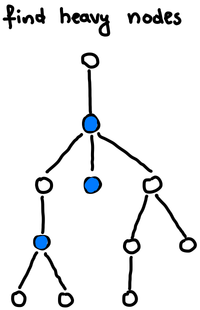

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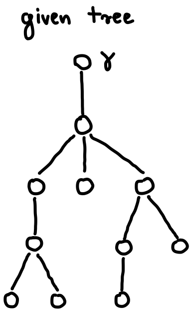

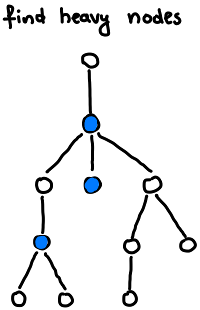

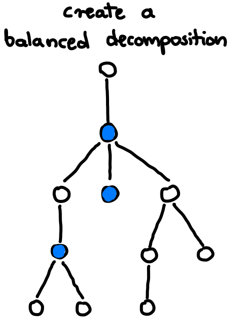

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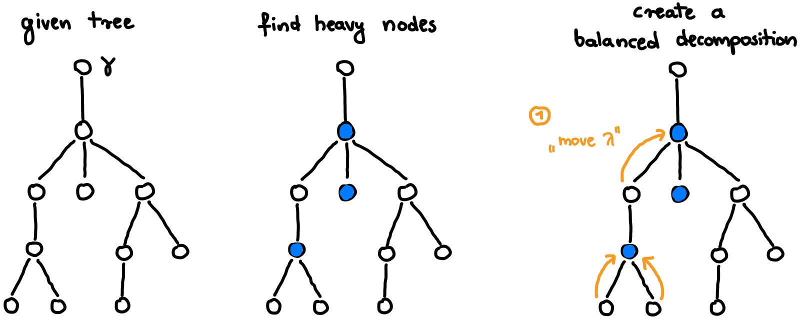

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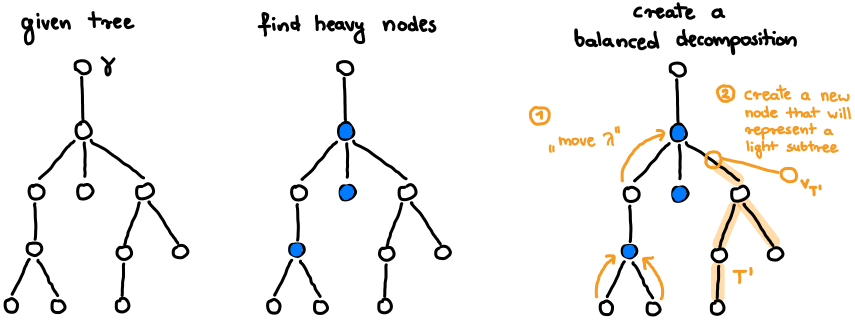

Open problems

1. does the **greedy** algorithm achieve a **constant ratio** of expectations?

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2. how to define and analyse **similar problems** (Facility Location, Online Service) in the stochastic environment?

Thank you!