Universal Optimization for Non-Clairvoyant Subadditive Joint Replenishment

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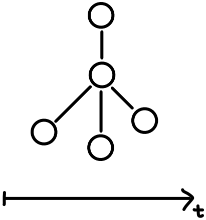

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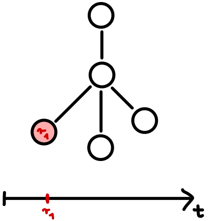

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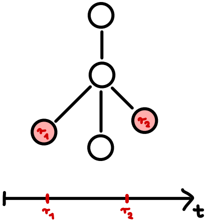

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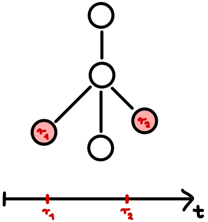

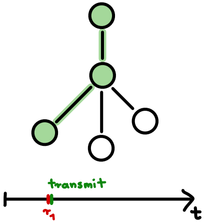

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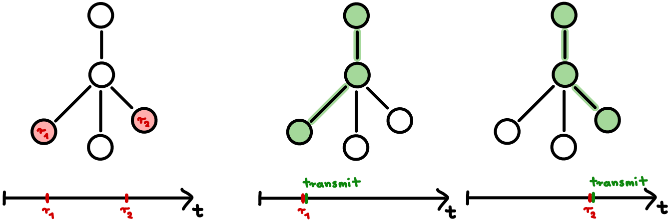

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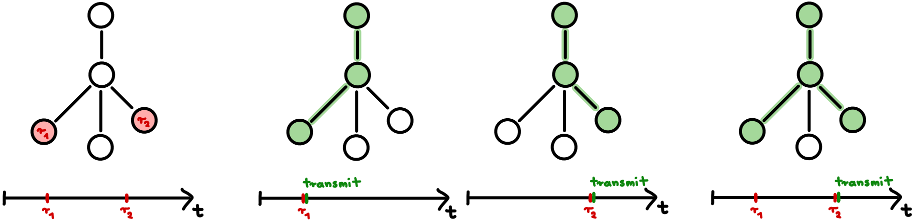

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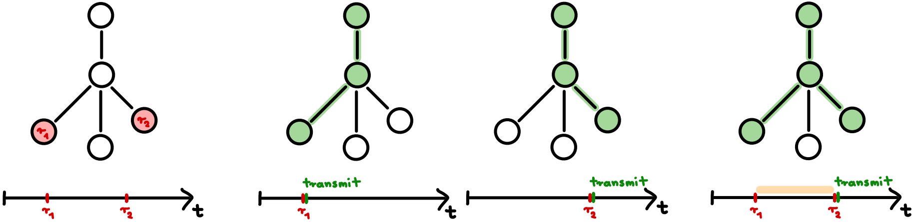

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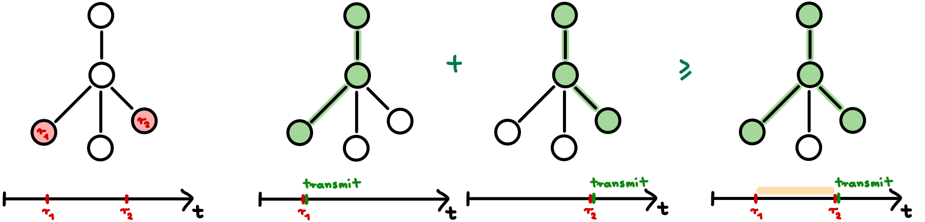

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Our results:

- we provide a **simpler, modular framework** that matches the competitive ratio established by Touitou for the same class of generalized JRP
- we obtain tight $O(\sqrt{n})$-competitive algorithms for two significant problems: **Multi-Level Aggregation** and **Weighted Symmetric Subadditive JRP**

Towards the Reduction to Disjoint TCP

Theorem [Jia et al., STOC 2005]

For every subadditive service function f, there is a **disjoint service function** g that $O(\sqrt{n \log n})$-**approximates** function f.

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1. function g **partitions** the universe U of request types into a **family of non-overlapping sets** $S_1, S_2, \ldots, S_k$ and given a set $S \subseteq U$ assigns it the cost of

$$g(S) = \sum_{i=1}^{k} f(S_i) \cdot \mathbb{1}\{S_i \cap S \neq \emptyset\}$$

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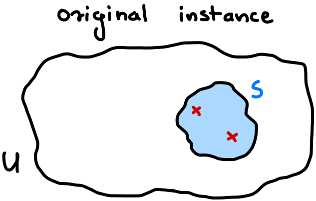
2. it holds that $f(S) \leq g(S) \leq \sqrt{n \log n} \cdot f(S)$ for every $S \subseteq U$

Subadditive JRP vs Disjoint TCP Acknowledgement



service cost $f: S \subseteq U \to \mathbb{R}_+$

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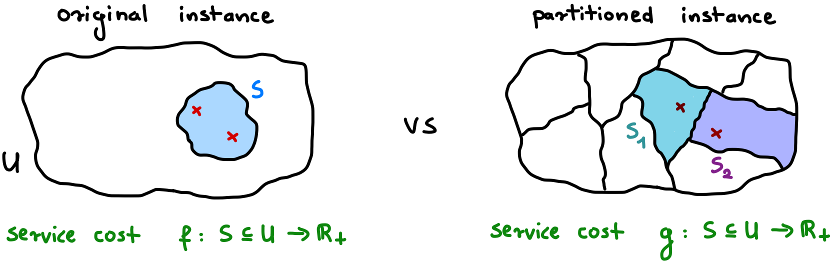

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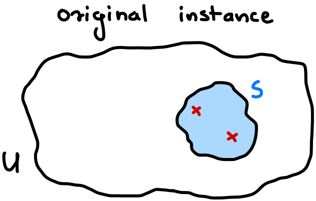
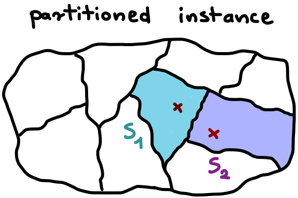


vs



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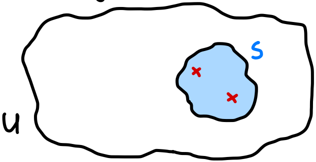
$f(S)$

$g(S)$

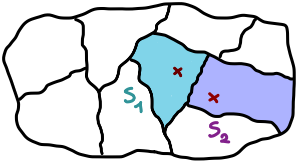
Subadditive JRP vs Disjoint TCP Acknowledgement

original instance



vs

partitioned instance



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$$\underbrace{g(S)}_{=}$$

$$f(S_1) + f(S_2)$$

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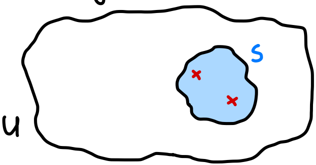

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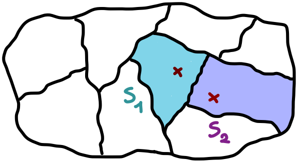

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Subadditive JRP

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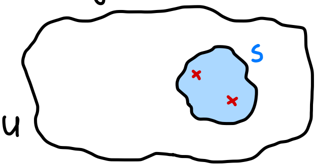

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vs

Disjoint TCP Ack

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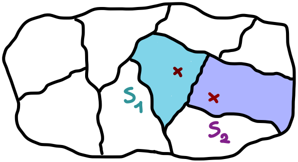

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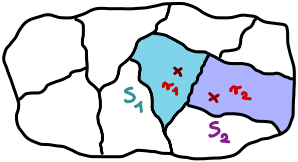
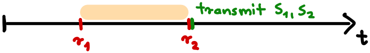
Cost Comparison for Disjoint TCP Solutions

1. solution corresponding to Subadditive JRP (transmit S)





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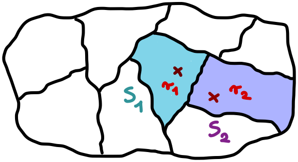

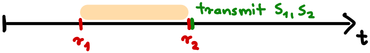

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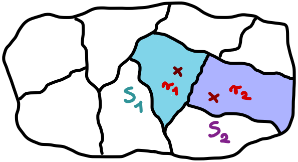

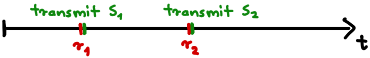

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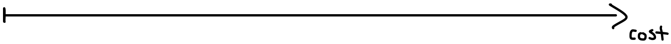

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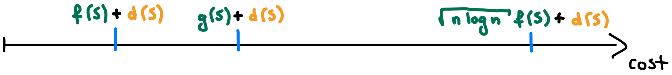

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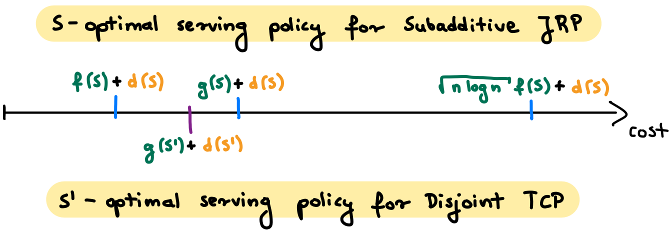

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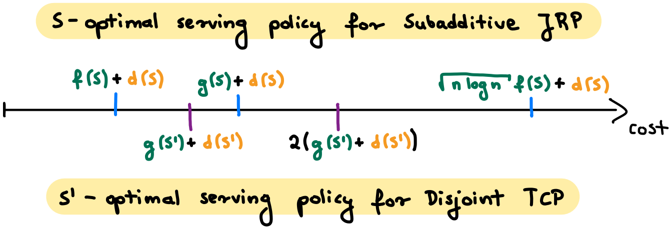

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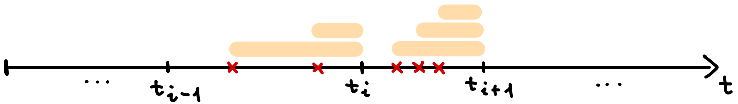

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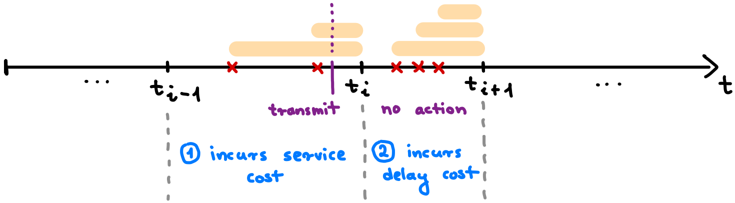

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Theorem (Multi-Level Aggregation)

There exists an efficient deterministic $O(\sqrt{n})$-competitive algorithm for the Non-Clairvoyant **Multi-Level Aggregation** problem.

Multi-Level Aggregation

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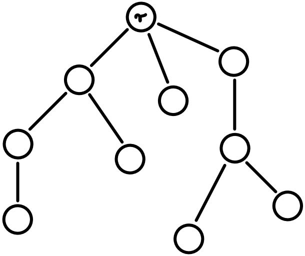

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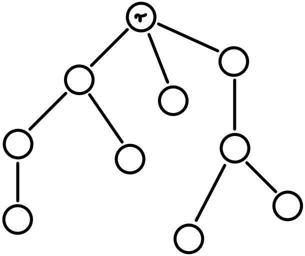

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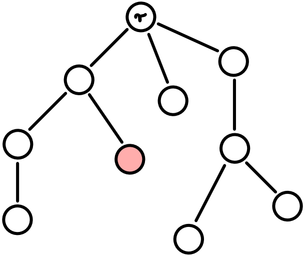

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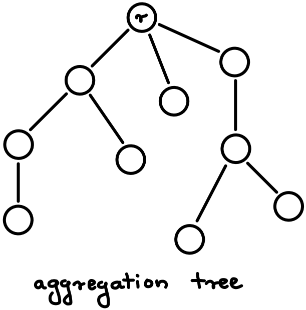

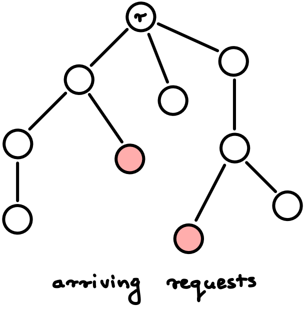

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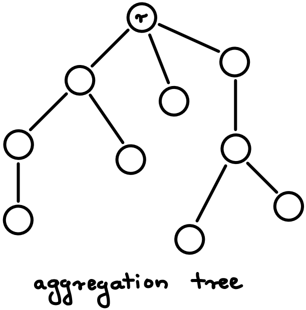

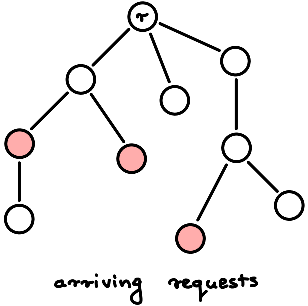

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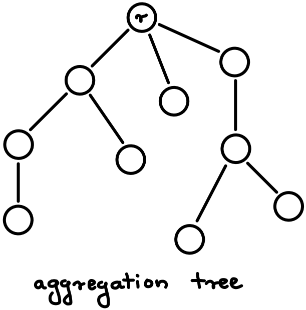

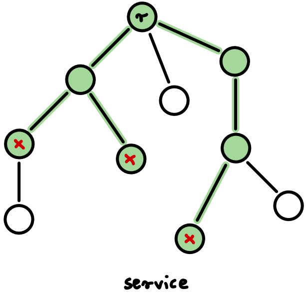

Service Cost Comparison

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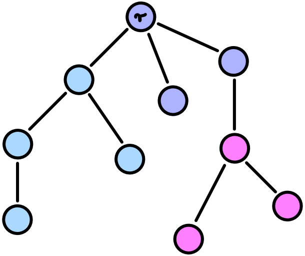

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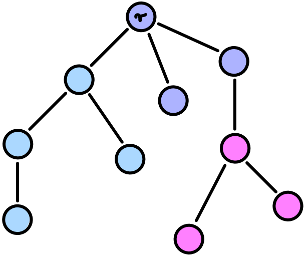

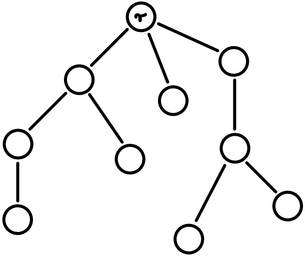

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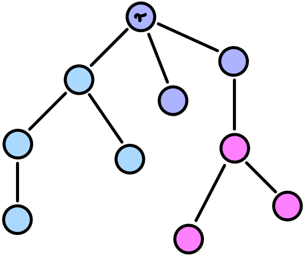

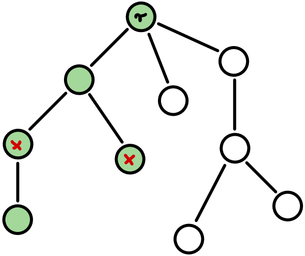

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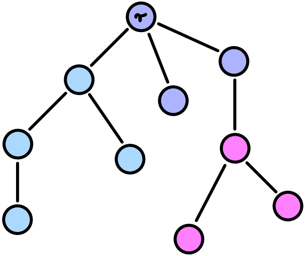

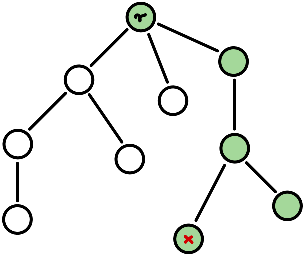

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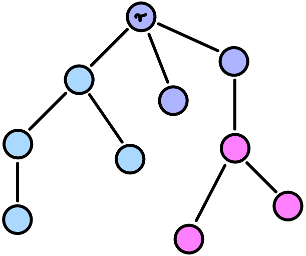
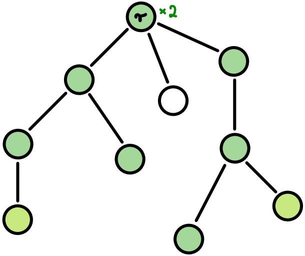
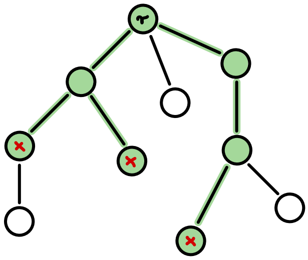




Heavy Clusters

Goal: find a node v for which both the cost of its **subtree** and the cost of the **path** leading to the root are at most $O(\sqrt{n})c(v)$



optimal solution

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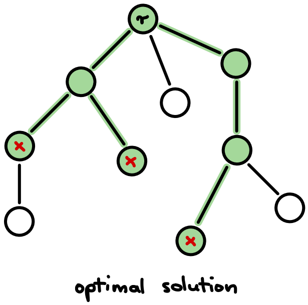

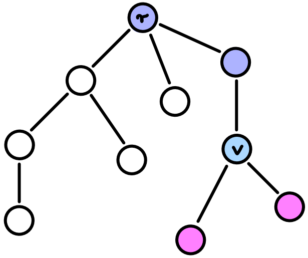

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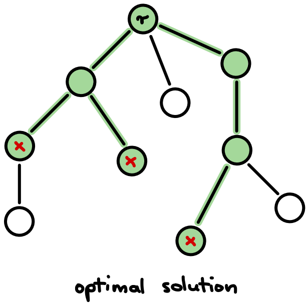

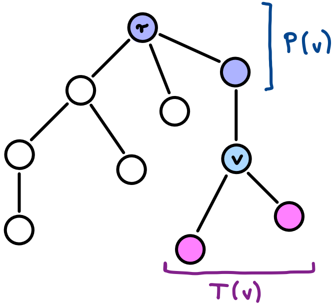

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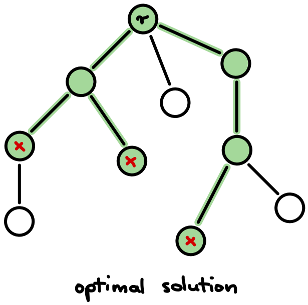

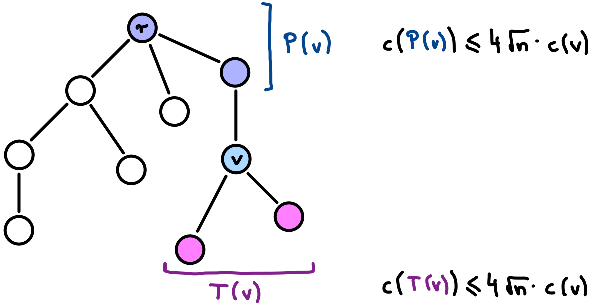

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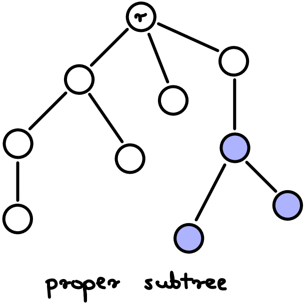

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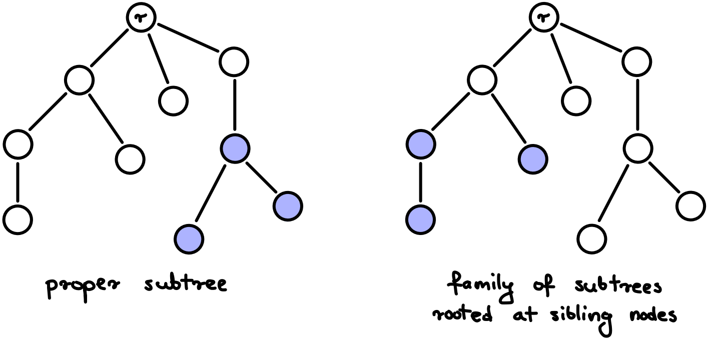

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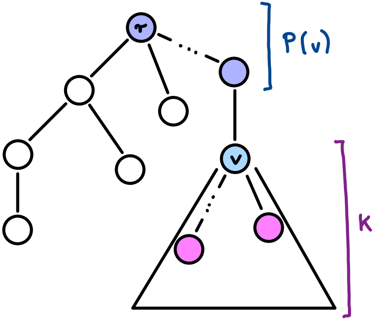

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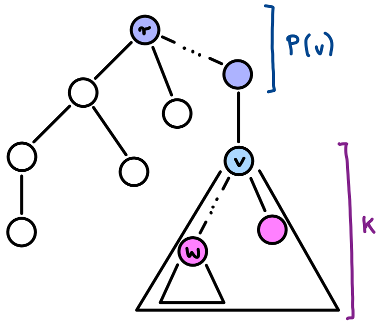

take the heaviest node $w \in K$

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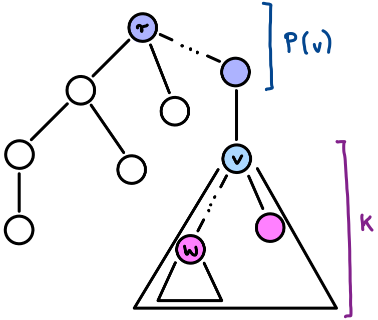

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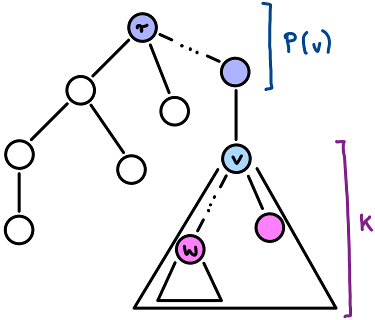

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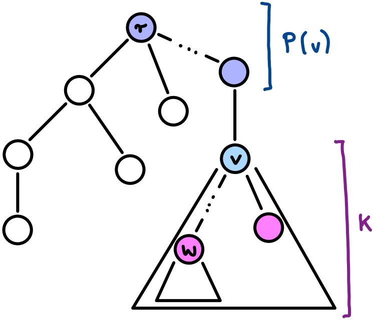

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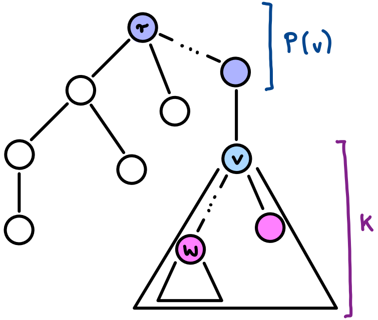

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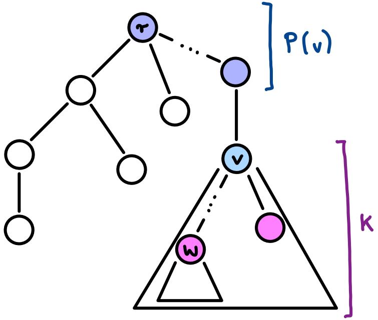

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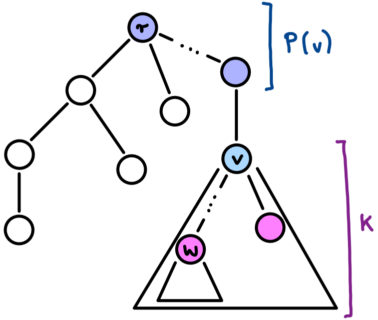

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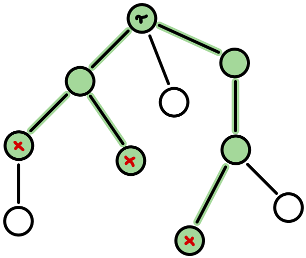
thus: $c(P(v)) \geq c(K)$

Approximation Factor Analysis

Theorem (Multi-Level Aggregation)

For any MLA service function f, there exists a **disjoint** service function g that $O(\sqrt{n})$-**approximates** f. It can be found in time **polynomial** with respect to the MLA instance defining f.



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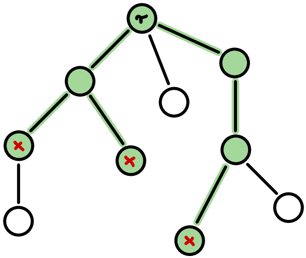

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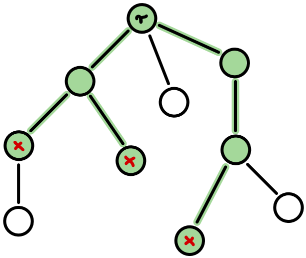

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at most $\sqrt{n}$ of them

$f(\text{path}) \geq f(\text{subtree})$

Conclusions

Our results

- we provide a **simpler, modular framework** that matches the competitive ratio established by Touitou for the same class of generalized JRP
- we obtain tight $O(\sqrt{n})$-competitive algorithms for two significant problems: **Multi-Level Aggregation** and **Weighted Symmetric Subadditive JRP**

Thank you!