Multi-Level Aggregation with Delays and Stochastic Arrivals

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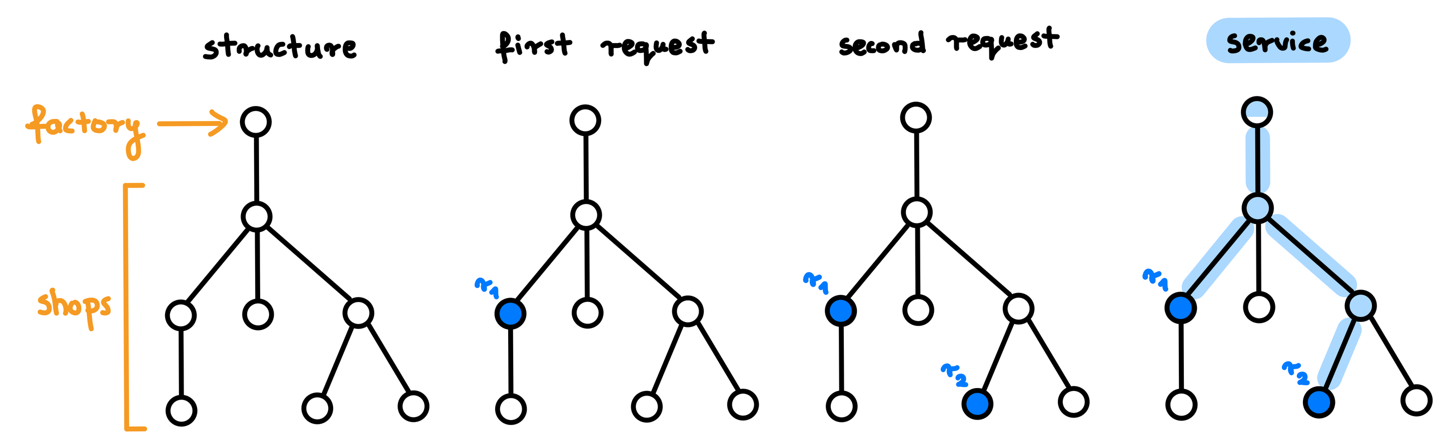
Motivation

Imagine a factory that needs to **schedule product deliveries** (services) to keep the cooperating shops running. Once a product is in **shortage** for a shop owner (agent), they inform the factory for **replenishment** (send a request). Each time the factory **schedules a service** to deliver some products, a **truck has to travel** from the factory to the locations of chosen agents. This incurs a cost proportional to the total travelling distance (we call it the service cost). Thus, to save on the delivery costs, it is beneficial to **accumulate** the replenishment **requests** from many stores and then deliver the ordered products altogether in one service. However, this **accumulated delay** in delivering the products may cause some agents **unsatisfied**. Typically, we measure this factor by looking at the **time gap between ordering and delivering** each requested product (and call it the delay cost of this request).

Goal: **plan the delivery schedule** in an online manner such that the total service cost and the total delay cost are minimized



Problem statement

Multi-Level Aggregation (MLA) with Delays [2]

- **edge-weighted** tree T rooted at γ
- V represents the set of T's nodes, $n := |V|$
- sequence σ of m **requests arriving online**
- request r: **location** $\ell(r) \in V$, **arrival time** $t(r) \in \mathbb{R}^+$
- service s issued at time t to serve a set R of requests incurs the **delay cost** of $\sum_{r \in R}(t - t(r))$ and the **service cost** being the weight of the minimum spanning tree containing all locations $\ell(r)$ for $r \in R$

Target: **minimize the total cost** produced by the online algorithm for serving all the arriving requests

Previous results

- in **adversarial model** → $O(d^2)$-competitive [1]
- **any algorithm** in this model → $\Omega(2 + \phi)$-competitive [3]
- offline problem is NP-hard, 2-approximation exists

Beyond worst-case

Assumption: the **waiting time** between any two consecutive requests arriving at any node u **follows an exponential distribution** $\mathrm{Exp}(\lambda_u)$ with parameter $\lambda_u \geq 0$

- the factory needs to **minimize the expected cost** it produces when dealing with a random input sequence of requests arriving over some time horizon τ for large τ
- to evaluate the performance of any algorithm A on stochastic input, we use the **ratio-of-expectations** → the ratio of the expected costs of A and the optimal offline solution (OPT)

Our main result

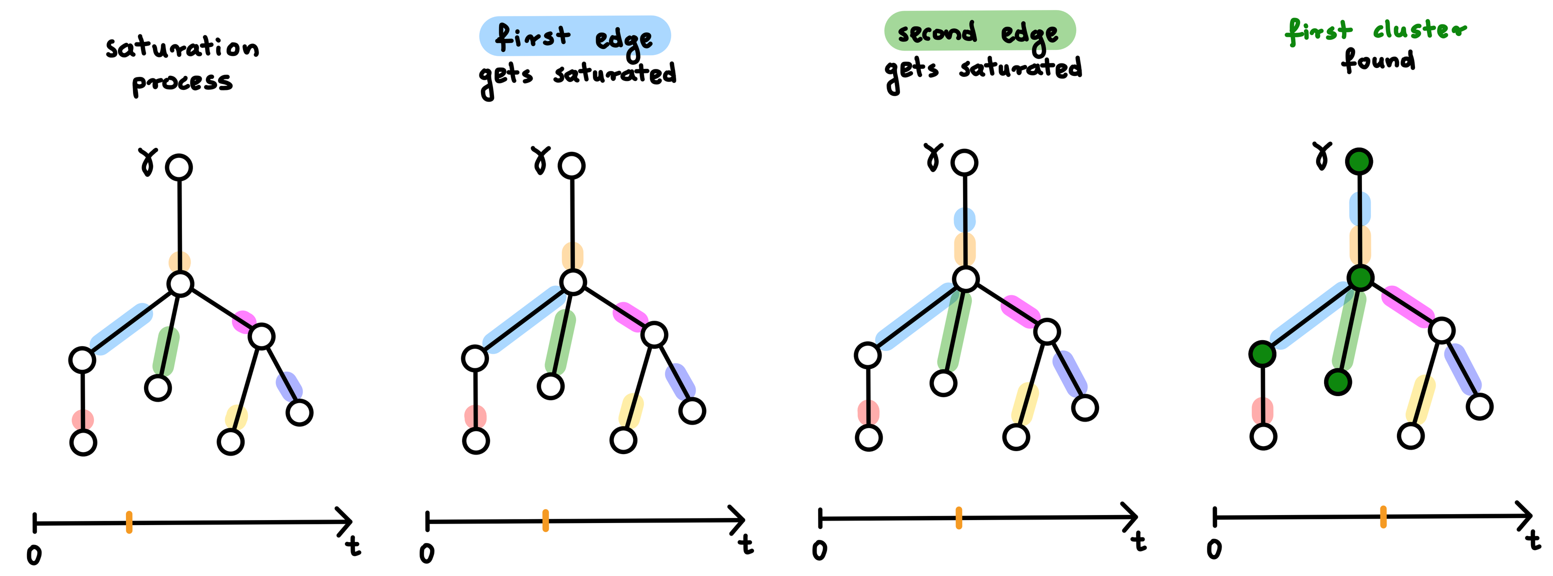
Theorem. *For MLA with delays in the Poisson arrival model, there exists an algorithm with a* ***constant*** *ratio-of-expectations.*

Single-edge case

- T has only **one edge** (u, γ) of weight w
- denote the arrival rate of u by λ → and consider **two strategies**
- **instant strategy**: serve each request as soon as it arrives
- efficient when the requests are **not so frequent** → $w \leq 1/\lambda$
- **periodic strategy**: group several requests arriving within a selected **period** p and **serve them together**
- efficient when requests **are frequent** enough → $w > 1/\lambda$
- we choose the period such that **the expected delay** cost generated by all the requests arriving within $[0, p]$ **equals** w, i.e., $p = \sqrt{2w/\lambda}$

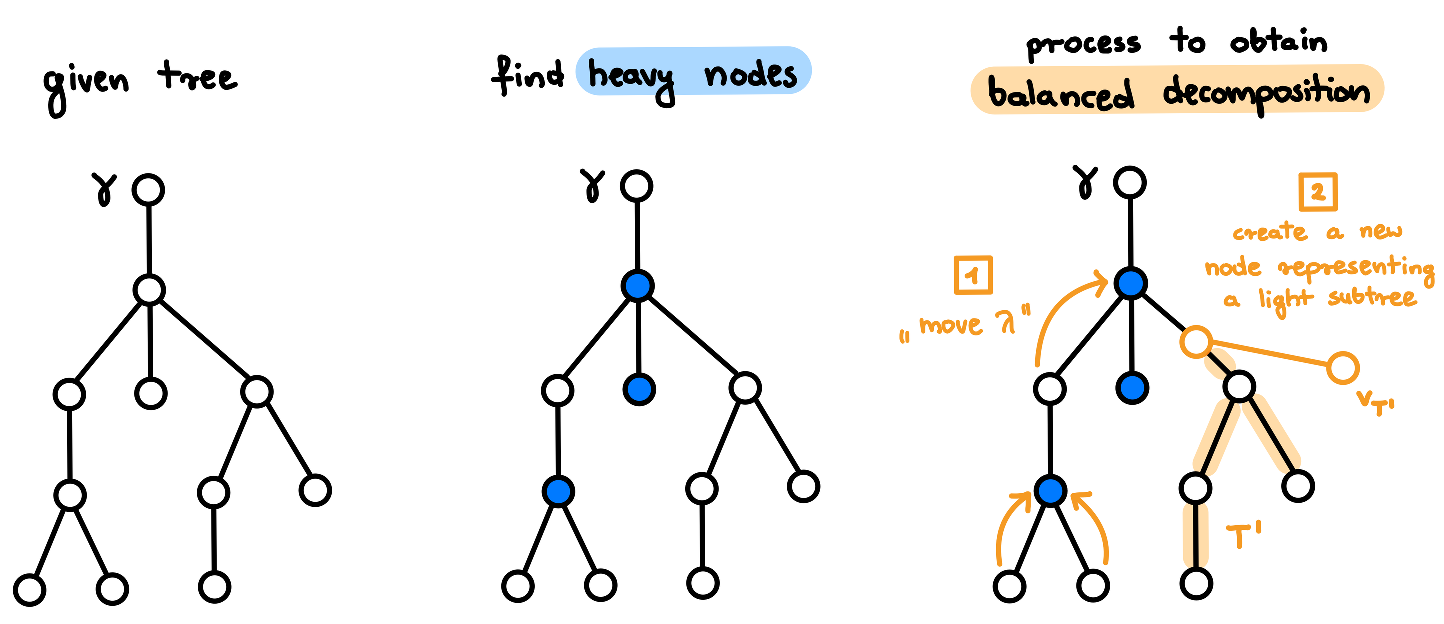
Extending to more complex trees

- **instant** strategy → trees for which the **average node-root distance**, weighted by the arrival rates, **is smaller** than the expected waiting time between two consecutive requests arrivals
- **periodic** strategy → trees for which **each edge satisfies the single-edge case condition**; here, we use an edge-saturation-based process to assign each node its period



General trees

- some of the nodes located **close to the root** should be served using **instant** strategy
- for the remaining ones, we need to **transform the tree** they form into a corresponding **heavy tree** and assign periods accordingly



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